Nanomechanics of hierarchical biological materials

Lecture 6

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Outline

1. **Introduction to Mechanics of Materials**
   Basic concepts of mechanics, stress and strain, deformation, strength and fracture
   Monday Jan 8, 09-10:30am

2. **Introduction to Classical Molecular Dynamics**
   Introduction into the molecular dynamics simulation; numerical techniques
   Tuesday Jan 9, 09-10:30am

3. **Mechanics of Ductile Materials**
   Dislocations; crystal structures; deformation of metals
   Tuesday Jan 16, 09-10:30am

4. **The Cauchy-Born rule**
   Calculation of elastic properties of atomic lattices
   Friday Jan 19, 09-10:30am

5. **Dynamic Fracture of Brittle Materials**
   Nonlinear elasticity in dynamic fracture, geometric confinement, interfaces
   Wednesday Jan 17, 09-10:30am

6. **Mechanics of biological materials**
   Monday Jan. 22, 09-10:30am

7. **Introduction to The Problem Set**
   Atomistic modeling of fracture of a nanocrystal of copper.
   Wednesday Jan 22, 09-10:30am

8. **Size Effects in Deformation of Materials**
   Size effects in deformation of materials: Is smaller stronger?
   Friday Jan 26, 09-10:30am

Outline and content (Lecture 5)

- **Topic:** Elasticity in biological materials: Entropic versus energetic contributions

- **Examples:** Deformation of collagen, vimentin, …: Protein mechanics

- **Material covered:** Covalent bonding and models, chemical complexity, reactivity, molecular potentials: CHARMM

- **Important lesson:** Models for bonding in proteins, entropic vs. energetic elasticity

- **Historical perspective:** AFM, single molecule mechanics

■ **Very brief review:**

Material covered in last 2-3 lectures
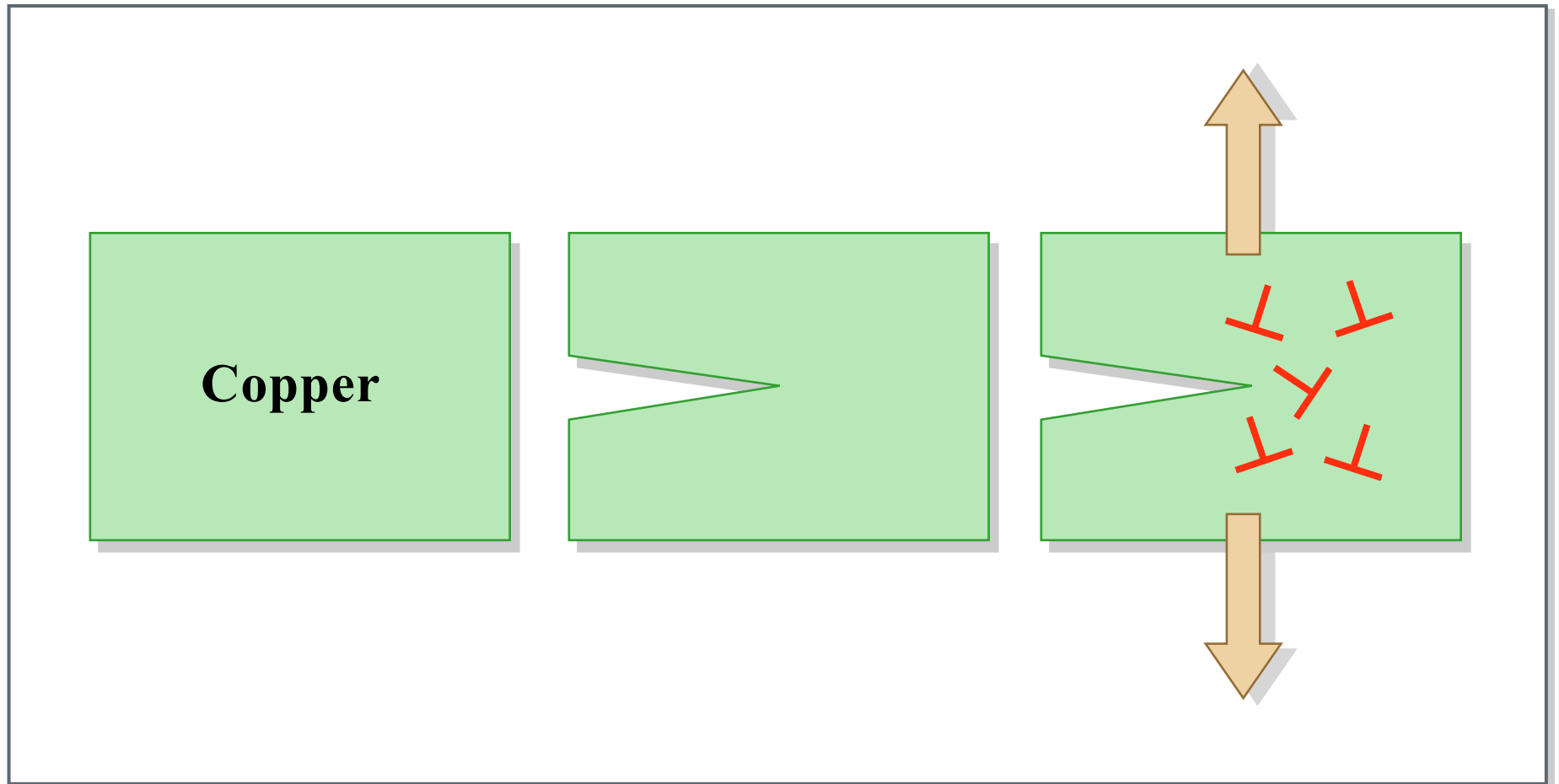
Dislocation nucleation from a crack tip

Copper

Ductile
vs. brittle
Depends on atomic
behavior at crack tip

Figure by MIT OCW.

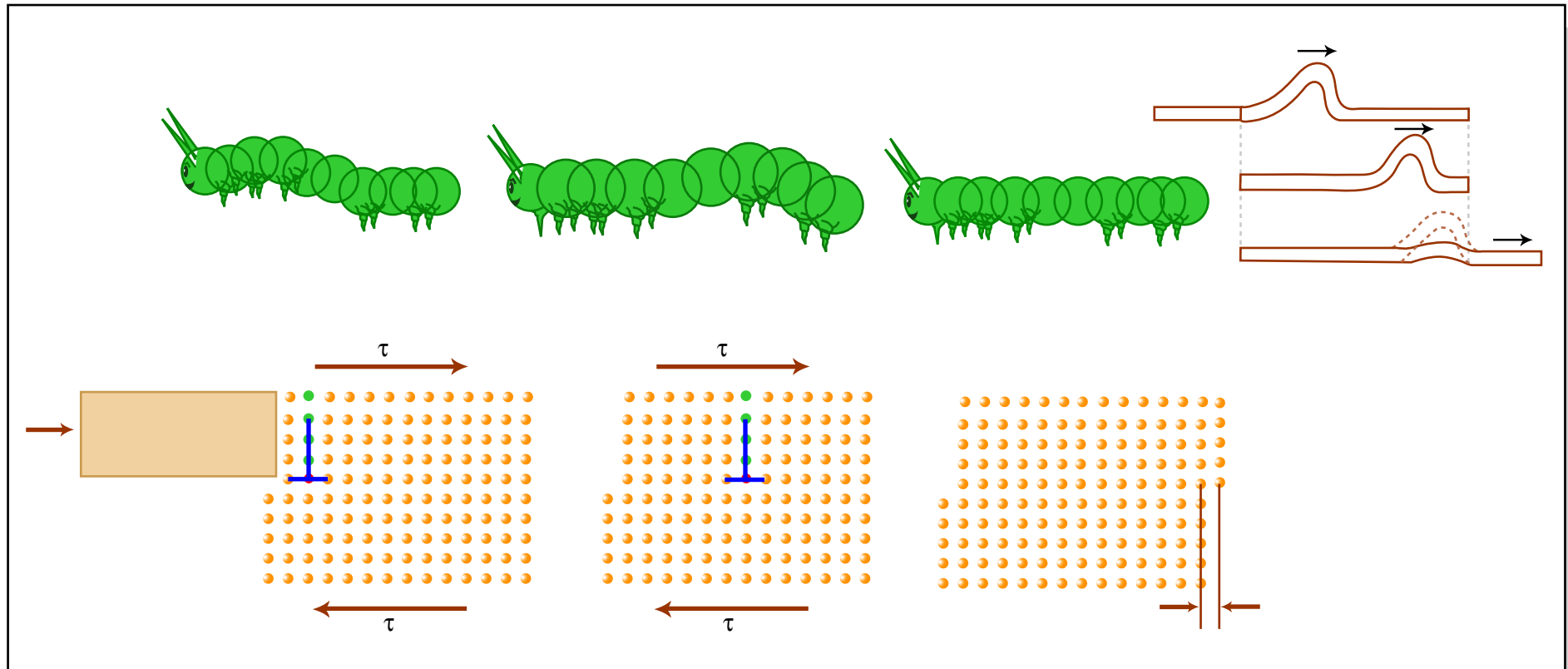# Ductile materials are governed by the motion of dislocations: Introduction

Figure by MIT OCW.

Dislocations are the <u>discrete</u> entities that carry plastic (permanent) deformation; measured by "Burgers vector"
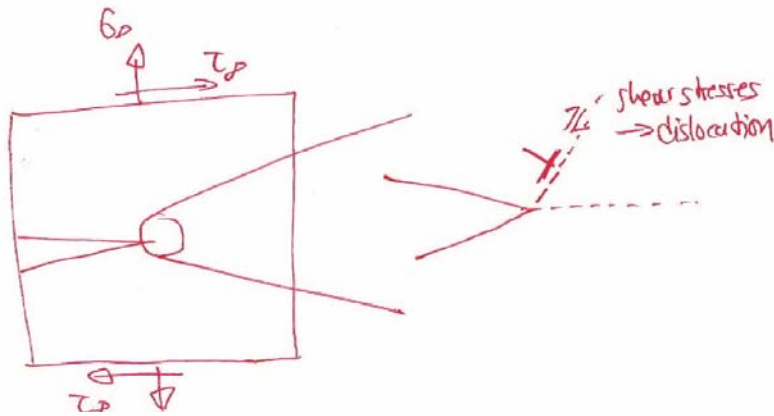
Brittle or ductile?

$$\begin{Bmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \\ \sigma_{r\theta} \end{Bmatrix} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \begin{Bmatrix} 1 + \sin^2\frac{\theta}{2} \\ \cos^2\frac{\theta}{2} \\ \sin\frac{\theta}{2} \cos\frac{\theta}{2} \end{Bmatrix}$$

