12.113 Structural Geology

Part 1: Continuum mechanics and rheology

Fall 2005

Contents

Chapter 1

Stress and strain

1.1 Stress (T&M pp.128 - 164)

1.1.1 Introduction

In geology, we seldom observe the forces responsible for the deformations that we are interested in. In fact, it turns out that you can't actually measure stress directly (stress measurements are made by observing the deformation of reference materials whose response to stress is known). Nevertheless, one of the main objectives of earth science is to try and understand the mechanisms whereby observed deformations (faults, folds, mountain ranges and so on) are produced.

The key concept in this regard is **stress**, which is related to the familiar concept of a **force** in a reasonably straightforward way:

Stress = **Force** / Area

By this definition, it is clear that a given force acting on a larger area results in a smaller stress than the same force acting on a smaller area. Since Force is a vector quantity (i.e. its direction and magnitude are defined), we denote the above relation as

$$\vec{\sigma} = \frac{\vec{F}}{A} \tag{1.1}$$

$\vec{\sigma}$ is termed a stress vector, or a traction, and is defined with reference to a particular plane. Stress has units of force (Newtons) divided by area (square meters). A $\frac{N}{m^2}$ is a Pascal (Pa). In the Earth, most stress of interest are of the order of 10^6 - 10^9 Pa, and stresses are commonly reported in megapascals (MPa).

In engineering and material science, the convention is that positive stresses are tensional, and compressional stresses are negative. In the Earth, true tensional stresses are exceedingly rare, however, so we use the opposite sign convention.

1.1.2 The stress tensor

An arbitrary stress acting on a plane can be resolved into three components: one normal to the plane (normal stress) and two mutually orthogonal components tangential to the surface of the plane (shear stresses). Since we are interested in continuous volumes (the interior of the earth), this is further generalized:

Consider a small volume whose faces are oriented with the coordinate axes. We keep track of the faces by identifying each face x, y, z with the coordinate axis normal to that face. The reason we deal with this volume is that the surface stresses on each pair of faces is independent of the surface stresses on the other two pairs of faces. As it turns out, if we know the surface stresses on any three orthogonal faces defining an infinitesimal volume, we have completely characterised the stress and we can calculate the surface stress on **any** plane crossing this region.

Chapter 2

Strain: part one

2.1 Reading assignment

Twiss and Moores: chapter 15, in particular, pages 292 – 302. Discussion of specific special types of strain (pure shear, simple shear) begin on page 303. Pages 304 – 310 deal with progressive strain, and are useful background material for the lab.

W. Means (1976) *Stress and Strain* is a great text, very clear, well written and reads easily. J. Ramsay and M. Huber (1983) *The Techniques of Modern Structural Geology, Volume 1: Strain Analysis* is amazingly detailed, with many, many examples of detailed strain analysis. It can, however, be "a bit much".

2.2 Strain I: displacement, strain and terminology

Given enough differential stress, a material responds by deforming. We distinguish: between **rigid body deformations** and **non-rigid body deformations**. The first includes translation and rotations of a body. The second includes distortion and dilation. Other important distinctions are: **continuous** vs. **discontinuous** strain and **homogenous** vs. **heterogeneous** strain. Whether strain is homogenous or heterogenous is often a function of the scale of observation. Also, when strain in natural systems is analyzed, a common approach is to identify **structural domains** wherein the strain is continuous and approximates homogeneity. The point of doing so is that we can use the tools of continuum mechanics – the physics of continuous deformation.

2.2.1 Measurement of strain

1. Changes in the lengths of lines

2. Changes in angles

3. Changes in areas or volumes

Changes in line length:

3 important measures:
Elongation

$$e \equiv \frac{\Delta l}{l_i} = \frac{l_f - l_i}{l_i} = \frac{l_f}{l_i} - 1$$

Stretch

$$S \equiv \frac{l_f}{l_i} = 1 + e$$

Quadratic elongation

$$\lambda \equiv S^2 = (1+e)^2$$

So $\lambda = 1$ means no change in length; $\lambda < 1$ reflects shortening and $\lambda > 1$ is extension.

Changes in angles

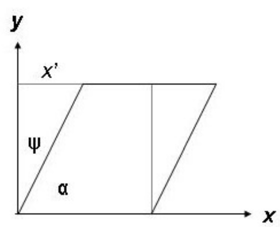


Figure 2.1: Tracking strain of two initially perpendicular lines

1. Consider 2 originally perpendicular lines. The change in angle between those lines is

$$90 - \alpha = \psi \equiv \textbf{angular shear}$$

2. Consider a particle on the y-axis. Measure displacement at some distance y from the origin, in the x direction:

$$\frac{x}{y} = \gamma \equiv \textbf{shear strain}$$

Note that:

$$\gamma = \tan\psi$$

Change in volume (or area)

The **dilation** is similar to the definition of elongation:

$$\Delta \equiv \frac{V_f - V_i}{V_i} = \frac{\Delta V}{V_i}$$

Note that all these measurements take the undeformed state as the point of reference. It is equally feasible to take the length or angles in the deformed state as the reference state. There are no particularly good reasons for doing one or the other, neither is good for large strains. Alternatively, the **infinitesimal strain** is often a very useful concept, corresponding to the strain accrued in a vanishingly small instant of deformation.

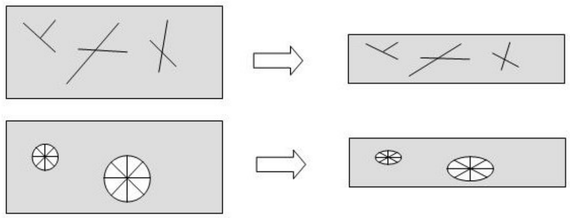


Figure 2.2: Distortion of material lines and strain markers

2.3 The strain ellipsoid

In a deforming body, material lines will rotate and change shape. We want to be able to characterize the rotation and elongation of any arbitrary line. A circle (sphere in 3D) allows us to keep track of all possible orientations of lines. As it turns out, any homogenous strain turns a circle into an ellipse and a sphere into an ellipsoid.

