Essentials of Geophysics 12.201/501
Problem Set 4

Due Monday, November 15, 2004.

1. In an ideal fluid, there are no shear tractions on any plane. Show that the stress tensor $\mathbf{T}$ is given by

$$\mathbf{T} = -p\mathbf{I} \tag{1}$$

where p is the pressure.

2. (a) Show how $\sigma_{ij} = c_{ijkl}\epsilon_{kl}$ together with $c_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$ lead to

$$\sigma_{ij} = c_{ijkl}\epsilon_{kl} = \lambda\delta_{ij}\epsilon_{kk} + 2\mu\epsilon_{ij} = \lambda\delta_{ij}\Delta + 2\mu\epsilon_{ij} \tag{2}$$

(b) Verify how

$$\rho\ddot{\mathbf{u}} = (\lambda + 2\mu)\nabla(\nabla \cdot \mathbf{u}) - \mu(\nabla \times \nabla \times \mathbf{u}) \tag{3}$$

leads to

$$\rho\frac{\partial^2(\nabla \cdot \mathbf{u})}{\partial t^2} = (\lambda + 2\mu)\nabla^2(\nabla \cdot \mathbf{u}) \tag{4}$$

and

$$\frac{\partial^2(\nabla \times \mathbf{u})}{\partial^2 t} = \frac{\mu}{\rho}\nabla^2(\nabla \times \mathbf{u}) \tag{5}$$

3. Verify Fowler's derivation of the expressions for the elastic parameters (Appendix 2) and use the definitions to answer the following questions.

(a) One of the simplest ways to determine the elastic constants of a rock is to measure its density and the travel times of P and S-waves across a small sample. Suppose that you cut a core 2 cm in diameter and 6 cm long out of a homogeneous hand specimen of mitmite. The weight of the sample is 61.45 g. A compressional

impulse given at one end arrives at the other end after 8.6 μs; for a shear impulse, you find a travel time of 14.5 μs. What (in S.I. units) are the Young's modulus E, the Poisson's ratio s, and the rigidity μ of mitmite?

(b) Consider two half spaces separated by a surface Σ. The material constants for the two media are as follows:

	P-wave speed [km/s]	Poisson's ratio [?]	ρ [g/cm^3]
med 1	5.6	0.20	2.7
med 2	8.1	0.30	3.2

A P-wave is incident from medium 2 at an angle of incidence of 25°. What types of waves are produced upon transmission and/or reflection? Why? Compute all the angles of incidence and draw all the reflected and refracted (transmitted) rays.