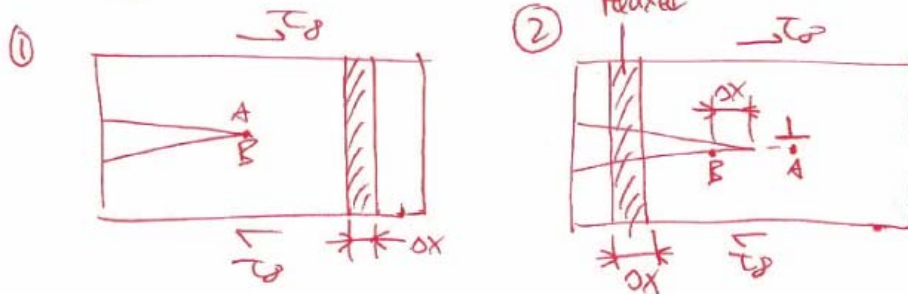Critical conditions for dislocation nucleation / beginning of fracture

Rice-Thomson model

$$\sigma_\infty = \sqrt{\frac{2b}{a} \frac{1}{f(\theta)} \frac{\mu}{4\pi(1-\nu)}}$$

$$\sigma_\infty \sim \sqrt{\frac{b}{a}} \mu.$$

Rice-Peirls model

$$\tau_{\infty,\text{disl}} = \sqrt{\frac{2\gamma_{us}\mu}{H}}.$$

$$\tau_{\infty,\text{surf}} = \sqrt{\frac{4\gamma_s\mu}{H}}.$$
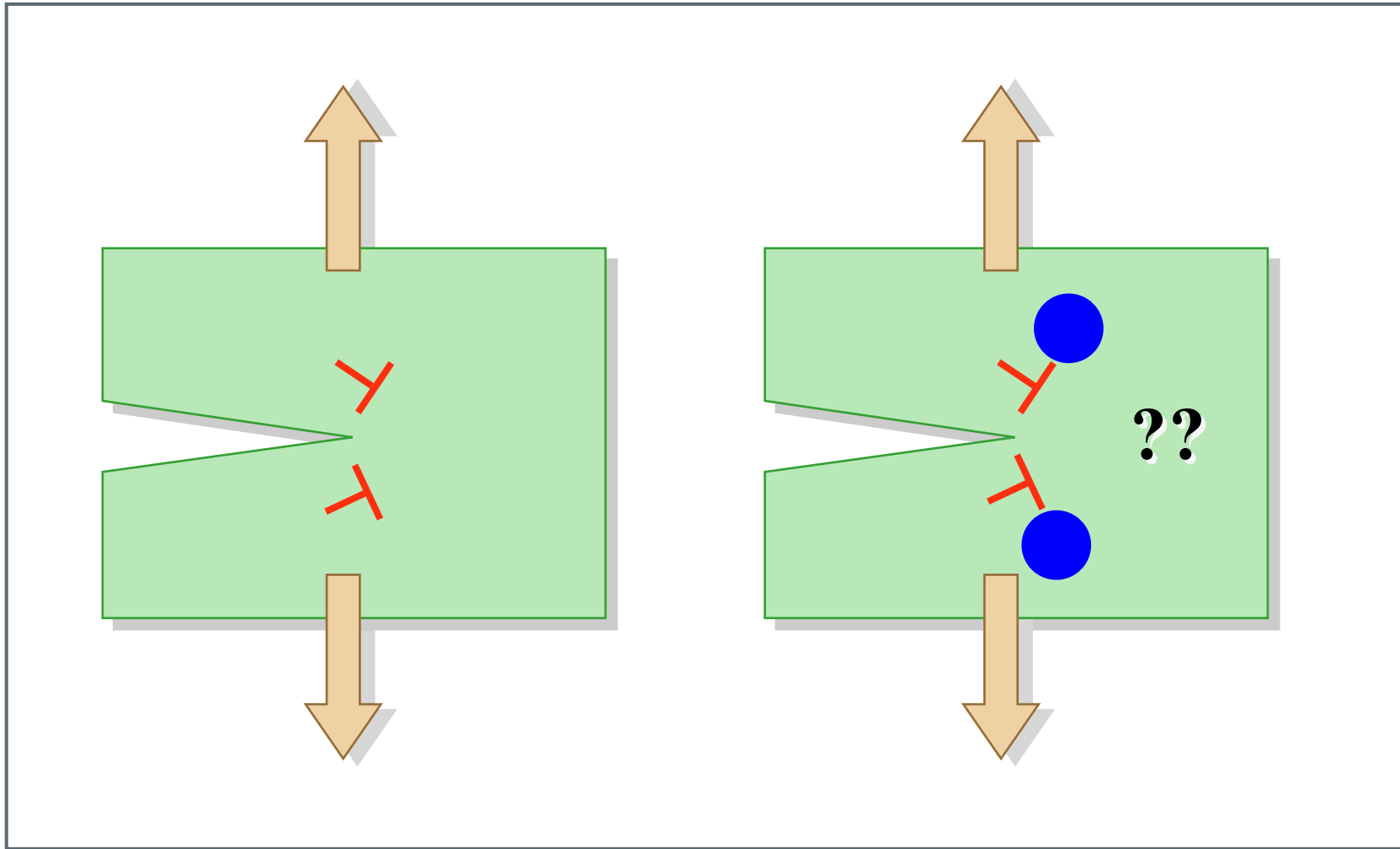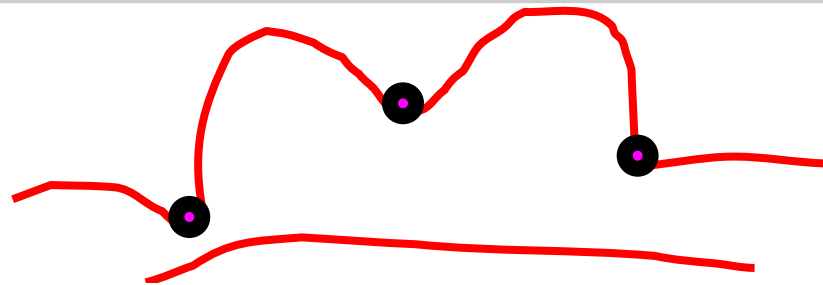
$$\alpha = \frac{\gamma_{us}}{2\gamma_s}$$

Figure by MIT OCW.

Final sessile structure

Image removed due to copyright restrictions.
See:
1. Buehler, M. J., et al. "The dynamical complexity of work-hardening: a large scale molecular dynamics situation." *Acta Mechanica Sinica* 21, no. 2 (2005): 103-111.

2. Buehler, M. J., et al. "Atomic plasticity: description and analysis of a one-billion atom simulation of ductile materials failure." *Computer Methods in Applied Mechanics and Engineering* 193, no. 48-51 (2004): 5257-5282.
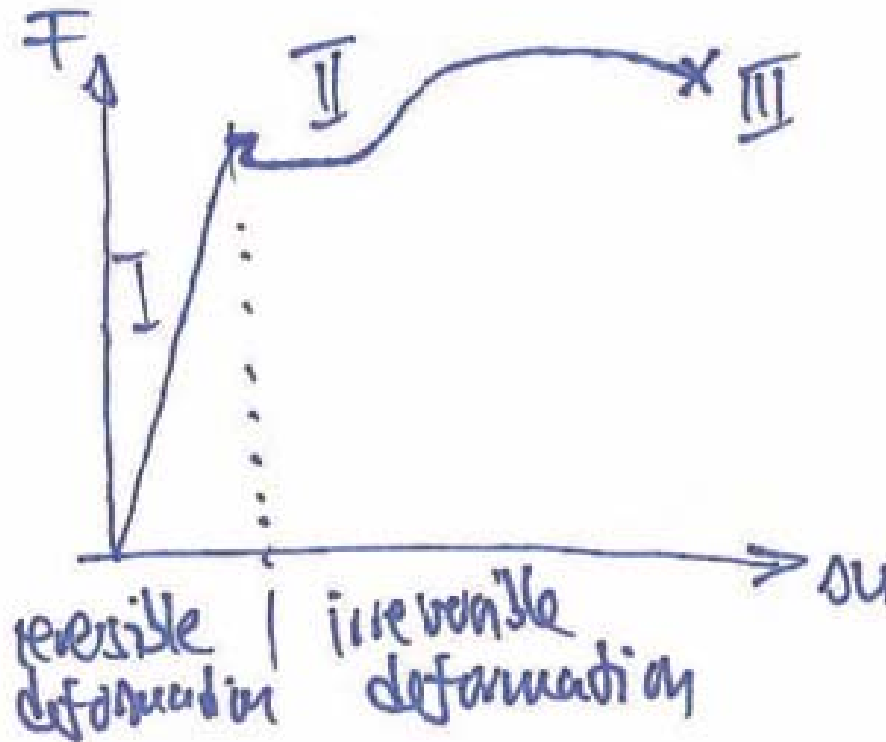
$$F = k_s \Delta u$$

Energy approach to elasticity

1st law of TD

$$\frac{dU}{dt} = \delta W + \delta Q$$

External work rate

$$\delta W = \dot{x} F_e$$

Applied force

2nd law

$$\frac{dS}{dt} \geq \frac{\delta Q}{T}$$

Change in entropy is always greater or equal than the entropy supplied in form of heat; difference is due to internal dissipation

$$\frac{dD}{dt} = T\frac{dS}{dt} - \delta Q \geq 0 \qquad \delta Q = dU/dt - \delta W$$

Dissipation rate

Dissipation rate after consider 1st law of TD:

$$\frac{dD}{dt} = \delta W - \frac{d}{dt}(U - TS)$$

Dissipation rate=External work rate -change in usable energy U-TS`

$F = U - TS$ is defined as free energy or Helmholtz energy,

Energy approach to elasticity

Elastic deformation (no dissipation by definition):

$$\frac{\mathrm{d}D}{\mathrm{d}t} = 0 \qquad\qquad \delta W - \frac{\mathrm{d}F}{\mathrm{d}t} = 0 \quad \text{Assume only internal energy change}$$

$$\delta W = \dot{x}F_e$$

$$\dot{x}\left(F_e - \frac{\mathrm{d}F}{\mathrm{d}x}\right) = 0. \qquad \text{Expand equation} \qquad \dot{x}F_e - \frac{\mathrm{d}F}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t} = 0$$

$$\text{dU/dt = dU/dx\ dx/dt}$$

Therefore: If applied force equals change in free energy of the system, have elastic deformation`

$$F_e = \frac{\mathrm{d}F}{\mathrm{d}x} \qquad F_e = \frac{\mathrm{d}U}{\mathrm{d}x}$$

With strain energy density: $\qquad \Psi = F/V \qquad \Psi = U/V$

$$\sigma_{ij} = \frac{\mathrm{d}\Psi}{\mathrm{d}\varepsilon_{ij}} \qquad c_{ijkl} = \frac{\mathrm{d}^2\Psi}{\mathrm{d}\varepsilon_{ij}\mathrm{d}\varepsilon_{kl}}$$

Cauchy-Born rule

- Idea: Use thermodynamic approach to develop expression of elasticity of a atomic microstructure, provided a specific interatomic potential

- Assuming locally homogeneous deformation, take a unit cell representation (typically PBCs) and express free energy density as a function of strain tensor

- This provides direct link between interatomic potential and constitutive behavior

- For example: FCC crystal and EAM potential – can calculate elastic properties directly

