12.005 Lecture Notes 11

Sandbox tectonics – Simple theory (after Dahlen '84)

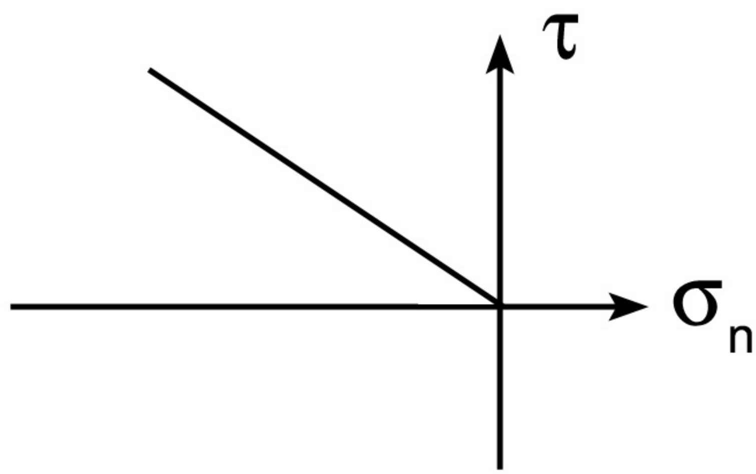

Figure 11.1

Figure by MIT OCW.

Assumptions

- Coulomb failure $\tau = -\mu(1-\lambda)\sigma_n$
- Based decollement may have different μ_b (or λ_b) (?) fluids in count?
- Material is on the verge of failure everywhere
- Mohr circle construction
- Inertia negligible $\Rightarrow \underset{\sim}{F} = m\underset{\sim}{a} = 0$

  4 parameters $\mu, \lambda, \mu_b, \lambda_b$ $\Rightarrow$ (?), 2 variables
- Neglect complications of being under water (?)

Figure 11.2
Figure by MIT OCW.

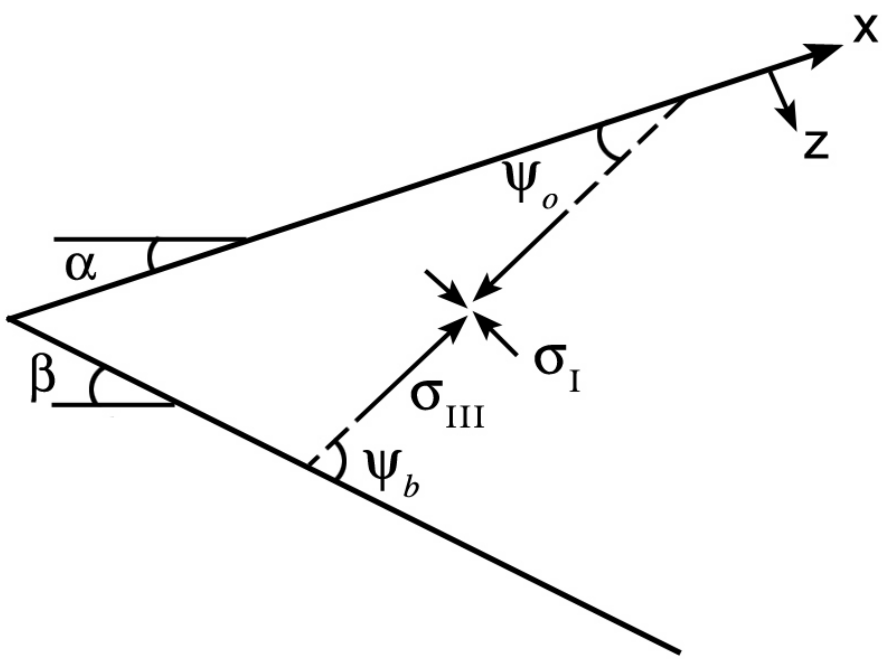

Figure 11.3
Figure by MIT OCW.

$\alpha \equiv$ surface slope
$\beta \equiv$ basal slope
$\psi_0 \equiv$ angle between σ_{III} and surface (or more fundamentally, σ_I and z axis)
$\psi_b \equiv$ angle between σ_I and base

Equilibrium equations: $\tau_{ij,j} + f_i = 0$

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} - \rho g \sin\alpha = 0$$

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} + \rho g \cos\alpha = 0$$

At z = 0

$\sigma_{xz} = 0$

$\sigma_{zz} = 0$

For now, ignore effects of pore fluid pressure – will go back to this later.