Consider a circle of unit radius, deformed into an ellipse oriented such that the major and minor semi-axes are parallel with the coordinate axes. The elongation, stretch and quadratic elongation of the major semi-axis are given by:

$$e_x = \frac{l_{fx} - 1}{1}$$
$$S_x = 1 + e_x = l_{fx}$$
$$\lambda_x = l_{fx}^2$$

Alternatively, $l_{fx} = \sqrt{\lambda_x}$. Similarly, $l_{fz} = \sqrt{\lambda_z}$. Since the equation of an ellipse is $\frac{x^2}{a^2} + \frac{z^2}{b^2} = 1$, where a and b are the lengths of the semi-axes, the equation of the strain ellipse is just

$$\frac{x^2}{\lambda_x} + \frac{z^2}{\lambda_z} = 1$$

In three dimensions:

$$\frac{x^2}{\lambda_x} + \frac{y^2}{\lambda_z} + \frac{z^2}{\lambda_z} = 1$$

More generally, the semi-axes of the strain ellipse (ellipsoid in 3D) are the **principal strains**, analogous to the principal stresses we saw earlier. The length of the semi-axes (S_1, S_2, S_3) are the magnitudes of the principal strains. The strain ellipsoid can have various shapes, corresponding to **uniaxial, biaxial** or **triaxial** strain. Circular sections through biaxial strain ellipses will be undistorted; in triaxial strain they will be distorted, though equally shortened or lengthened in all directions.

If you're lucky, the rock body you are looking at will contain initially circular or spherical markers. Deformation will turn these into ellipses, whose long and short axes are the principal strain axes. Examples of reasonably nice spherical strain markers are ooids or (perhaps) pebbles in a conglomerate. The spherical cross sections of worm tubes can also be used in this way. Initially elliptical or ellipsoidal markers can also be used to characterize strain, though this is more complicated.

2.4 Displacement vector fields and strain

One approach to analyzing strain is to keep track of particle displacements. The position (x, y) of a particle before deformation to its position after deformation (x', y')

can be related by a set of coordinate transformation equations of the form:

$$x' = ax + by$$
$$y' = cx + dy$$

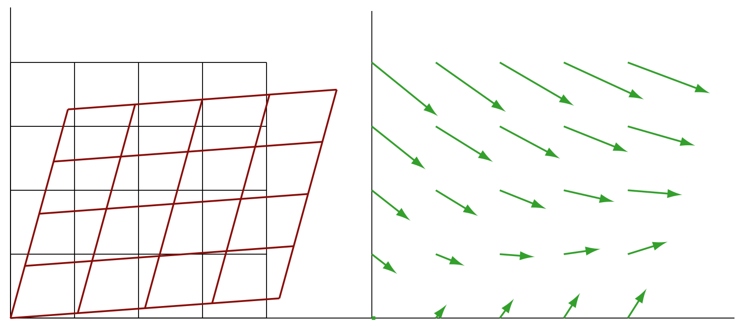


Figure 2.3: Displacement field for general, homogenous strain

If a, b, c, d are constants, then the strain is homogenous (fig 2.3). Two particularly important strain regimes are **simple shear** and **pure shear** (fig. 2.4). Their coordinate transformation equations, expressed in matrix notation are:

$$\text{simple shear:} \quad \begin{bmatrix} 1 & \gamma \\ 0 & 1 \end{bmatrix} \quad \text{pure shear:} \quad \begin{bmatrix} k & 0 \\ 0 & 1/k \end{bmatrix}$$

Deformation is usually not instantaneous: strains accumulate over time. One kind of strain can follow one another. Mathematically, this is equivalent to multiplying the strain matrices. Note that matrix multiplication is not commutative, so, for example, simple shear followed by pure shear does not yield the same final result as pure shear followed by simple shear.

If the orientations of the principal strain axes do not rotate during deformation, then the strain is said to be **irrotational** or **coaxial**.

2.5 Mohr circles for strain I : Infinitesimal strain

The **infinitesimal strain** is a useful concept that can be thought of as representing the instantaneous material response to stress. The accumulation of infinitesimal strain increments over geological time results in the deformation that the geologist observes and tries to understand in the outcrop – known as the **finite strain**. For our purposes we can consider "pretty small" strains – say, less than 1% – to be infinitesimal. For the purposes of deriving the Mohr circle equations, this case also allows the use of small angle approximations, in particular, $\gamma = \psi$.

As with stress, we want a Mohr circle construction that allows us to read off (1) the elongation (for infinitesimal strain, call it ϵ) and (2) the shear strain (γ) that affects a line of some given orientation. The principal strains are denoted $\epsilon_1, \epsilon_2, \epsilon_3$. A