Cauchy-Born rule

- Idea: Express elastic energy (strain energy density) for a atomistic representative volume element as a function of macroscopic applied strain

$$\Phi = \frac{1}{\Omega_0} \int_{\Omega_0^*} U(l) \, D_\Omega \, d\Omega$$

U(l) a function of deformation gradient

- D_Ω: Mapping function, e.g. $\quad D(l_0, \theta, \phi) = \delta_D(l_0 - l_0^*) D_{\theta\phi}(\theta, \phi)$

- Impose macroscopic deformation gradient on atomistic volume element, then calculate atomic stress – this corresponds to the macroscopic stress
- Strictly valid only far away from defects in periodic lattice (homogeneous deformation, perfect lattice, amorphous solid-average)

- Allows direct link of potential to macroscopic continuum elasticity

1D example: Cauchy-Born rule

- Impose homogeneous strain field on 1D string of atoms
- Then get $\sigma_{ij} = c_{ijkl}\varepsilon_{kl}$ from that
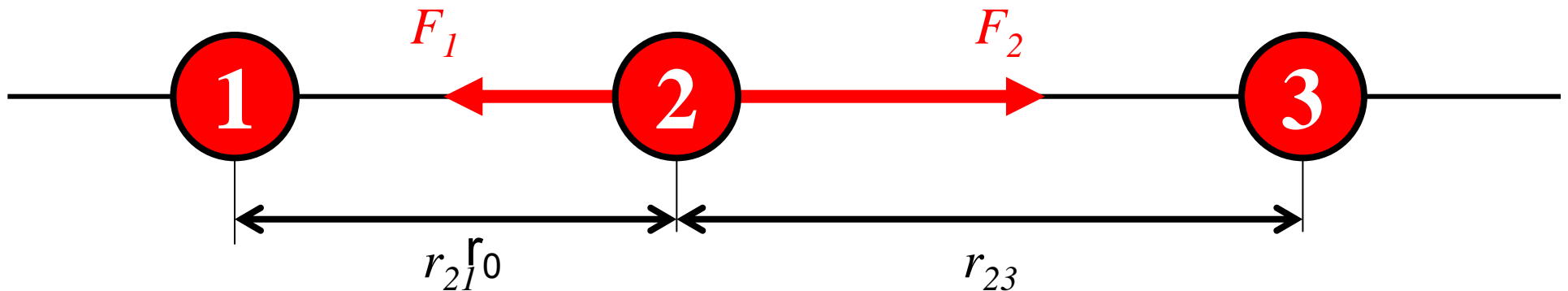
$$\Phi(\varepsilon) = \frac{1}{r_0 \cdot D}\phi(r) = \frac{1}{r_0 \cdot D}\phi((1+\varepsilon)\cdot r_0)$$ Strain energy density function

$r_0 \cdot D$ Atomic volume

■ Detailed write-up on 1D / 2D Cauchy-Born rule is included on line

Review

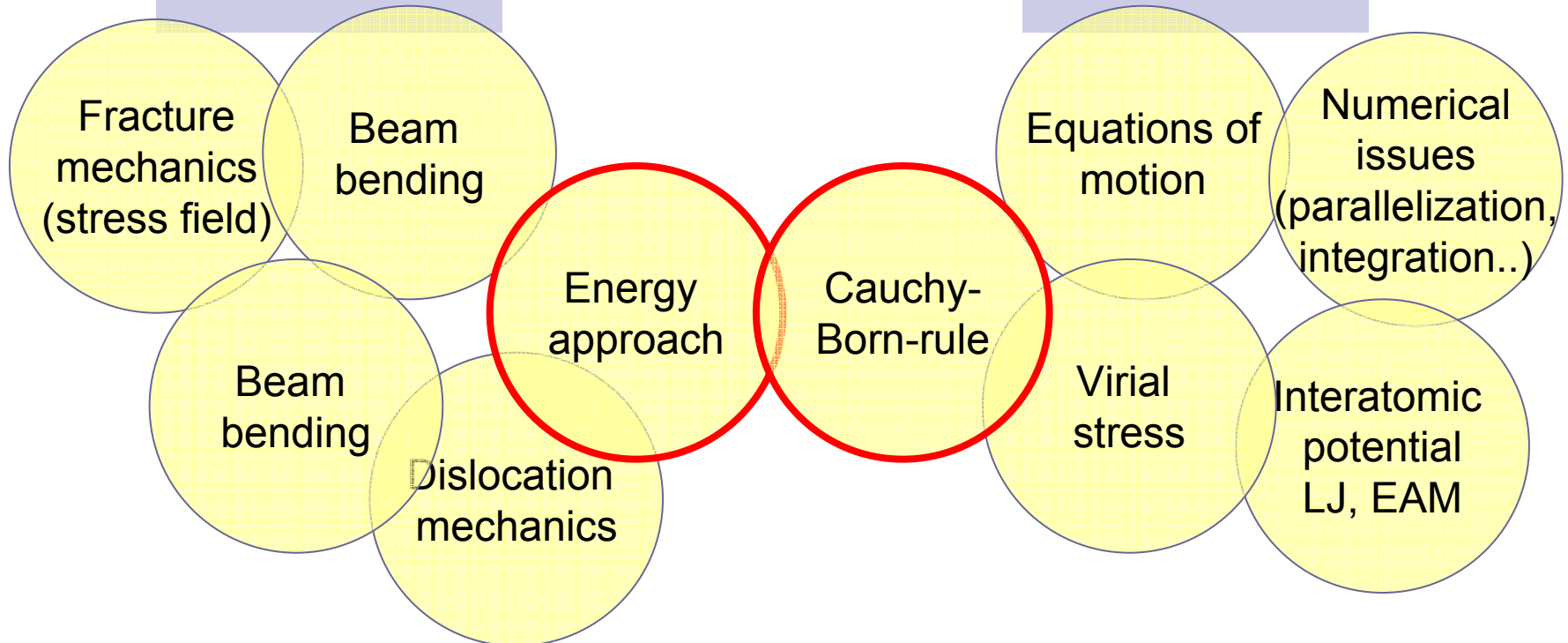© 2007 Markus J. Buehler, CEE/MIT
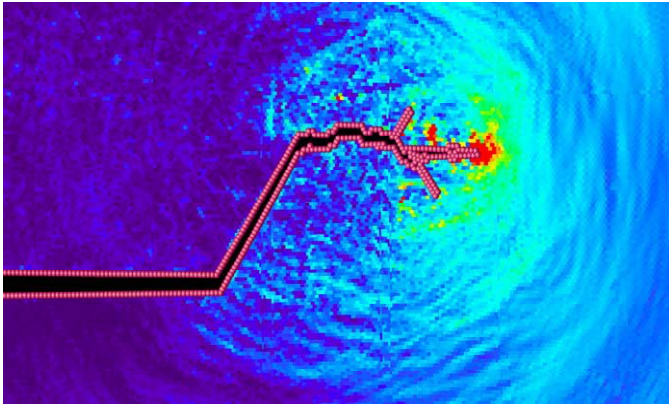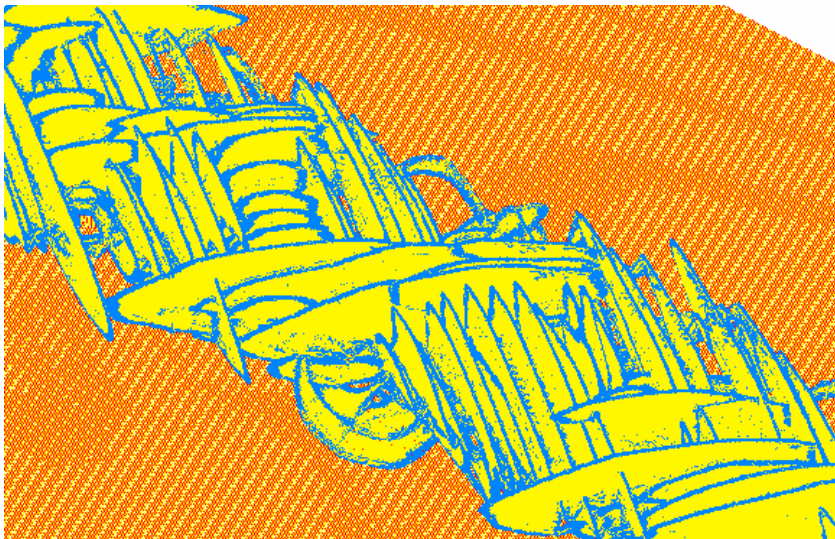
Mechanics of biological materials

(a) Brittle

(b) Ductile

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Example: Stretching of proteins

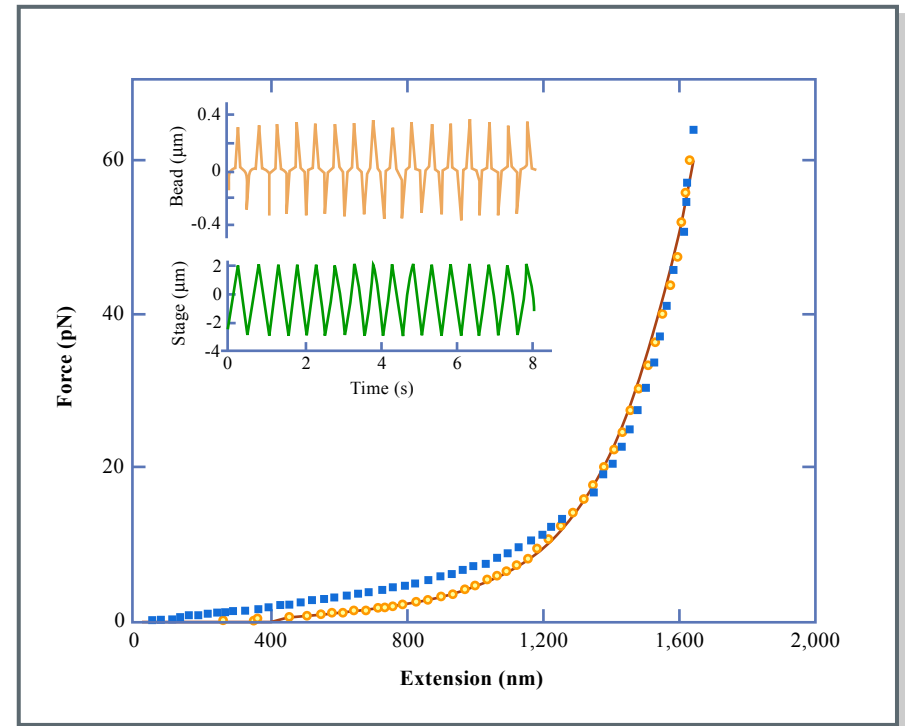Images removed due to copyright restrictions.

Figure by MIT OCW.