Dry sand

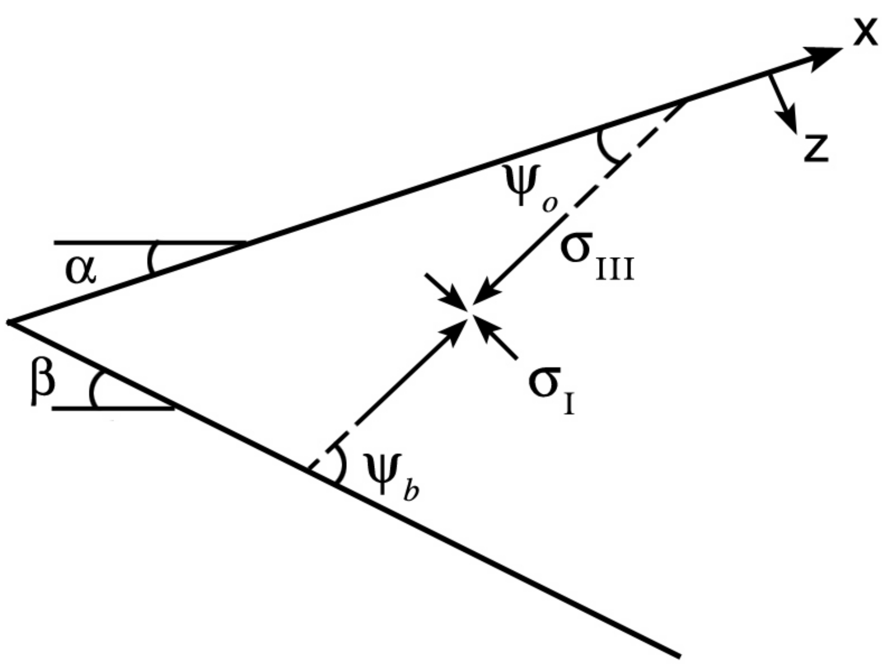

Figure 11.4 "Physical" space

Figure by MIT OCW.

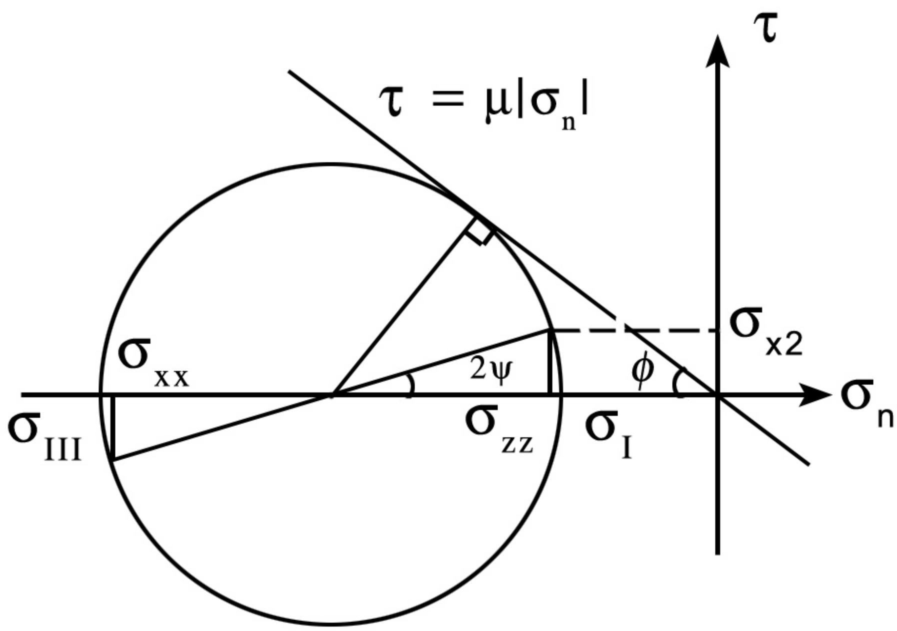

Figure 11.5 "Mohr" space

Figure by MIT OCW.