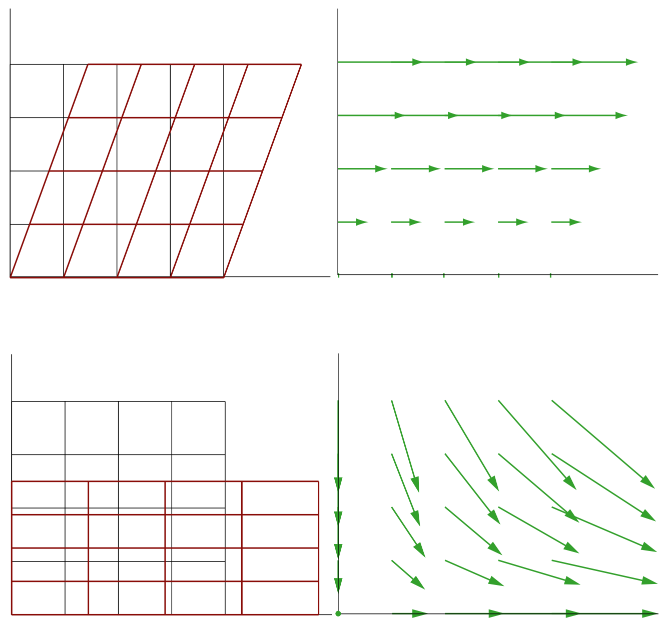


Figure 2.4: Displacement fields for pure and simple shear

fairly tedious derivation (for details, you might check out Hobbes (1976)) yields:

$$\epsilon = \left(\frac{\epsilon_1+\epsilon_2}{2}\right) + \left(\frac{\epsilon_1-\epsilon_2}{2}\right)\cos 2\alpha$$
$$\frac{\gamma}{2} = \left(\frac{\epsilon_1-\epsilon_2}{2}\right)\sin 2\alpha$$

From which a Mohr circle construction can be made (see figure 3.1). α is the angle between a material line and the principal strain axes. Note that this Mohr circle is drawn in ϵ vs. $\gamma/2$ coordinates. Examination of the Mohr circle for infinitesimal strain yields the following important relations:
1. There are two lines that experience the maximum shear strain, and they are located at 45°to the principal strain axes.
2. The maximum shear strain is given by $\gamma/2 \pm (\epsilon_1-\epsilon_2)/2$, i.e. $\gamma \pm (\epsilon_1-\epsilon_2)$.
3. Any 2 lines perpendicular to one another are 180°apart on the Mohr circle, so they suffer shear strains equal in magnitude but opposite in sign.

2.6 Mohr circles II: Finite strain

As the result of finite strain, lines are lengthened or shortened and the angles between intersecting lines are usually changed. Considering a unit circle deformed into an ellipse whose axes are parallel with the coordinate frame, we can derive relationships that track the elongations and rotations (shear strains) for any line. As you might expect, a Mohr circle construction is the ticket to the big time. The derivations of these are so tedious that even Ramsay and Huber relegate them to an appendix (cf. Ramsay and Huber, appendix D if you can't help yourself). Two separate constructions are available: one identifies lines according to the angles they make with the principal strain directions in the "unstrained state" – this is of somewhat limited use since we rarely know what the orientations of lines used to be. The other deals with the orientations in the strained state. Again, as with stress and infinitesimal strain, the orientation of a line is defined by the angle it makes with the principal strain axes; the Mohr circle constructions are created by deriving equations that relate elongations, shear strains and line orientation that also form parametric equations for a circle.

2.6.1 Unstrained state reference frame

The unstrained reference frame refers back to the pre-deformation orientation of a line P. The line makes an angle θ with the principal strain axes. Upon deformation, it suffers an elongation and rotates into a new position P' (in general, we will use primes to distinguish the reference frames) making an angle θ' with the principal strain directions. The elongation and shear strain of an arbitrary line (in 2D) are given by equations that should provoke a sense of *deja vue*:

$$\lambda = \frac{(\lambda_1 + \lambda_2)}{2} + \frac{(\lambda_1 - \lambda_2)}{2}\cos 2\theta$$
$$\gamma = \frac{\lambda_1 - \lambda_2}{2\sqrt{\lambda_1\lambda_2}}\sin 2\theta$$

Note: these fail to produce a circle (instead, you get a Mohr *ellipse*) unless $\sqrt{\lambda_1\lambda_2} = 1$, i.e. there is no dilation.

2.6.2 Strained reference frame

Usually, we are confronted with good rocks gone bad, and so the angles we measure are those of lines in the strained state, i.e. we measure θ', not θ. Conversion between the two reference frames can be done using:

$$\sin\theta = \lambda^{1/2}\sin\theta'/\lambda_2^{1/2}$$
$$\cos\theta = \lambda^{1/2}\cos\theta'/\lambda_1^{1/2}$$

and pulling a little definitional slight of hand. In particular, we define:
1. A **new strain parameter** $\gamma' = \gamma/\lambda$**.**; and
2. The **reciprocal quadratic extensions** $\gamma'_1 = 1/\gamma_1$ and $\gamma'_2 = 1/\gamma_2$.

The Mohr equations become:

$$\lambda' = \frac{\lambda'_1 + \lambda'_2}{2} - \frac{\lambda'_1 - \lambda'_2}{2}\cos 2\theta'$$
$$\gamma' = \frac{\lambda'_2 - \lambda'_1}{2}\sin 2\theta'$$

Chapter 3

Week 3 notes: Progressive deformation

3.1 Reading assignment

Progressive strain histories are covered in section 15.5 (pages 308ff.) of Twiss and Moores. Also read pages 352 to 357 ("Strain in shear zones"). Nice treatment of this sort of thing is also found in the introductory chapters of Passchier et. al. (2005) *Microtectonics*.

3.2 Progressive strain in pure shear

In lecture, we discuss possible strain histories for lines of various orientations that undergo progressive strain. (In lab, we deal with the case of pure shear).

By **progressive strain**, we simply indicate that in our analysis we will break down the deformation history into many small steps. At each step we will consider both the **infinitesimal strain** that accrues at that step, and the **finite strain** that has accrued up to that point. Even in very simple cases, some rather complex behavior can result.

The analysis considers one of the "simplest" kinds of strain: **pure shear** in two-dimensions, i.e.:**plane strain**, $\epsilon_2 = 0$ and $\lambda_2 = 1$.

