12.009/18.352 Problem Set 6

Due Thursday April 16, 2015

Problem 1: 60 pts — (a,b,c,d)=(5,20,15,20)
Problem 2: 40 pts — (a,b,c)=(10,20,10)

1. *The Spatiotemporal Distribution of Solar Radiation*: We all (should!) know that 2) the equator is warmer than the poles because on average it receives more sunlight and 3) that seasons occur because the Earth's axial tilt exposes each hemisphere to a varying amount of sunlight over the year. In class, we derived the average incoming solar radiation received by the Earth, per unit area:

$$\overline{I(\phi, \lambda)} = S_0/4, \tag{1}$$

where we define $I(\phi, \lambda)$ as the *insolation* (units: W/m^2), ϕ is the latitude angle (zero at the equator, $\pm\pi/2$ at the poles), λ is the solar longitude angle, or angle between the actual longitude and the longitude of solar noon (varies between $\pm\pi$), and S_0 is the "solar constant", or energy flux on a plane normal to the solar beam, at the Earth's orbital distance (an inconvenient truth: S_0 is not actually constant with time, because the Earth-Sun distance changes, but we'll ignore that here!). The overline in eqn. 1 indicates that we have averaged over the whole surface of the Earth.

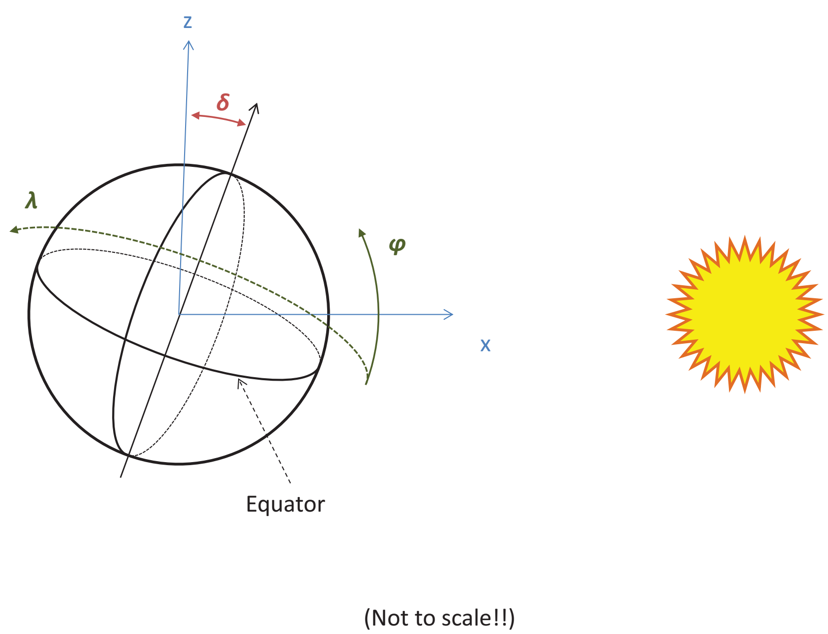

Figure 1: Geometry of Earth-Sun System

(a) We'll now explore how I depends on space and time. Using the geometry in Figure 1, argue that $I = \max[S_0(\hat{\mathbf{x}} \cdot \hat{\mathbf{n}}), 0]$, where $\hat{\mathbf{x}}$ is the unit vector pointing in the direction from the Earth to the Sun, $\hat{\mathbf{n}}$ is the unit normal vector pointing out from the surface of the Earth, $\cdot$ denotes the vector dot product, and the max function is used to indicate that I cannot be negative.

(b) Using the fact that the rotation axis of the Earth projects into the xz plane by an angle δ towards the Sun (we will assume δ is a given parameter), show that:

$$\hat{\mathbf{x}} \cdot \hat{\mathbf{n}} = \cos\phi\cos\lambda\cos\delta + \sin\phi\sin\delta, \tag{2}$$

(where $\lambda = 0$ indicates the longitude that is currently experiencing solar noon).

(c) Given that every point on Earth experiences all 'solar longitudes' over the course of the day, express the solar longitude λ in terms of the local solar time in hours h, with $h = 12$ being solar noon, and $h = 24$ being solar midnight. Note that at sunrise and sunset, the local insolation just equals zero. Determine the local solar time of sunrise/sunset by calculating the time at which $\cos\phi\cos\lambda\cos\delta + \sin\phi\sin\delta = 0$.

(d) We just made use of the fact that λ varies over the course of a day. Now we will use the fact that δ varies over the course of a year. For planets with relatively small obliquity γ, or absolute axial tilt relative to a vector normal to the ecliptic, it can be shown (you do not need to show this!) that $\delta = \gamma\cos(2\pi t/P)$, where P is the period of revolution about the sun, and $t = 0$ represents the northern hemisphere summer solstice. We will use $\gamma = 23.5°$ and $P = 1$ year for the Earth. Make a plot of I as a function of time for latitudes ϕ =0, 30, 60, 75, and 90°. Make sure to include plots of at least one full year, and to not allow negative values of I. You should see variations in insolation on both daily and yearly timescales, though it will be difficult to capture both on the same plot.

2. *Observations of Solar Radiation at the Earth's Surface*: Now we will look at some actual data. In the file `HarvardForest.mat` are observations of surface solar radiation, air temperature, and net carbon uptake from the Harvard Forest, a long-term ecological research station in central Massachusetts. Solar radiation at the surface is given by the `Solar_Rad` variable, which we will call Q_S here, in units of W/m^2, for 48 measurements a day (half-hourly timestep) over a 14-year period from 1993-2006 (2002 is missing due to instrumentation errors).

(a) Pick a year and plot Q_S. Make a plot of the full year, and then zoom in on a week to see the day-to-day and finer-scale structure.

(b) How do your plots of bottom-of-atmosphere solar radiation compare to the top-of-atmosphere insolation I that you derived in Problem 1? Make a scatter plot of Q_S against a calculation of I at 42.5° N latitude (approximately the latitude of Harvard Forest). Note that in order to do a proper comparison, you will have to introduce phase shifts into the expressions for δ and λ, to account for the fact that $t = 0$ in the data file is local midnight, and January 1, not the summer solstice. Discuss why the observed and theoretical values differ.

(c) *"Writing about music is like dancing about architecture"* – Try *listening* to the full timeseries of Q_S data with MATLAB®, using the `soundsc` command, which converts a timeseries into an audible signal, and scales the volume by the maximum and minimum in the timeseries. The `soundsc` command, by default, samples at a rate of 8192 points per second, which means that the maximum frequency that can be heard, by a timeseries that goes 0-1-0-1-... would be 4192 cycles per second, or Hertz. Test that your version of MATLAB interacts with your speakers and ears properly by issuing the following commands on the MATLAB command prompt:

```
>> testsound = zeros(8192,1);
>> testsound(2:2:end)=1;
>> soundsc(testsound)
```

You should hear an annoying high-pitched sound that lasts a second (you may want to have your hearing checked if you do not find it annoying).

Now, describe what you hear when you listen to the Q_S data, and how it relates to the graph you made in part 2a). Using the default sampling rate of 8192 data points per second, calculate the dominant frequency that you hear (in Hertz). Also listen to a calculated timeseries of I, with the same timestep and length as the Q_S data, and describe how it sounds similar or different.