Physical origin?
Soft matter
Nonlinear
Viscoelastic (time dependent)
…

Book Chapters

T. Courtney, Mechanical Behavior of Materials
Chapter 2

D.I. Bower
An Introduction to Polymer Physics (Chapter 6 [+7 and 3])

D. Boal
Mechanics of the Cell (Chapters 1+2)

D. Whitford
Proteins – Structures and Function (Chapters 1-4)

Physical fundamentals of elasticity

- **Interactions between atoms, molecules and/or particles govern the elastic response of materials**

- **For a fundamental understanding of elasticity, need to consider atomic bonding (chemistry or quantum mechanics)**

- Elastic response is governed by change of free energy as a function of deformation (yields stress versus strain); second derivative of free energy with respect to strain yields Young's modulus
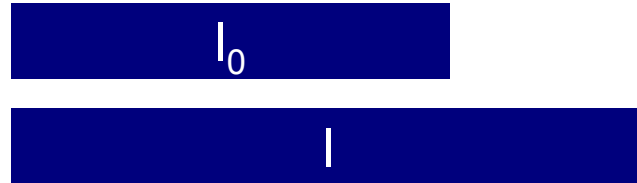
$$F(T,V) = U - TS$$

- Polymers
- Biological structures and materials (proteins, DNA, …)

Extension ratios: Large-strain deformation

$$\varepsilon_{ii} = \frac{\partial u_i}{\partial x_i}$$

l_0

l

Extension ratio

$$\lambda = l / l_0$$

$$\lambda_i = 1 + \Delta_i$$

$$\varepsilon_{ii} = \varepsilon_i = \Delta_i$$

In small-strain elasticity

$$\lambda_i^2 = 1 + 2\Delta_i + \cancel{\Delta_i^2}^{\ 0}$$

$\Delta_i \to 0$ Leads to

(small strains)

$$\lambda_i^2 = 1 + 2\Delta_i$$

λ_i Extension ratios in three directions; pure tensile stress state (directions of principal stress)

Strain energy functions

- Phenomenological theory that uses the concept of strain-energy functions

$$U = f(\lambda_i)$$

- Shear strains are assumed to be zero (coordinate system of principal stresses)

- *U* is (physically) **not** equivalent to the thermodynamic internal energy function;
instead it is a function that maps changes in entropy and internal energy into a mathematical function (phenomenological model) – **free energy density!**

- **Examples**: Rivlin, Neo-Hook, …

Example: Neo-Hookean Solid

$$U = C\left(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3\right)$$

Express "strain energy density" as a function of extension ratios λ_i

C is a constant (parameter) that is related to the
Young's modulus (will be derived later)

The SED function is used to calculate the stress for a given
deformation state
(note: 2nd derivatives=modulus=not constant!!)

Uniaxial deformation

Incompressible (retain volume during deformation)

$$U = C\left(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3\right)$$

Orthogonal to pulling direction:

$$\sigma_1 = \sigma_2 = 0 \qquad \sigma_3 = \sigma$$

$$\underbrace{\lambda_1 \lambda_2 \lambda_3 = 1 \qquad \lambda_1 = \lambda_2 \qquad \lambda_3 = \lambda}$$

$$\lambda_3 = \lambda_2 = 1/\sqrt{\lambda}$$

λ_3

$$U = C\left(\lambda^2 + 2/\lambda - 3\right) \quad \text{for uniaxial tension}$$

$$dU/d\lambda = 2C\left(\lambda - 1/\lambda^2\right) = \sigma$$

Then, the relation between extension ratio and stress can be written as

$$\sigma = 2C(\lambda^2 - 1/\lambda)$$

Consider the nominal stress force/unit area of the undeformed medium:

$$\sigma = 2C(\lambda - 1/\lambda^2) \qquad \text{For small strains:} \quad \lambda_i = 1 + \varepsilon_i$$

$$\sigma = 2C((1+\varepsilon)^2 - 1/(1+\varepsilon)) \approx 2C[\cancel{1} + \cancel{2}\varepsilon - (\cancel{1} - \cancel{\varepsilon}) + O(0)]$$
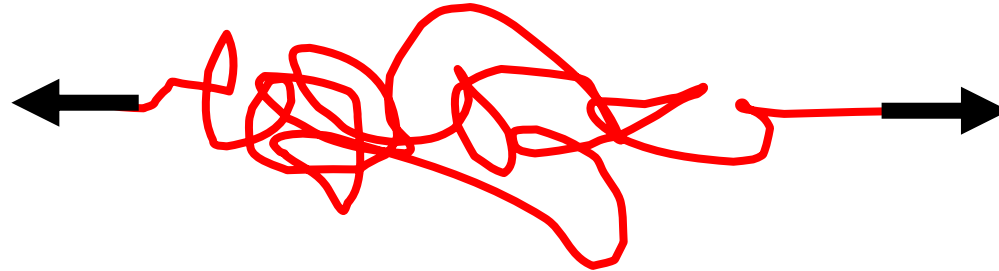
3

$$\boxed{\sigma = 6C\varepsilon = E\varepsilon} \quad \text{Hooke's law (\textit{linear} elasticity)} \quad \boxed{C = E/6}$$
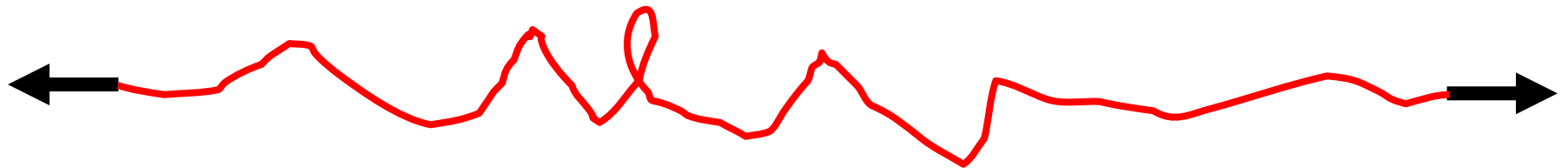
Entropic change as a function of stretch

High entropy

Low entropy
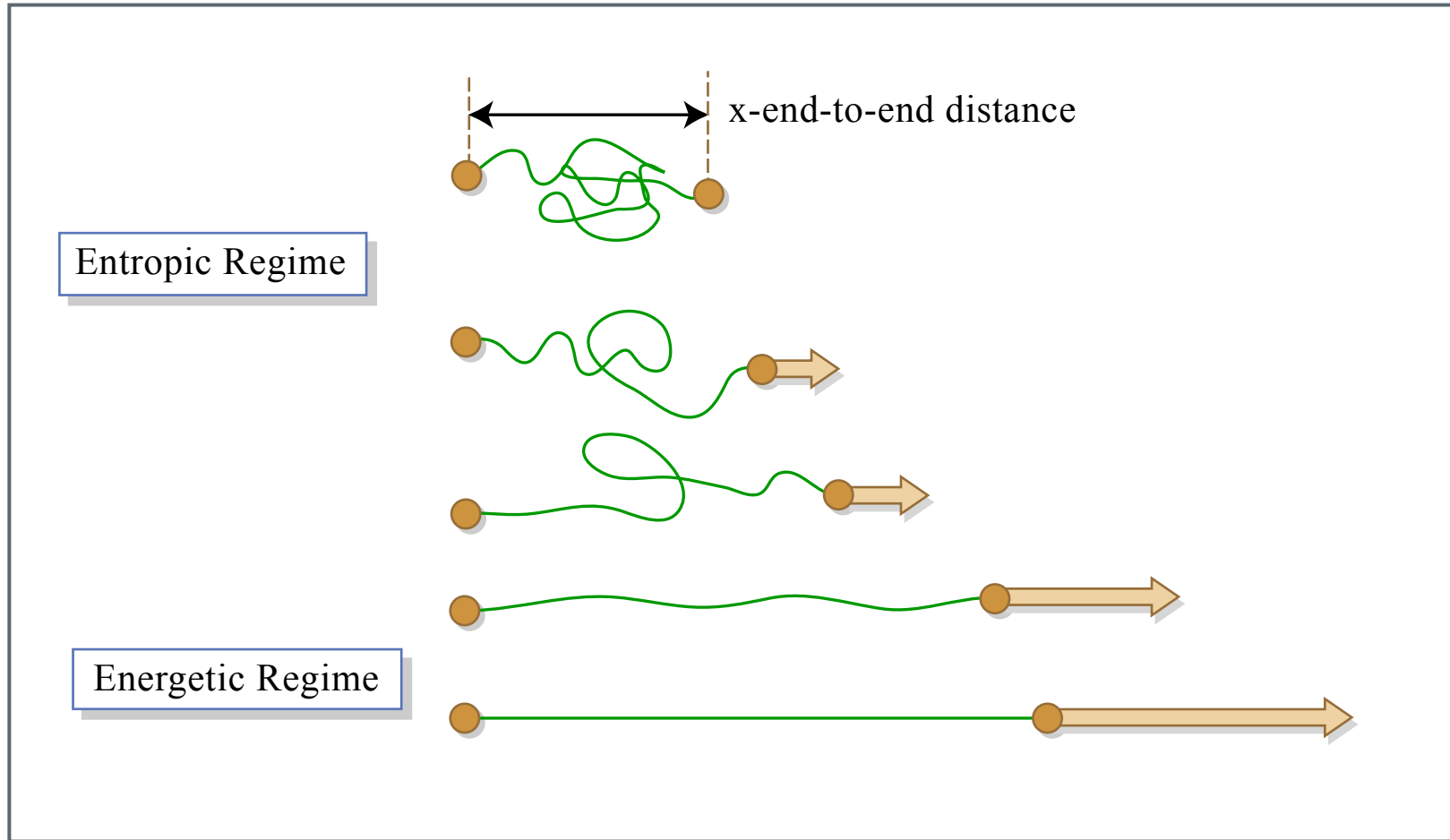
Entropic change as a function of stretch

Figure by MIT OCW.