Argue:

No strength: $s_0 = 0 \Rightarrow$ no scale length

$$\Rightarrow \frac{\partial}{\partial x} = 0 \quad \sigma_{\text{I}} \ \& \sigma_{III} \text{ functions of z only}$$

ψ_0 is a constant

Plug in to equilibrium equations:

$$\begin{Bmatrix} \sigma_{zz} = -\rho g z \cos\alpha \\ \sigma_{xz} = \rho g z \sin\alpha \end{Bmatrix}$$ satisfies equilibrium equations and boundary conditions

How to relate ψ_0 and α?

If stress is critical everywhere, can use Mohr circle.

Say $\sigma_c = \dfrac{\sigma_I + \sigma_{III}}{2} \qquad \sigma_r = \dfrac{\sigma_I - \sigma_{III}}{2}$

Geometry of Mohr circle $\Rightarrow$

$$\csc\phi = \frac{\sigma_c}{\sigma_r}$$

$$\tan 2\psi_0 = \frac{\sigma_{xz}}{\sigma_{zz} - \sigma_c}$$

$$\sec 2\psi_0 = \frac{\sigma_r}{\sigma_{zz} - \sigma_c}$$

Equilibrium $\Rightarrow \dfrac{\sigma_{xz}}{\sigma_{zz}} = -\tan\alpha$

Solving 4 equations $\Rightarrow \tan\alpha = \dfrac{\tan 2\psi_0}{\csc\phi \sec 2\psi_0 - 1}$

The March of Science revisited (after S. V. Panasyuk)

To introduce the notation, let's draw a sketch:

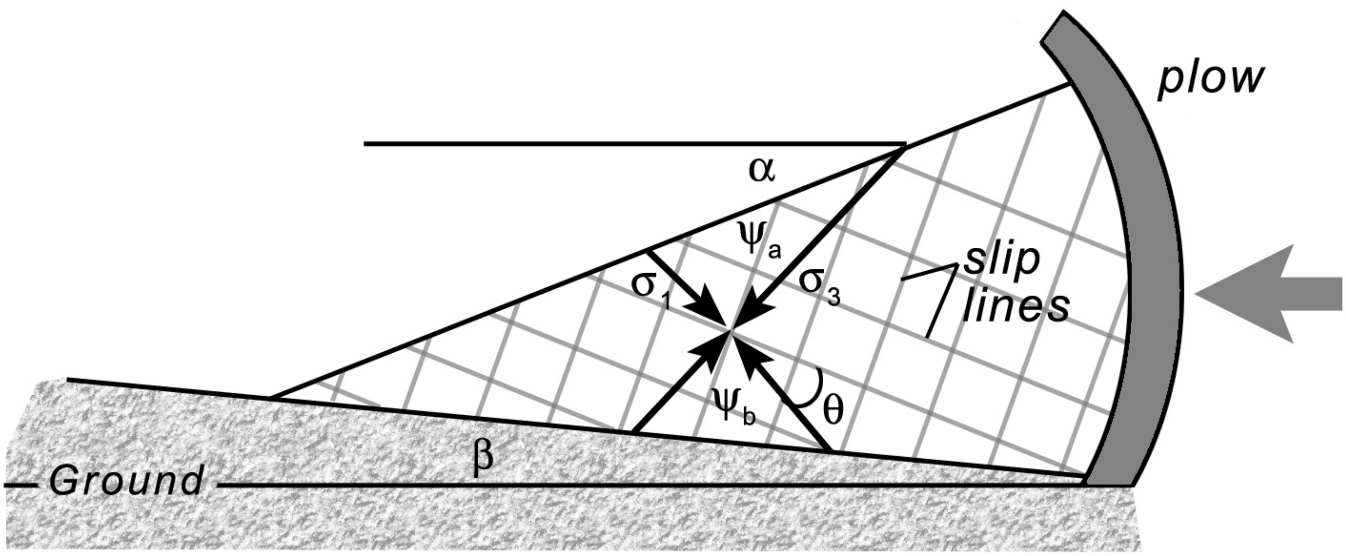

Figure 11.6

Figure by MIT OCW.