The following four figures accompany the in-class discussion. These figures are meant to be incomplete. Use these to help you follow the class discussion; you should annotate and complete these.

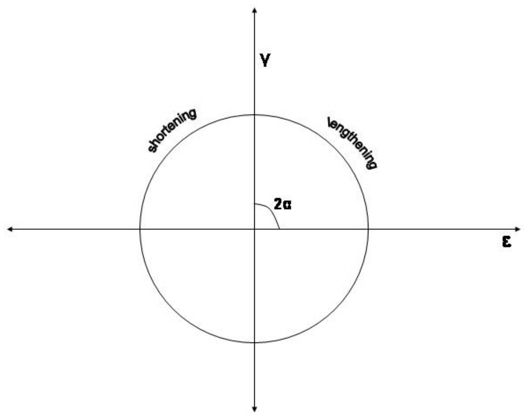


Figure 3.1: Mohr circle for Infinitesimal strain

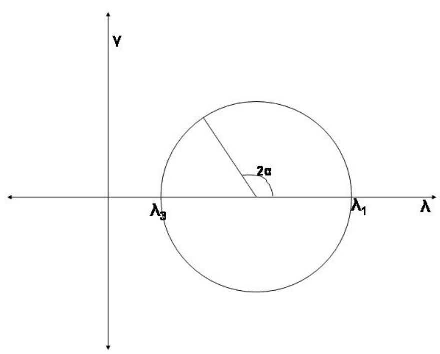


Figure 3.2: Mohr circle for finite strain, unstrained state reference frame

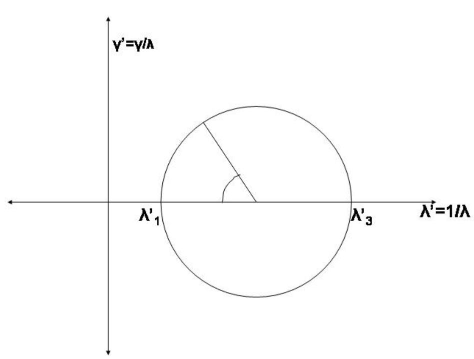


Figure 3.3: Mohr circle for finite strain, strained state reference frame

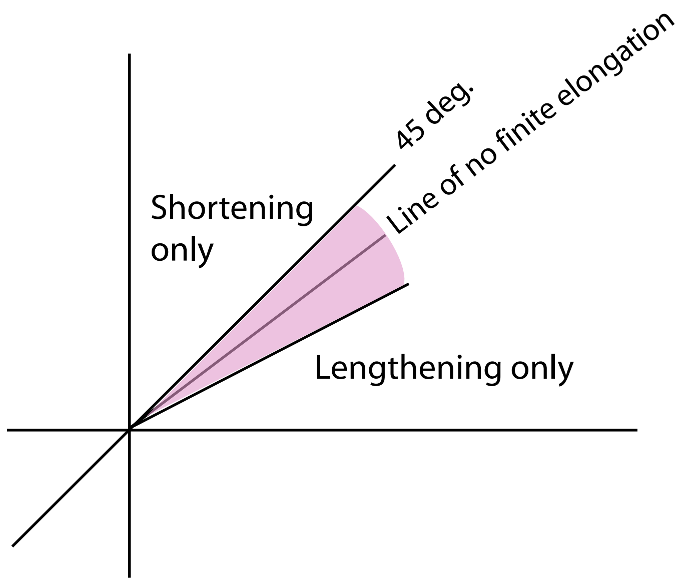


Figure 3.4: "Real world" reference frame. Shaded area shows region where material lines have suffered shortening followed by lengthening. Line of no finite elongation separates sub-regions defined by whether extension exceeds shortening.

Chapter 4

Rheology part 1: Ideal material behaviours

4.1 Reading assignment

Chapter 18 in Twiss and Moores, pages 361 – 385, deals with both ideal models for rock deformation as well as experimental investigation of ductile flow. Chapter 9, pages 165 – 190, deals with the mechanics of brittle fracture. This is good stuff, believe.

4.2 Ideal behaviours

The next section of the course will review what we know about the relationship between stress and strain; that is, the laws that govern deformation. We start with "ideal" behaviours, and then compare these to experimental results. A material's response to an imposed stress is called the **rheology**. A mathematical representation of the rheology is called a **constitutive law**.

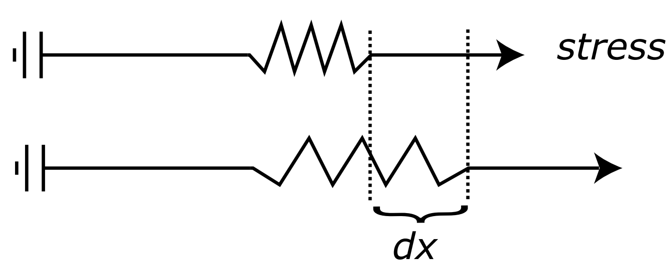


Figure 4.1: Simple, 1-D Hooke body (linear elasticity)

Figure 4.2: In a Newtonian fluid, stress and strain-rate are linearly proportional to each other. The slope of the stress-strain rate line is the viscosity.

4.2.1 Elastic behaviour – Hooke's law

The characteristics of elasticity are: 1. strain is **instantaneous** upon application of stress; 2. stress and strain are **linearly** related; 3. strain is perfectly **recoverable**.

$$\sigma \propto \epsilon$$

The constant of proportionality is called the **elastic modulus**; the exact modulus you use depends on whether the strain is volumetric, uniaxial or in shear. Respectively, these are:

$$\sigma_h = K\epsilon_v$$
$$\sigma_n = E\epsilon_n$$
$$\sigma_s = 2\mu\epsilon_s$$

Where K, E, μ are the bulk modulus, Young's modulus and the shear modulus. Another important elastic constant is ν, Poisson's ratio. Poisson's ratio relates the elastic strain for orthogonal directions.