- Based on the assumption that the rubber entropy S can be calculated in terms of

$$\lambda = l / l_0$$

- For example, it can be shown that

$$U = C\left(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3\right) \qquad C = E / 6$$

$$\sigma_n = C(\lambda - 1/\lambda^2)$$

This is the result for a neo-Hookean solid

(see Courtney, p. 64)

Statistical theory of rubber elasticity

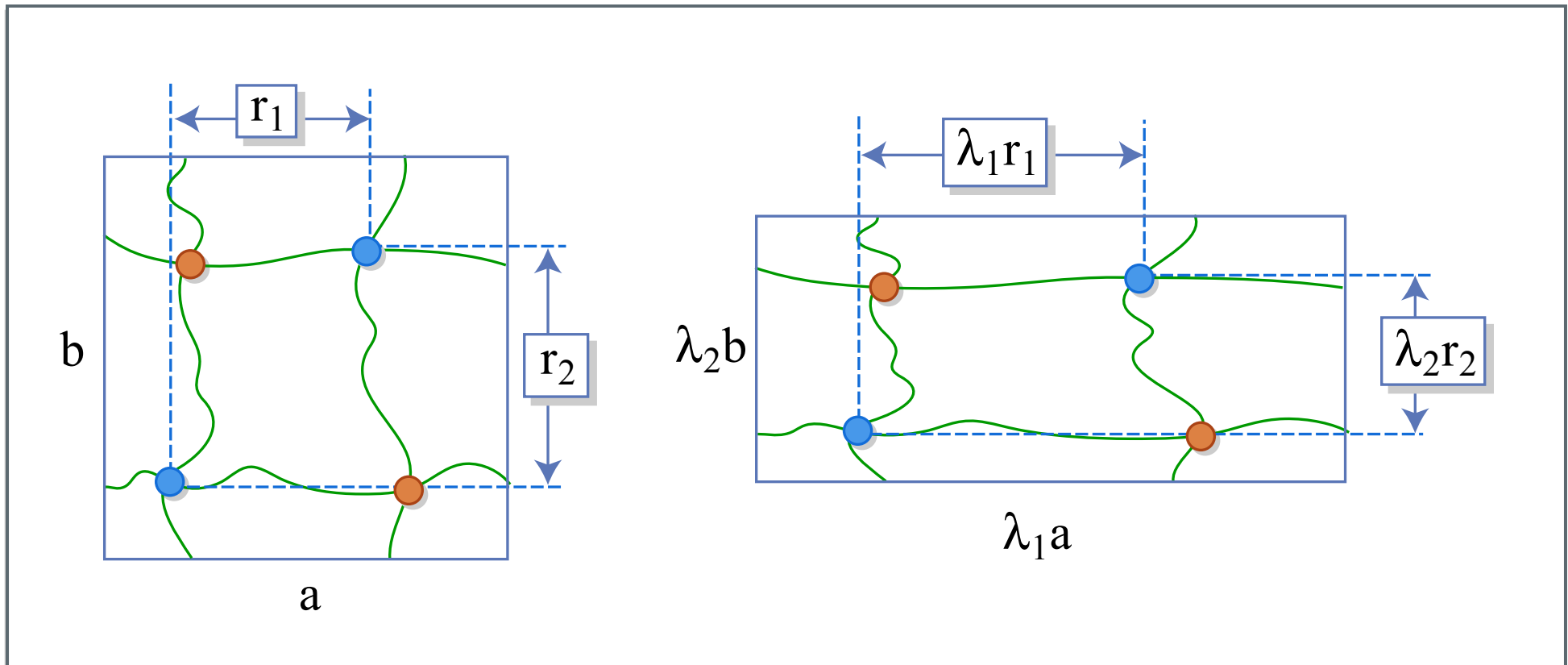Note: No change in elastic energy of molecules

Figure by MIT OCW.

Needed to understand elasticity: Expression of free energy as a function of the applied strain!

Here: Entropic elasticity – therefore change in entropy
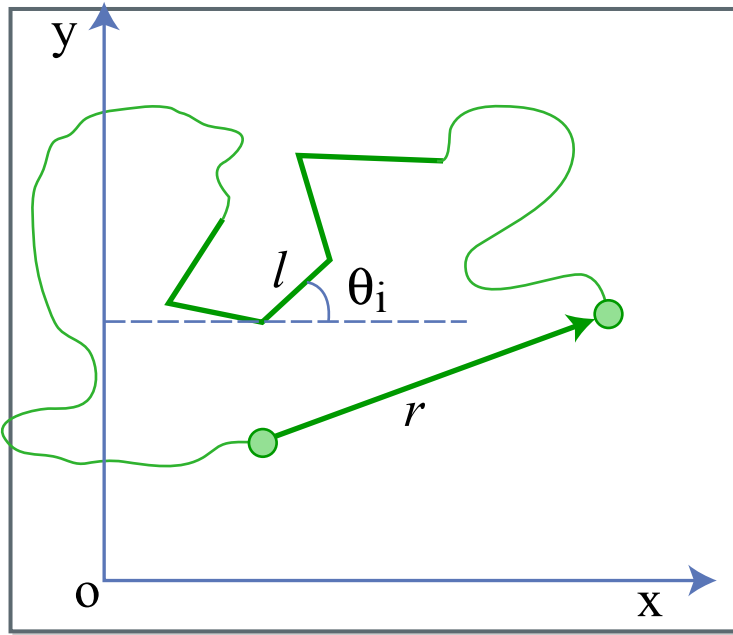
Single freely jointed chain

Figure by MIT OCW.

$$S = c - kb^2 r^2$$

$$b^2 = \frac{3}{2nl^2}$$

Total length: $nl = L$

$r^2 = \; <r_b^2> = n \cdot l^2$ RMS length of the chain (no force applied)

$\sqrt{n} \cdot l \; \rightarrow nl$ Maximum extension due to force

Physical meaning of l: Length at which molecular bending is uncorrelated

Freely jointed Gaussian chain with *n* links and length *l* each (same for all chains in rubber)

$$S = c - kb^2 r^2 \quad \text{where} \quad b^2 = \frac{3}{2nl^2} \quad r \quad \text{end-to-end distance of chain}$$
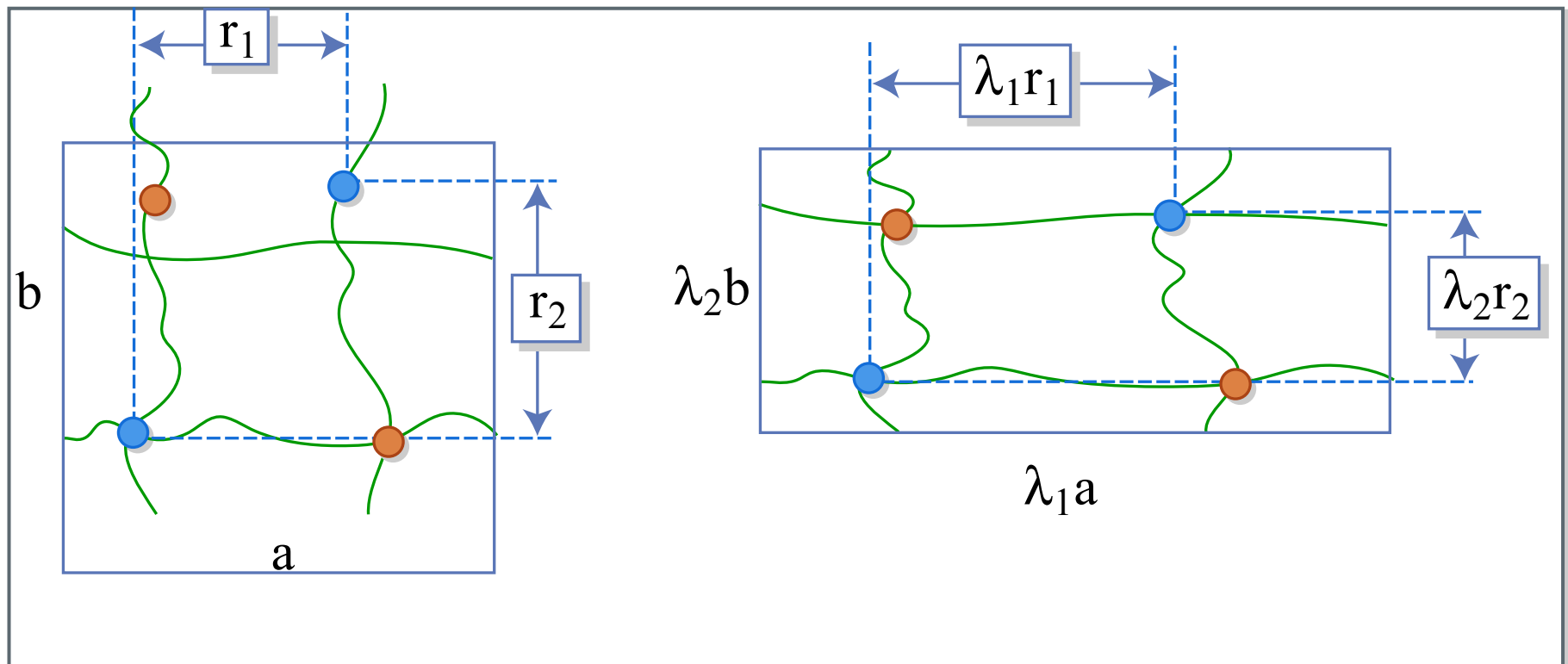
$$\Delta S = -kb^2 \sum_{N_b} \left(\lambda_1^2 - 1\right)x^2 + \left(\lambda_2^2 - 1\right)y^2 + \left(\lambda_3^2 - 1\right)z^2$$

$$\Delta S = -kb^2 \sum_{N_b} \left(\lambda_1^2 - 1\right)x^2 + \left(\lambda_2^2 - 1\right)y^2 + \left(\lambda_3^2 - 1\right)z^2$$

$$\Delta S = -kb^2 N_b \left[\left(\lambda_1^2 - 1\right) < x^2 > + \left(\lambda_2^2 - 1\right) < y^2 > + \left(\lambda_3^2 - 1\right) < z^2 >\right]$$

$< .. >$ Average values over all N_b chains
    – need multiply by N_b to get total energy

Isotropic solid: End-to-end distances of the Nb chains are
directed equally in all directions; therefore

$$< x^2 > = < y^2 > = < z^2 > = \tfrac{1}{3} < r_b^2 >$$
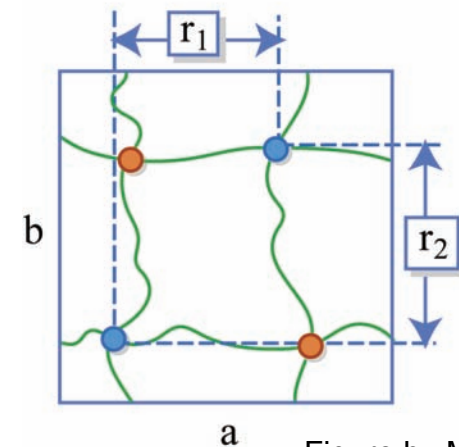
Figure by MIT OCW.

The length $<r_b^2>$ in the unstressed state is equal to the mean square length of totally free chains.