We can use the well known Mohr's circle technique to describe the wedge behaviour. Given coefficients of friction for the base μ_b and the μ, and the principal stresses in the wedge (σ_1, σ_3), we can construct the Mohr's circle diagram and the Coulomb failure criterion as:

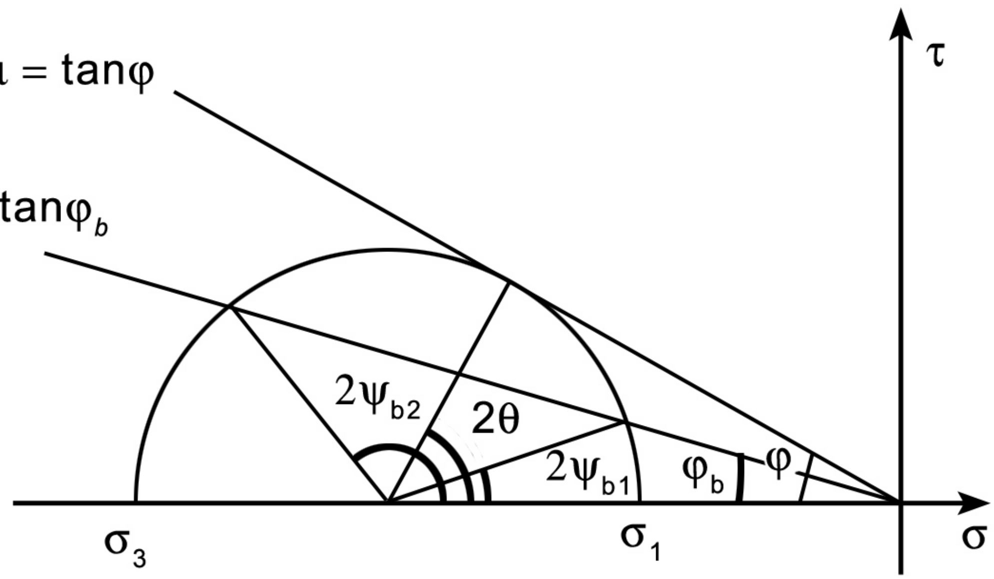

Figure 11.7
Figure by MIT OCW.

Using this diagram it is possible to determine the orientation of the principal stresses with respect to the base (ψ_{b1} and ψ_{b2}) and the orientation of the fault planes, i.e. the slip lines in the wedge (θ) with respect to the principal directions (σ_1, σ_3). Since motion on the slip planes participates in the creation of the actual slope of the wedge as well, this slope might differ from that of the slip planes. To analyse the behaviour of the wedge in front of a bulldozer, we derived equations to connect all angles ($\alpha, \beta, \psi_0, \psi_b, \varphi, \varphi_b$) with each other:

i) from the picture of the wedge (Fig. 1), we have the purely geometrical relationship between angles:

$$\alpha + \beta = \psi_b - \psi_0 \tag{1}$$

ii) from the equations of static equlibrium and boundary conditions (see previous lecture notes):

$$\tan(\varphi_b) = \frac{\tan 2\psi_b}{\csc\varphi \cdot \sec 2\psi_b - 1} = \mu_b, \quad \text{or} \quad \varphi_b = \arctan\left(\frac{\tan 2\psi_b}{\csc\varphi \cdot \sec 2\psi_b - 1}\right) \tag{2}$$

$$\tan(\alpha) = \frac{\tan 2\psi_0}{\csc\varphi \cdot \sec 2\psi_0 - 1}, \qquad \text{or} \quad \alpha = \arctan\left(\frac{\tan 2\psi_0}{\csc\varphi \cdot \sec 2\psi_0 - 1}\right) \tag{3}$$

Note that in the second form of (2) and (3), angles (e.g.) are given directly, which will be useful below.

iii) from reasonable sense:

$$0 \leq \alpha + \beta \leq \pi \tag{4}$$

Having a system of 4 equations in terms of the 4 unknowns guarantees a solution, but does not make apparent the "obvious" connection between angles. There are at least two ways to solve this system and present results in a viewable form: the physical approach (choosen by Nature and by the TA) and the mathematical approach (choosen by the Princeton Professor).