4.2.2 Viscous behaviour – Newtonian fluids

Strain in a viscous fluid is **time dependant** and **non-recoverable**. Time-dependance is the fundamental difference with elastic behaviour: instead of a linear relationship between stress and strain, viscous materials exhibit a linear relationship between stress and **strain-rate**.

$$\tau = \eta\dot{\epsilon}$$

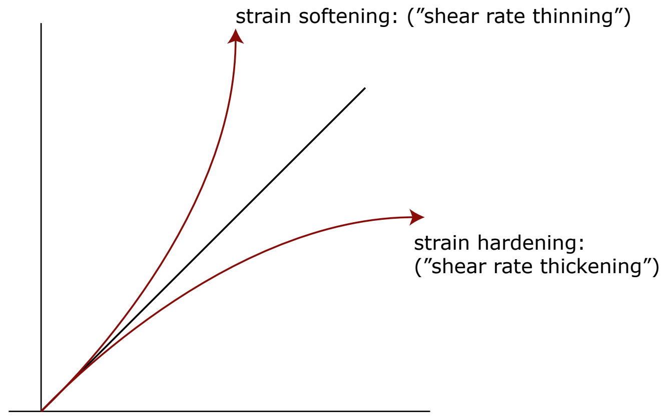


Figure 4.3: Strain-rate softening or hardening

The constant of proportionality, η, is the **viscosity**, and has units of Pascal-seconds. A plot of strain against time is a line, provided that the viscosity remains constant. Viscosity is a measure of the strength of a material: higher viscosity materials are stronger. Typical geological viscosities are:

Material	Viscosity (Pa-s)
Water	10^{-3}
Lava	0.1 - 10
Glacier ice	10^{13}
Salt	10^{14} - 10^{20}
Window glass	10^{21}
Athenospheric mantle	10^{21}

The effective viscosity can change during the course of deformation. If the viscosity increases, we call the process **strain hardening** or **shear rate thickening**; conversely, if the material becomes weaker and the viscosity decreases, the process is **strain softening** or **shear rate thinning**.

4.2.3 Visco-elasticity I: Maxwell bodies

If we represent elastic behaviour with a spring, and viscous behaviour with a dashpot[1], we can also combine dashpots and springs in a number of different ways to simulate various possible ideal material responses or rheologies.

[1]What the heck is a dashpot? They show up a lot in material science, geodynamics and differential equations, and are invariably introduced as though they are a familiar, intuitive part of one's day to day experience. Except that they aren't, really. Ask the random person on the street what a dashpot is and you'll get a pretty blank look. Dashpots are simple pistons combined – usually – with a hydraulic fluid. Examples of objects that include dashpots are car shocks, bike shocks, and are found on some doors.

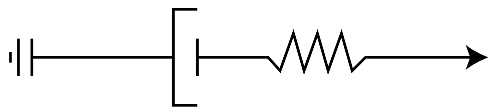


Figure 4.4: Maxwell visco-elastic body

The most basic combination is to put a spring and a dashpot in series. This combination is known as **Maxwell viscoelasticity**. The constitutive relationship for Maxwell viscoelasticity comes from the linear addition of the relationships for elasticity and viscous fluids[2]:

$$\dot{\epsilon} = \frac{\dot{\sigma}}{2\mu_M} + \frac{\sigma}{2\eta_M}$$

The subscript M just indicates that we're dealing with Maxwell-rigidity and Maxwell-viscosity. This equation can be manipulated by stipulating either constant stress ($\sigma = \sigma_0$) or constant strain ($\dot{\epsilon} = 0$). Under constant stress:

$$\epsilon = \frac{\sigma_0}{2\mu_M} + \frac{\sigma_0}{2\eta_M} t$$

On a strain vs. time plot (fig. 4.5), this shows instantaneous elastic strain, followed by steady state linear viscous strain. The case of constant strain is a little more difficult to intuit, but the equation, and plot of stress against time are clear enough.

$$\sigma = \sigma_0 \exp\left(-\frac{\mu_M}{\eta_M} t\right)$$

A plot of stress against time demonstrates the viscous stress relaxation of a Maxwell body (fig. 4.5). That is, a Maxwell body should relax to an isotropic (i.e. hydrostatic) stress state on a time scale that is captured by the ratio of $\eta_M : \mu_M$. Expressed as a fraction, this ratio has units of time, and is known as the **Maxwell time**. The Maxwell time of the asthenospheric mantle is on the order of a thousand years and is what sets the time scale of phenomena like post glacial rebound.

4.2.4 Visco-elasticity II: Kelvin bodies

Another way to combine a spring and a dashpot (that is, an elastic response and a viscous response) are to put them in parallel, rather than in series. This material is known as a **Kelvin** body, and sometimes called **fermoviscous** behaviour. This is the idealization of a phenomenon (that is, something that actually happens in the real world) called **elastic afterworking**, which is just that, in the real world, most springs

[2]The notation I use differs somewhat from what Clark used in class. Mine follows standard texts such as Ranalli, G. (1995) *Rheology of the Earth*, and Turcotte, D. and Schubert (2002) *Geodynamics*.