It can be shown that

$$r_{RMS} = \sqrt{n} \cdot l = < \sqrt{<r_b^2>}$$

$$<r_b^2> = n \cdot l^2$$

$$<x^2> = <y^2> = <z^2> = \tfrac{1}{3} n \cdot l^2 = \frac{1}{2b^2}$$

$$\Delta S = -kN_b / 2 \left[(\lambda_1^2 - 1) + (\lambda_2^2 - 1) + (\lambda_3^2 - 1) \right]$$

No explicit dep. on b any more

$$U = -T\Delta S = \tfrac{1}{2} N_b kT (\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3)$$

$$C = E / 6$$

$$U = C(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3)$$

$$\sigma = (E/3)(\lambda^2 - 1/\lambda)$$

For SED: *Free energy density*

Young's modulus

$$U = -T\Delta S = \tfrac{1}{2} N_b kT \left(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3 \right)$$

$$C = E/6$$

Predictions:

$$E = 3N^* kT \quad N^* = N_b / V$$

Stiffness is proportional to temperature

$$E \sim T$$

Stiffness is proportional to degree of cross-linking (for ideal network, N* equals twice the cross-link density)

$$E \sim N^*$$

Summary

- Developed rigorous link extension ratio and elastic properties of rubber-like materials

- Based on statistical theory and by considering changes in entropy due to deformation, arrived at an expression for the Neo-Hookeian solid

- This enables to link cross-linking density and temperature with Young's modulus
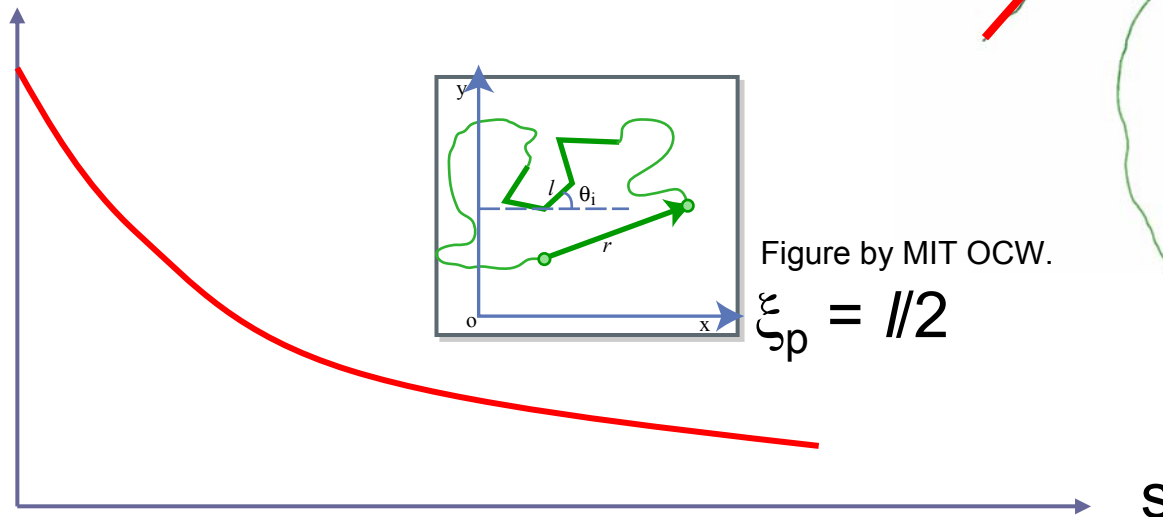
Persistence length

$$\langle \mathbf{t}(s) \cdot \mathbf{t}(s') \rangle = e^{-|s-s'|/\ \xi_p}$$

t(s) tangent slope

Figure by MIT OCW.

$\xi_p = l/2$

The length at which a filament is capable of bending significantly in independent directions, at a given temperature.
This is defined by a autocorrelation function which gives the characteristic distance along the contour over which the tangent vectors **t(s)** become uncorrelated
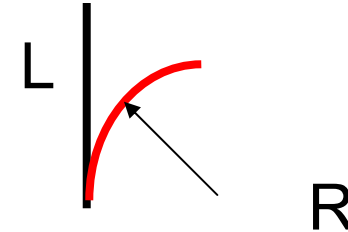
Persistence length: Illustration

- Bending deformation (R=radius, EI=flexural rigidity of the rod) - energy

$$E_{bend} = EI \frac{L}{2R^2}$$

L

R

- Thermal (kinetic) energy per molecule (kinetic theory of gases) - energy

$$E_{kin,mol} = \frac{3}{2} kT$$

- Example: kT~4E-21 J at room temperature

- Persistence length is defined as $\quad \xi_p = \frac{EI}{k\,T}$

(unit: length)

Contour length of molecules

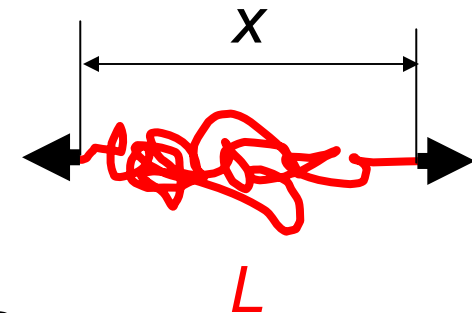- The contour length of a molecule is the total length in the stretched configuration, denoted as L

- When $$L << \xi_p$$

  No energetic interactions!

  a filament appears relatively straight.

- When $$L >> \xi_p$$

  a filament adopts more convoluted shapes

- To pull a highly convoluted molecule apart ($L >> \xi_p$), a force is necessary; define effective spring constant

$$k_{sp} = \frac{3kT}{2L\xi_p} \qquad F \sim k_{sp}x \qquad x << L$$

Worm-like chain model

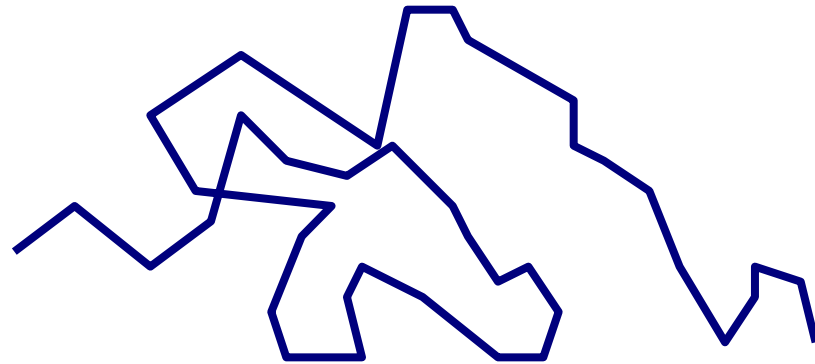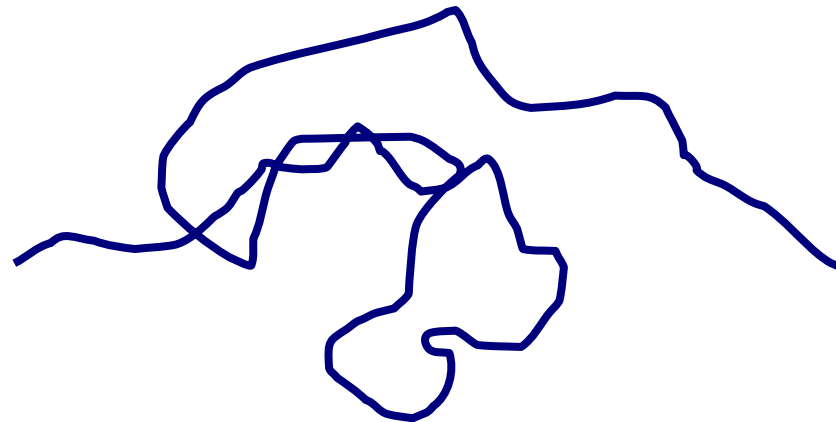**Freely-jointed rigid rods**

Image removed due to copyright restrictions.

Continuously flexible ropes

Worm like chain model

Worm-like chain model

- This spring constant is only valid for small deformations from a highly convoluted molecule, with length far from its contour length

$$x << L$$

- A more accurate model (without derivation) is the Worm-like chain model (WLC) that can be derived from the Kratky-Porod energy expression (see D. Boal, Ch. 2)

- A numerical, approximate solution of the WLC model:

$$F = \frac{kT}{\xi_p} \left(\frac{1}{4} \frac{1}{\left(1 - x/L\right)^2} - \frac{1}{4} + x/L \right)$$

Marko and Siggia, 1995

Proteins

- An important building block in biological systems are proteins

- Proteins are made up of amino acids

- 20 amino acids carrying different side groups (R)

- Amino acids linked by the amide bond via condensation

- Proteins have four levels of structural organization: primary, secondary, tertiary and quaternary

Protein structure

- **Primary structure:** Sequence of amino acids

- **Secondary structure:** Protein secondary structure refers to certain common repeating structures found in proteins. There are two types of secondary structures: <u>alpha-helix</u> and <u>beta-pleated</u> sheet.

- **Tertiary structure:** Tertiary structure is the full 3-dimensional folded structure of the polypeptide chain.

- **Quartenary Structure:** Quartenary structure is only present if there is more than one polypeptide chain. With multiple polypeptide chains, quartenary structure is their interconnections and organization.

```
A A S X D X S L V E
V H X X
```

Images removed due to copyright restrictions.

20 natural amino acids

Images removed due to copyright restrictions.
Table of amino acid chemical structures.
See similar image:
http://web.mit.edu/esgbio/www/lm/proteins/aa/aminoacids.gif.

Hierarchical structure of collagen

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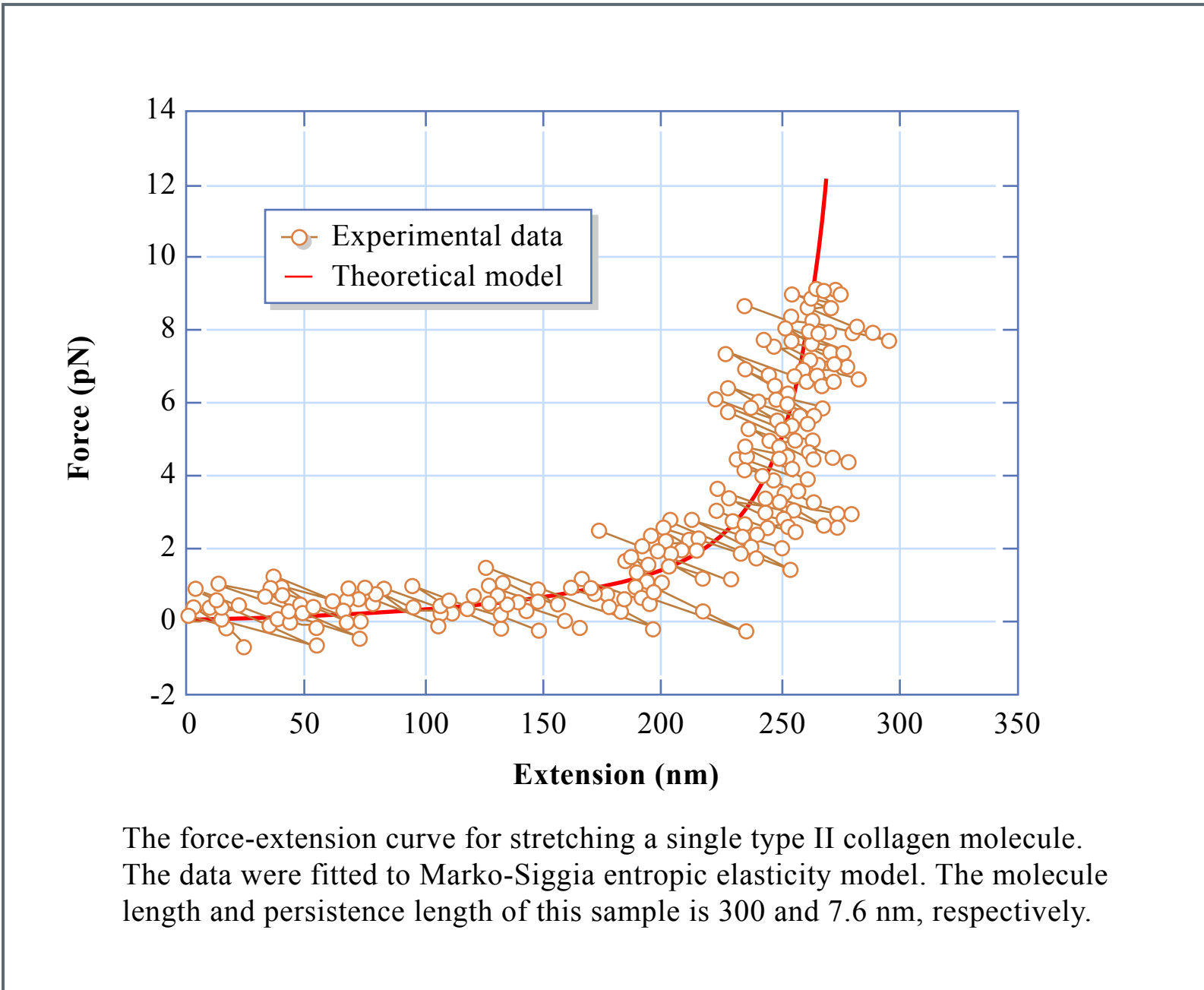Collagen features hierarchical structure

Goal: Understand the *scale-specific properties* and *cross-scale interactions*

Macroscopic properties of collagen depend on the finer scales

Material properties are scale-dependent

(Buehler, JMR, 2006)

Elasticity of tropocollagen molecules

The force-extension curve for stretching a single type II collagen molecule. The data were fitted to Marko-Siggia entropic elasticity model. The molecule length and persistence length of this sample is 300 and 7.6 nm, respectively.