I. From a physical point of view the decollement dip (the base slope = β) is a more fundamental quantity then the surface slope (the wedge slope = α). Together with the material properties (coefficients of friction μ and μ_b) it determines the surface slope. For example, suppose a sand wedge ($\mu = \tan(\varphi) = \tan(30^0)$) is placed on sloping, slippery ground ($\mu_b = \tan(\varphi_b) = \tan(10^0)$) which is inclined at β =35° to the horizon. Let's solve the system of equations graphically to get all possible values for the other angles α, ψ_0, ψ_b:

Equation 1) relates α and ψ_0: $\alpha = (\psi_b - \beta) - \psi_0$. Its graph is:

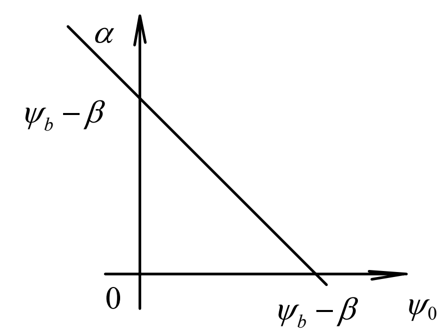

Figure 11.8

Note: in these coordinate axes, the slope of this straight line is 45°.

Equation 2) determines ψ_b values from $\varphi_b = \arctan\left(\frac{\tan 2\psi_b}{\csc\varphi \cdot \sec 2\psi_b - 1}\right)$. Its graph is:

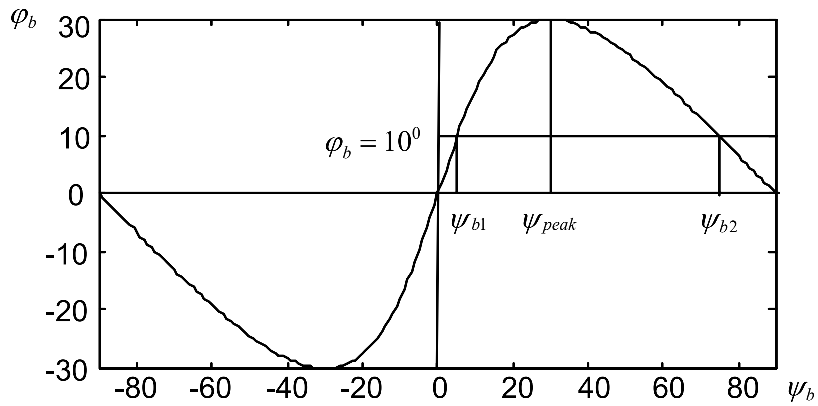

Figure 11.9

Note: This function reaches peaks at

$$\psi_{peak} = \pm(\pi/4 - \varphi/2) \text{ and } \varphi_b(\psi_{peak}) = \pm\varphi$$

The value φ_b intersects the curve at two points, ψ_{b1} and ψ_{b2} .

Equation 3) defines α in terms of ψ_0 as $\alpha = \arctan\left(\dfrac{\tan 2\psi_0}{\csc\varphi \cdot \sec 2\psi_0 - 1}\right)$. Its graph is:

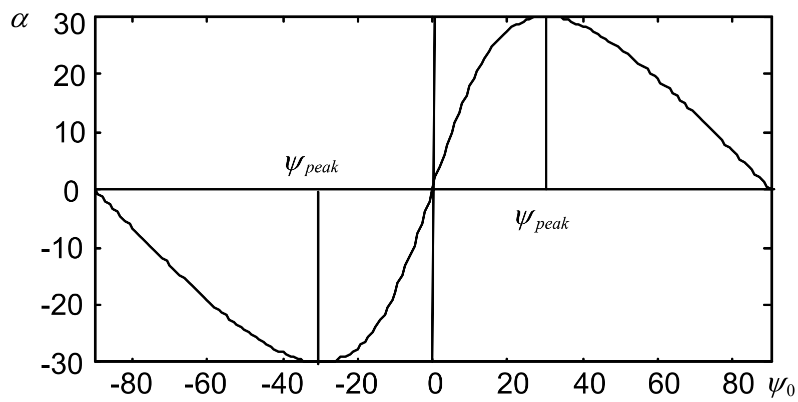

Figure 11.10

Note: This function reaches peaks at $\psi_{peak} = \pm(\pi/4 - \varphi/2)$ and $\alpha(\psi_{peak}) = \pm\varphi$