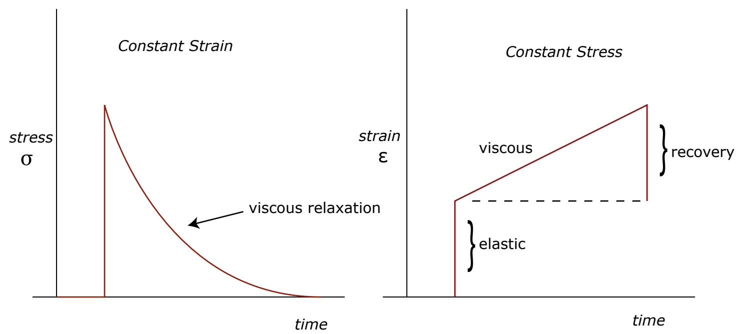


Figure 4.5: Stress and strain versus time for Maxwell viscolelastic bodies

and supposedly elastic materials don't always respond instantaneously to the imposed stress. A Kelvin body shows **time-dependant**, **recoverable** strain. Kelvin behaviour is given by

$$\sigma = 2\mu_K \epsilon + 2\eta_K \dot{\epsilon}$$

Upon loading, the elastic response of the spring is damped by the dashpot, and goes asymptotically to $\sigma_0/2\mu_K$:

$$\epsilon = \frac{\sigma_0}{2\mu_K}\left[1 - \exp\left(-\frac{\mu_K}{\eta_K}t\right)\right]$$

When the load is removed, the strain is recovered, but not instantaneously. Suppose, a strain ϵ_0 had been accummulated, then $\epsilon(t)$ is

$$\epsilon = \epsilon_0 \exp\left(-\frac{\mu_K}{\eta_K}t\right)$$

4.2.5 Other ideal rheologies

Four other ideal behaviours are worth mentioning. The first are two end-members of Newtonian viscosity (fig. 4.7): the **Pascal liquid** and the **Euclid solid**, which have viscosities 0 and ∞, respectively. The last two are types of **plastic** behaviour. A plastic material is one that has a yield stress, but otherwise behaves as a Newtonian fluid (or a non-Newtonian, power-law fluid). The **St. Venant** body (also called an **elastic-plastic** body) is idealized as a friction block being pulled by an elastic spring (fig. 4.8). Such a body is a material that exhibits linear elasticity up to a certain point, the yield point. Then, the material fails abruptly. This sort of rheology is relevant to understanding elastic strain accumulations that drive the earthquake cycle.

A **Bingham** body, also called **viscoplastic**, is idealized as a spring in series with a friction block *itself in parallel* with a dashpot [3]. Its behaviour is governed by the equations:

$$\sigma = 2\mu\epsilon \quad \sigma < \sigma_Y$$
$$\sigma = \sigma_Y + 2\eta_B \dot{\epsilon} \quad \sigma \geq \sigma_Y$$

[3] Note Twiss and Moores leave out the leading spring, which is a special case.

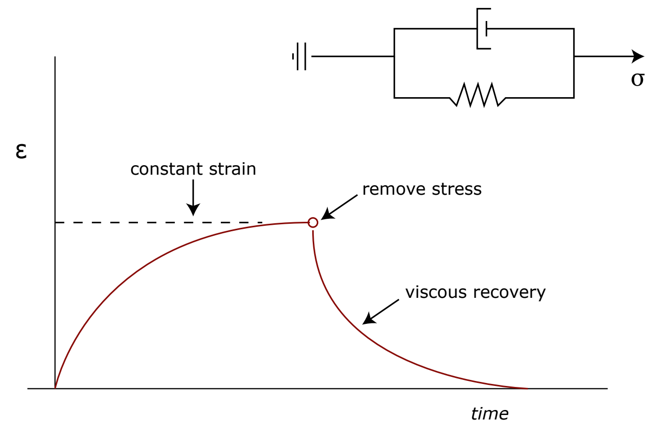


Figure 4.6: Kelvin visco-elasticity, strain versus time.

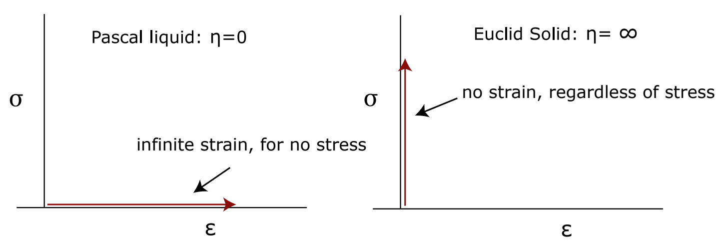


Figure 4.7: Idealized end-members of Newtonian viscosity

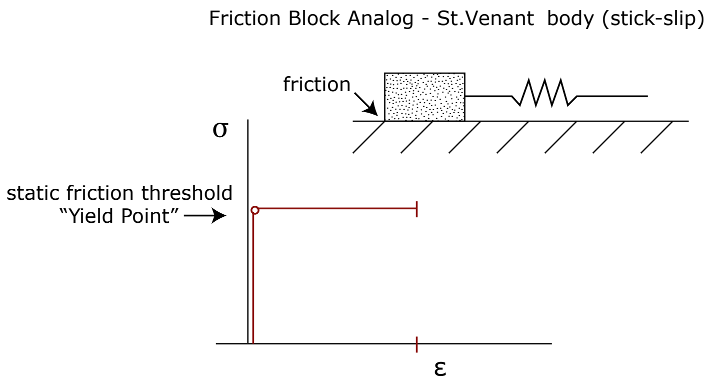


Figure 4.8: Model and stress-strain plot for St. Venant (or "stick-slip") rheology

The Bingham body behaves elastically at stresses lower than the yield stress, and flows as a linear fluid above the yield strength with a strain rate proportional to $\sigma - \sigma_Y$. Examples of Bingham materials are certain clays, some kinds of submarine debris flows, oil paintings, drilling muds, toothpaste and bread dough.