Modeling organic chemistry

Covalent bonds (directional)
Electrostatic interactions
H-bonds
vdW interactions

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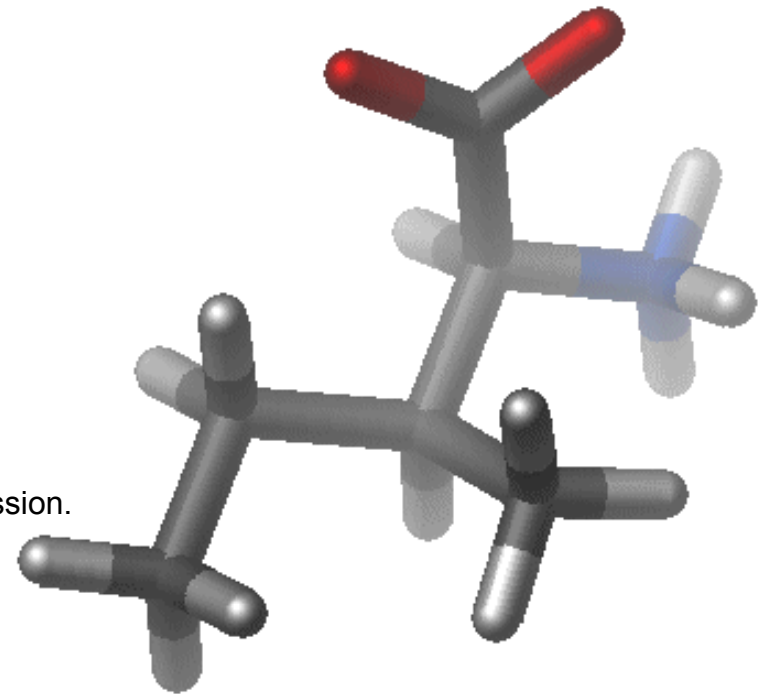
Model for covalent bonds

$$V(R) = E_{bonded} + E_{non-bonded}$$

$$E_{bonded} = E_{bond-stretch} + E_{angle-bend} + E_{rotate-along-bond}$$

Bonding between atoms described as combination of various terms, describing the angular, stretching etc. contributions

Courtesy of the EMBnet Education & Training Committee. Used with permission. Images created for the CHARMM tutorial by Dr. Dmitry Kuznetsov (Swiss Institute of Bioinformatics) for the EMBnet Education & Training committee (http://www.embnet.org)

http://www.ch.embnet.org/MD_tutorial/pages/MD.Part2.html
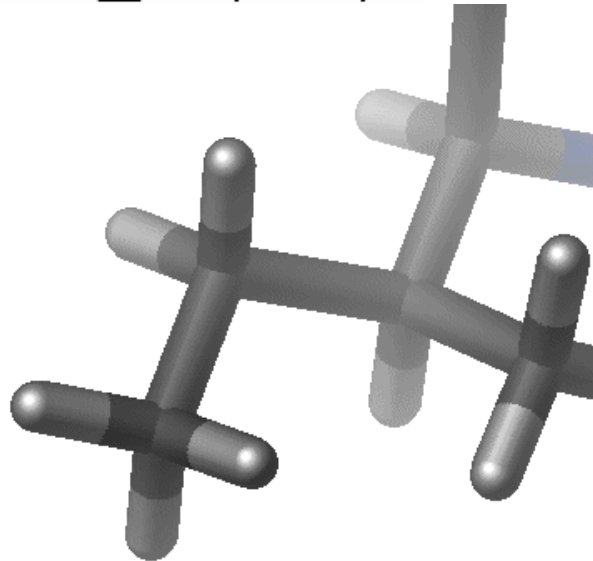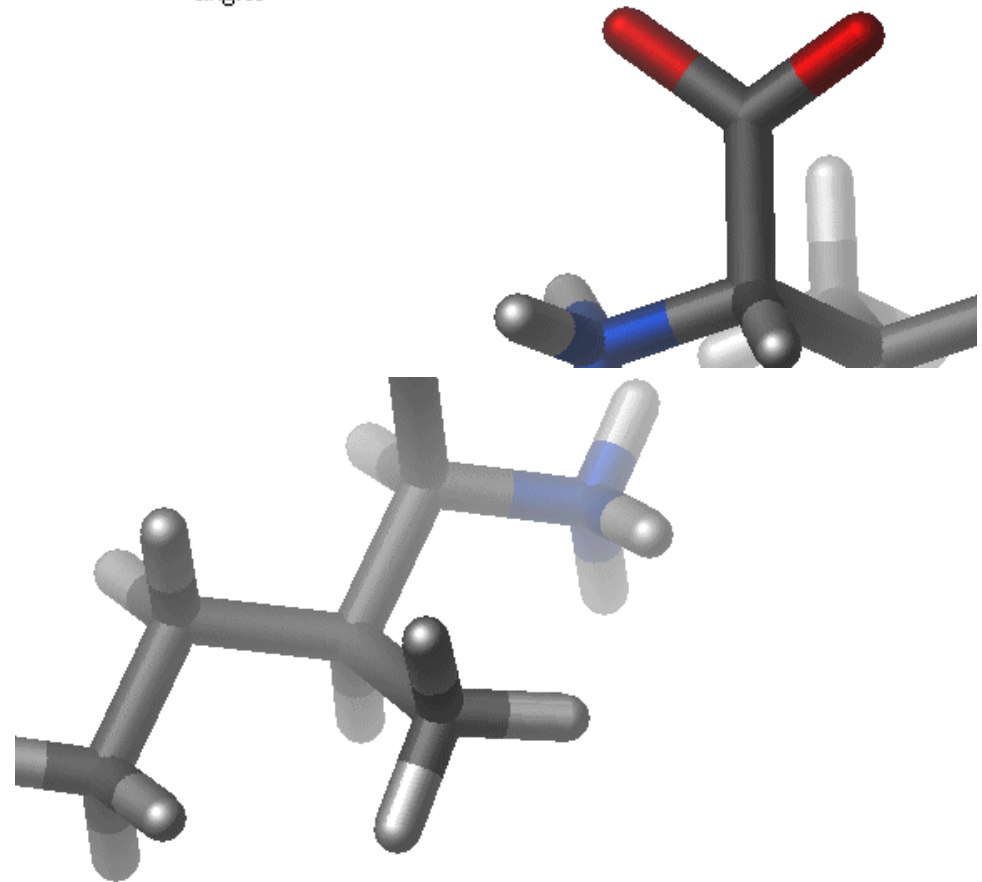http://www.pharmacy.umaryland.edu/faculty/amackere/force_fields.htm

Model for covalent bonds

$$E_{bond-stretch} = \sum K_b (b - b_0)^2$$

$$E_{bond-bend} = \sum_{angles} K_\theta (\theta - \theta_0)^2$$

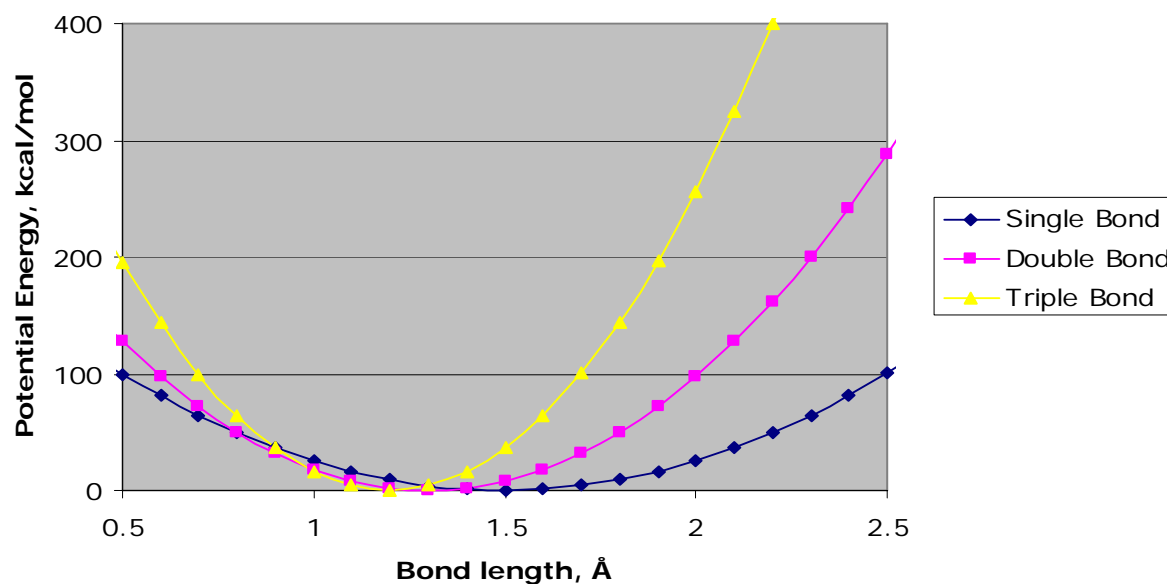$$E_{rotate-along-bond} = \sum_{1,4\,pairs} K_\phi (1 - \cos(n\phi))$$

Review: CHARMM potential

Chemical type	K_{bond}	b_o
C−C	100 kcal/mole/Å^2	1.5 Å
C=C	200 kcal/mole/Å^2	1.3 Å
C≡C	400 kcal/mole/Å^2	1.2 Å

Bond Energy versus Bond length

Different types of C-C bonding represented by different choices of b_0 and k_b;

Need to retype when chemical environment changes

$$V_{bond} = K_b \left(b - b_o \right)^2$$

http://www.ch.embnet.org/MD_tutorial/pages/MD.Part2.html
http://www.pharmacy.umaryland.edu/faculty/amackere/force_fields.htm

Review: CHARMM potential

$$E_{non-bonded} = E_{van-der-Waals} + E_{electrostatic}$$

$$E_{van-der-Waals} = \sum_{\substack{nonbonded \\ pairs}} \left(\frac{A_{ik}}{r_{ik}^{12}} - \frac{C_{ik}}{r_{ik}^{6}} \right)$$