Since the first and third graphs can be plotted on the same axes, the values of α and ψ_0 which satisfy both equations are the coordinates of the intersection points of two curves. Since the second and third graphs have the same dependence, we can use "combined" axes. Now we are ready to find graphically all possible values for other angles α, ψ_0, ψ_b by plotting all three graphs in one:

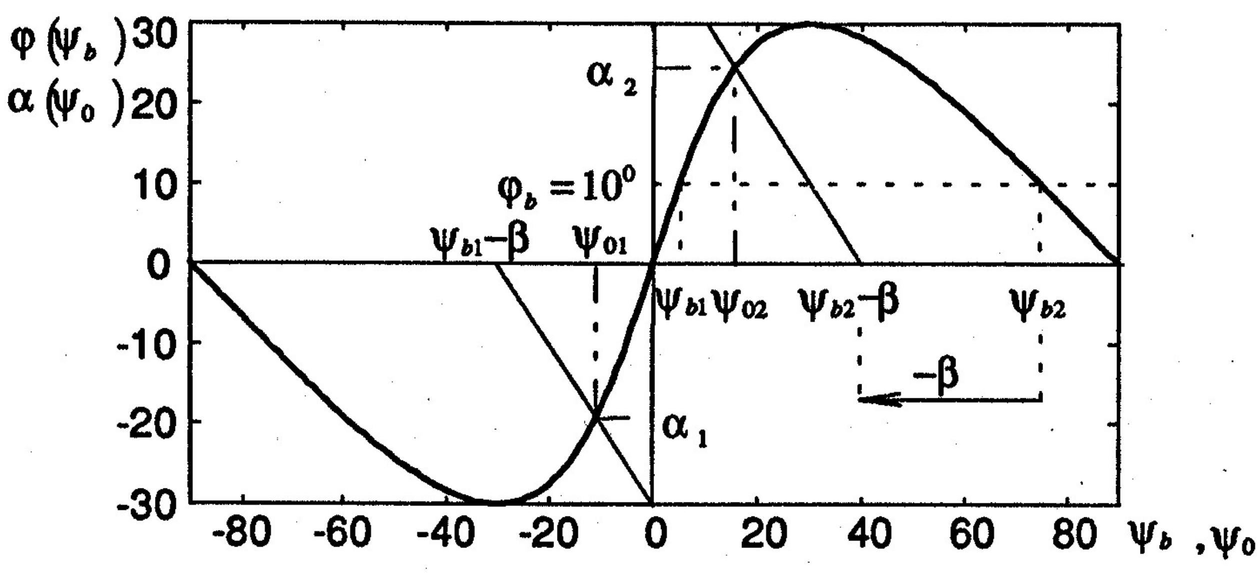

Figure 11.11

Figure by MIT OCW.

From the graph (for $\phi = 30°$, $\phi_b = 10°$, $\beta = 35°$):

From the first intersection line $\varphi_b = 10^0$ with the curve $\Phi(\psi)$, $\psi_{b1} \approx 5^0$, and $(\psi_{b1} - \beta) \approx -30^0$. Following down the $\alpha \Leftrightarrow \psi_0$ line (45 line in the equal unit axes) to its intersection with the $\Phi(\psi)$ curves gives $\psi_{01} \approx -11^0$ and $\alpha_1 \approx -19^0$. A similar approach for the second intersection line $\varphi_b = 10^0$ with the curve $\Phi(\psi)$, i.e. the second value $\psi_{b2} \approx 75^0$ gives $\psi_{02} \approx 15^0$ and $\alpha_2 \approx 25^0$.

II. From the "mathematical" point of view given in the approach is to choose a "test" value for α (from the stability diagram), determine ψ_0 from the curve intersection for $\tan(\alpha)$. The stability diagram is a function of the surface slope angle α with respect to the dip angle β, and its graph can be obtained in the following way:

From equation 2) we can get two values of ψ_b, which are ψ_{b1} and ψ_{b2} for the given friction coefficients in the base and wedge.

From equation 3) by varying the angle of ψ_0 (say from $-\pi/2$ to $\pi/2$) we can get the range of α and substitute all of these values into the equation 1) to get a formula for β:

$$\beta_{12} = \psi_{b1,b2} - \psi_0 - \alpha(\psi_0)$$

under the restriction 4).