4.3 Study and review questions

You should be aware of the differences between: linear elastic, linear viscous, Maxwell viscoelastic, and Kelvin behaviour. St. Venant and Bingham bodies are also relevant since they introduce the concept of a yield stress.

For each of the above, draw a spring and dashpot idealization, and draw curves that relate stress and strain (or strain-rate). For Maxwell and Kelvin bodies, also plot stress against time for constant strain, and strain (or strain rate) against time for constant stress. Think of a real world example or two for each behaviour.

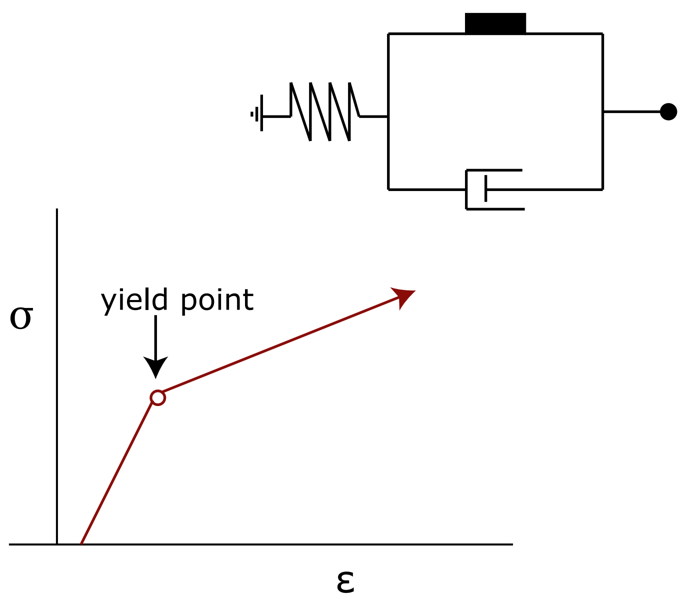


Figure 4.9: Bingham viscoplastic rheology

Chapter 5

Rheology part 2: Experimental rock deformation, stress-strain curves

5.1 Reading assignment

The latter part of Chapter 18 in TM is essential reading for this section (pp. 369 – 385). Other useful sources are chapter 5 in Ranalli, G. (1995) *Rheology of the Earth* and Nicholas, A. and Poirier, J.P. (1976) *Crystalline Plasticity and Solid State Flow in Metamorphic Rocks*. Hobbes, et. al. (1982) *An Outline of Structural Geology* is the closest textbook to the order and logic of the lectures as presented in class.

5.2 Experimental rock deformation

Most of the results of experimental rock deformation come from a fairly standard experimental set-up: a triaxial deformation apparatus.

The triaxial deformation apparatus is basically a pressure vessel surrounding a piston. This kind of apparatus allows careful control of the principal stresses, temperature and strain-rates. The effect of pore fluids or chemical solutions can also be controlled. Major limitations are: (1) the amount of finite strain that the apparatus can accommodate (on the order of 10%); (2) the slowest strain-rates possible (about 10^{-7}sec^{-1}. Geological strain rates, by comparison, range from 10^{-12} to 10^{-20}.

5.3 Experimental rock deformation: phenomenology

Figure 5.2 shows the relationship between stress (this is shorthand for "differential stress", i.e. $\sigma_1 - \sigma_3$) and (total, finite) strain. The first part of the curve shows a linear relationship between stress and strain – characteristic of elasticity. This strain is *recoverable*, meaning that the strain goes back to zero once the stress is removed. The second part of the curve is known as **primary creep**. Stress and strain are no longer linearly related; strain is recoverable, but the recovery turns out to be time-dependant. That is, this part of the curve shows behaviour characteristic of Kelvin visco-elasticity. The third part of the curve is referred to as **secondary creep**. When

Figure 5.1: Triaxial deformation rig: the so-called Patterson vessel. Pore pressure, confining pressure

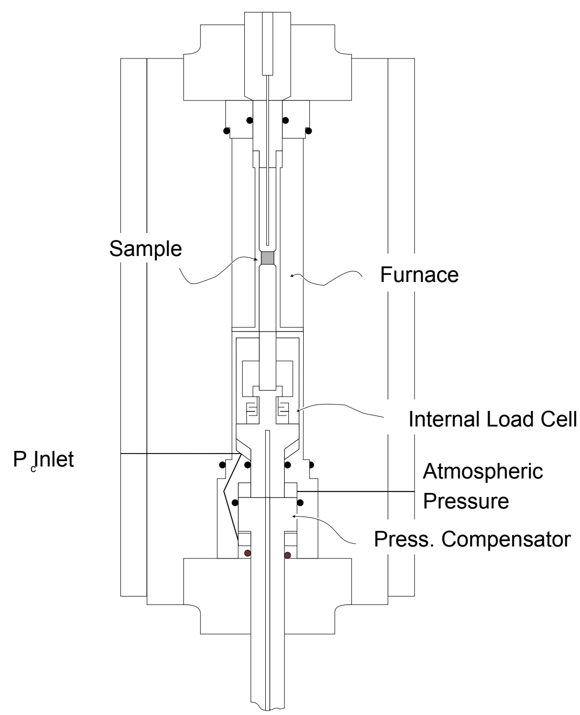
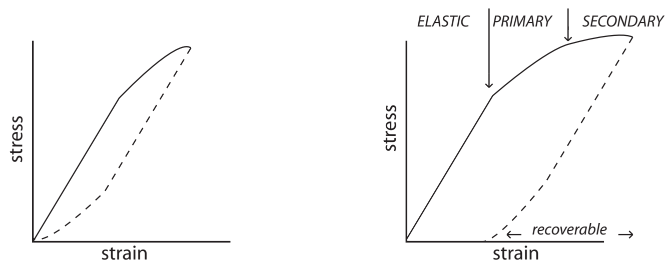


Figure 5.2: Stress-strain curves



stress is removed, you recover the elastic part instantaneously, the secondary part in a time dependant part, but you accumulate permanent strain.

5.4 Ductile deformation: the effects of strain rate and temperature

Consider a series of experiments (these results were first described by Heard and his colleagues from experiments on marble, see references in Hobbes, et. al. 1982). Figure 5.3 shows three plots. The first shows a family of curves corresponding to different temperatures at a constant strain-rate. The second shows a family of curves corresponding to different strain-rates at a constant (high) temperature. The first-order observation is that both increasing temperature and decreasing strain-rate have the effect of weakening the rock (less stress for the same strain). The third shows a plot of stress against strain rates for experiments on marbles. The thin black lines shows how the lines might be extrapolated to geological strain rates, but, more importantly since many of the lines are parallel, it suggests that for certain deformation mechanisms, temperature can substitute for strain-rate. Since we can heat up a sample much easier than we can control the passage of time, this represents a strategy for simulating natural deformation in the laboratory.

Experimental data can be fit by

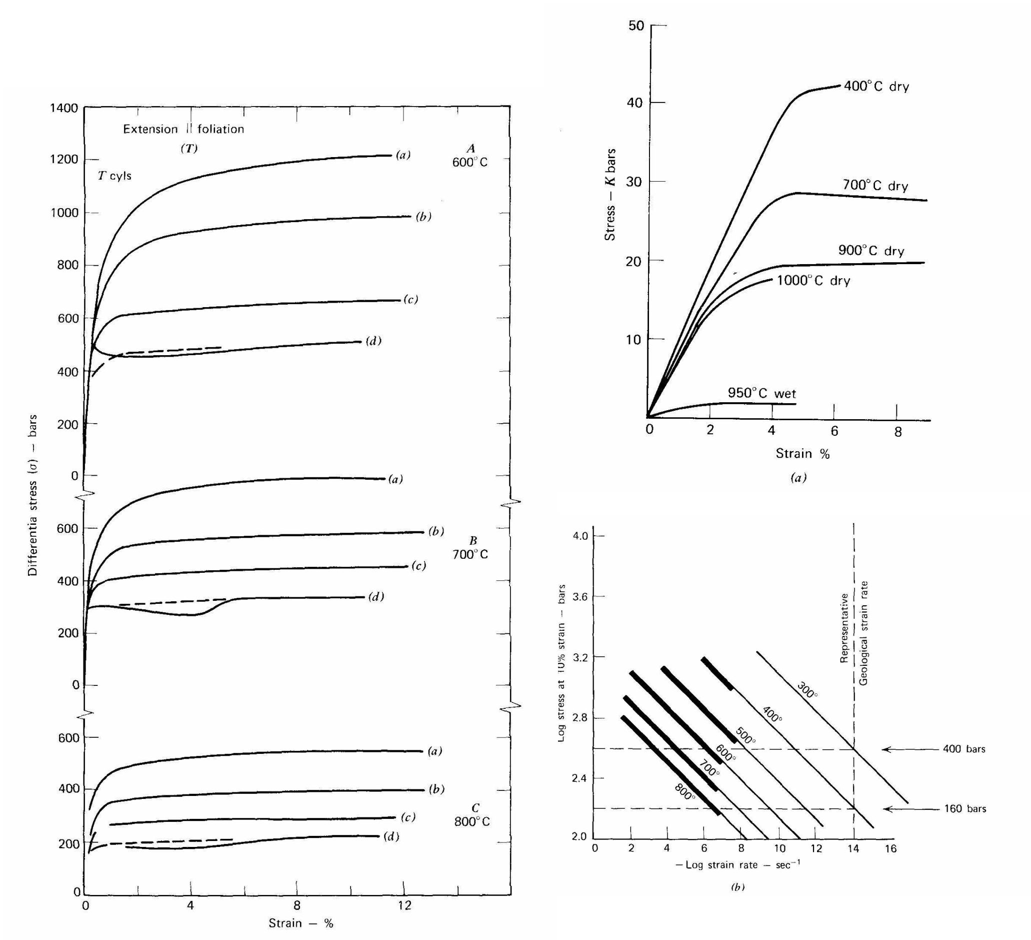
$$\dot{\epsilon} = A \exp - \left(\frac{E}{RT} \right) \sigma^N$$

where T is the absolute temperature, R is the gas constant, E is the activation energy, and N is a constant ranging from 1 to 8. For materials deforming such that $N > 1$, this is called "power-law creep", since stress and strain rate are related by a power N. If $N = 1$, then strain rate is linearly proportional to stress, which is the definition of a Newtonian viscous body. For constant strain-rates or constant stresses, the equation can be manipulated to yield an "effective viscosity", which at the very least is a useful shorthand for the strength of materials deforming by ductile flow. The effective viscosity of the upper mantle, as inferred from studies of post-glacial rebound are in the 10^{21-22} range.

5.5 Review questions

What are typical strain rates for tectonic systems? What are the slowest strain-rates typically achievable in laboratory experiments? How can we safely extrapolate ex-

Figure 5.3: Experimental stress-strain curves. These plots are taken from Hobbs, H., Means, W., and Williams, P. (1982) *An Outline of Structural Geology*, New York: Wiley and Sons, inc., pp. 64 – 66.



perimental results to natural systems?

In the plot with a family of temperature curves in stress vs. strain-rate space, not all the lines are, in fact parallel. This and similar plots are often subdivided into high, moderate and low stress regions.

Outline how the environmental variables of confining pressure, temperature and strain-rate affect the behaviour of rock deformation.