Image removed for copyright restrictions.
See the graph on this page:

http://www.ch.embnet.org/MD_tutorial/pages/MD.Part2.html

$$E_{electrostatic} = \sum_{\substack{nonbonded \\ pairs}} \frac{q_i q_k}{D r_{ik}}$$

Nonbonding interactions

vdW (dispersive)

Coulomb (electrostatic)

H-bonding

UFF "Universal Force Field"

- Can handle complete periodic table

- Force constants derived using general rules of element, hybridization and connectivity

Features:

- Atom types=elements

- Chemistry based rules for determination of force constants

$$E_R = \frac{1}{2}k_{IJ}(r - r_{IJ})^2$$

$$r_{IJ} = r_I + r_J + r_{BO} + r_{EN}$$

Pauling-type bond order correction

$$r_{BO} = -\lambda(r_I + r_J)\ln(n)$$

$$r_{EN} = r_Ir_J(\sqrt{\chi_I} - \sqrt{\chi_J})^2/(\chi_Ir_I + \chi_Jr_J)$$

$$k_{IJ} = \left(\frac{\partial^2 E_r}{\partial R^2}\right)_0 = 2G\frac{Z_I^*Z_J^*}{R^3} = 664.12\frac{Z_I^*Z_J^*}{r_{IJ}^3}$$

Rappé *et al.*

Common empirical force fields

Class I (experiment derived, simple form)

- CHARMM
- CHARMm (Accelrys)
- AMBER
- OPLS/AMBER/Schrödinger
- ECEPP (free energy force field)
- GROMOS

Harmonic terms;
Derived from vibrational spectroscopy, gas-phase molecular structures
Very system-specific

Class II (more complex, derived from QM)

- CFF95 (Biosym/Accelrys)
- MM3
- MMFF94 (CHARMM, Macromodel…)
- UFF, DREIDING

Include anharmonic terms
Derived from QM, more general

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http://www.ch.embnet.org/MD_tutorial/pages/MD.Part2.html
http://www.pharmacy.umaryland.edu/faculty/amackere/force_fields.htm
http://amber.scripps.edu/

Alpha helix and beta sheets

Hydrogen bonding
e.g. between O and H in H_2O
Between N and O in proteins…

Images removed due to copyright restrictions.

Image removed due to copyright restrictions.
See:
http://www.columbia.edu/cu/biology/courses/c2005/images/3levelpro.4.p.jpg

Image removed due to copyright restrictions.

Unfolding of alpha helix structure
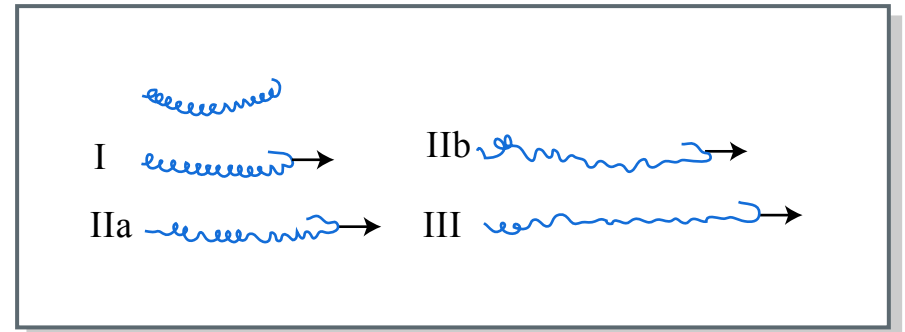
Figure by MIT OCW.
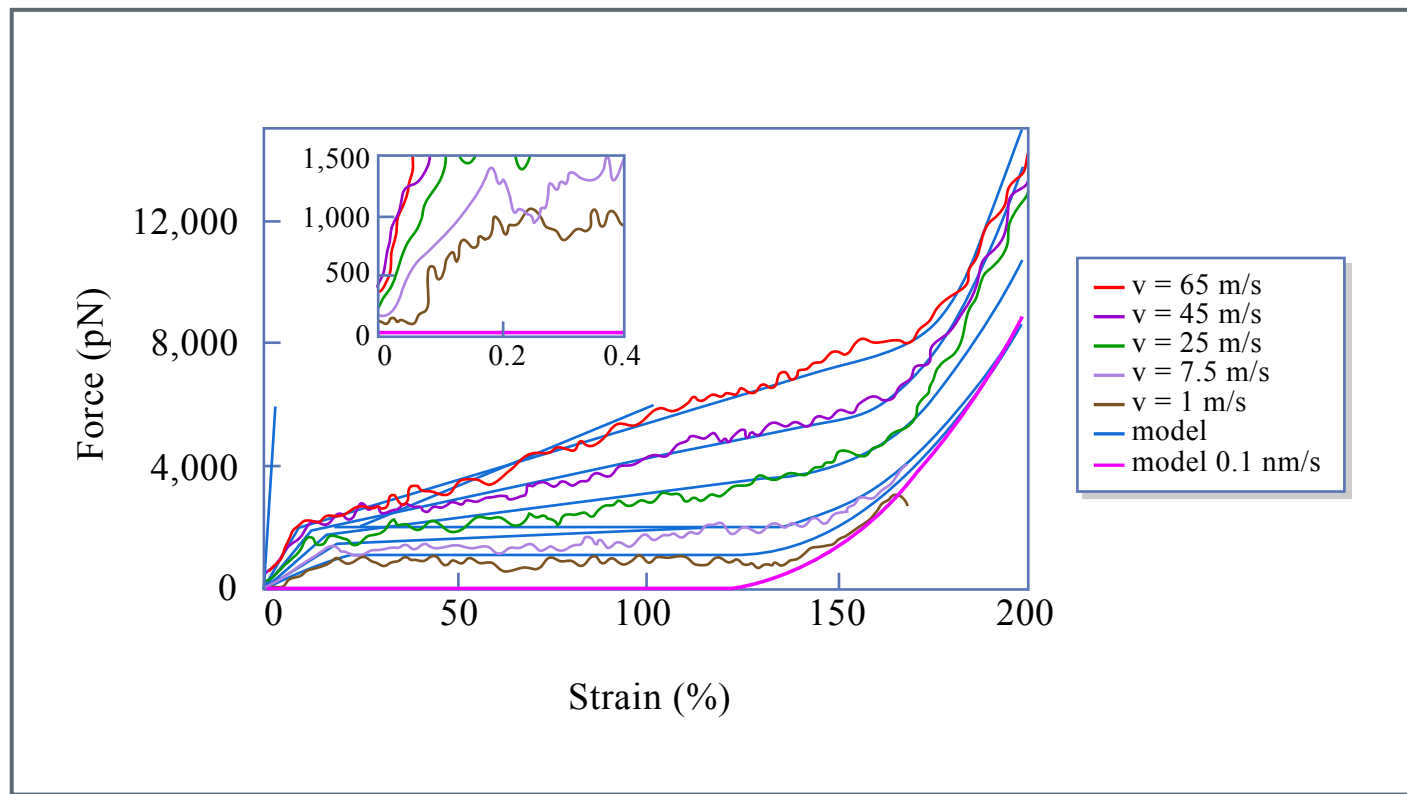
Unfolding of beta sheet
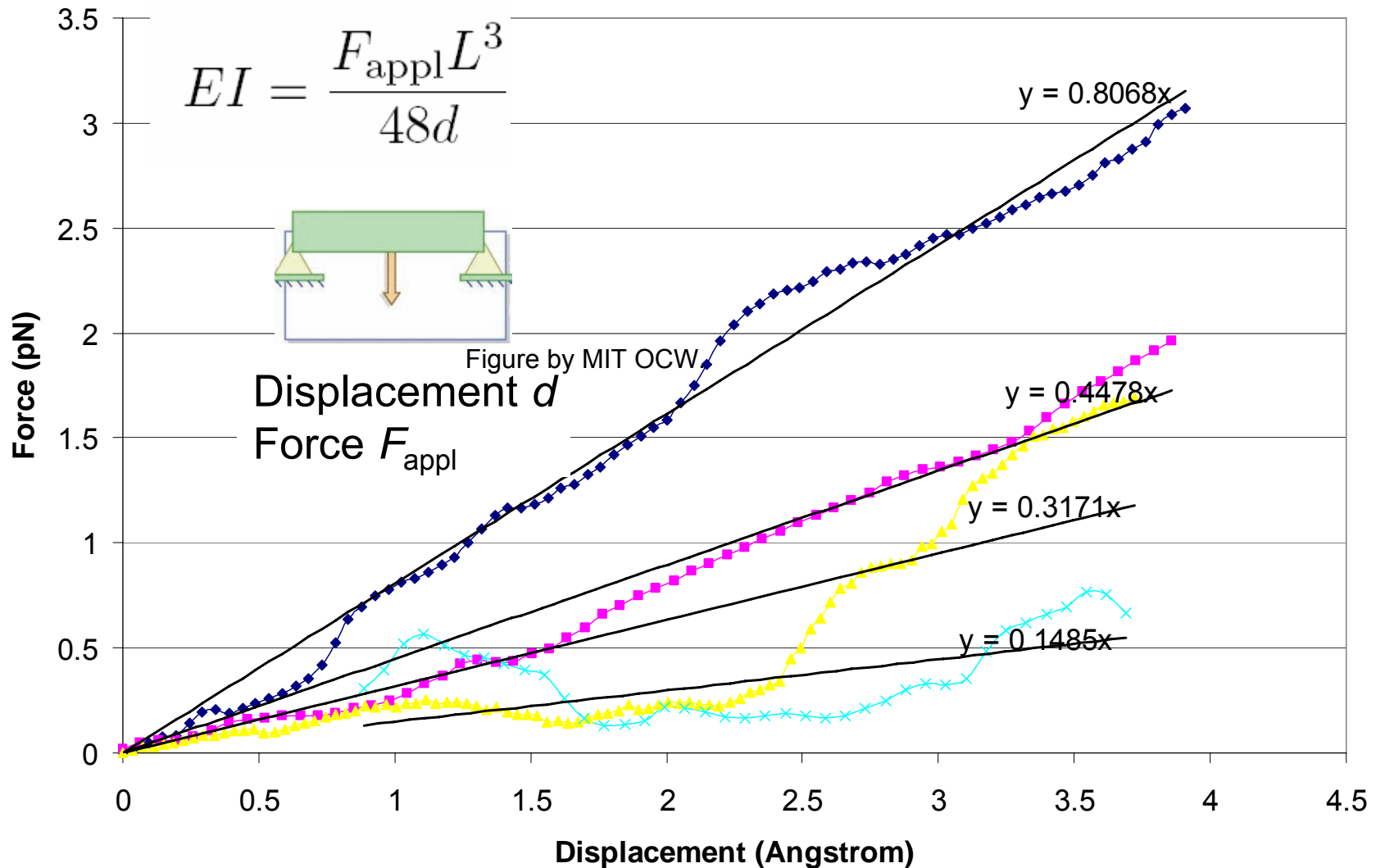
Force - Displacement Curve
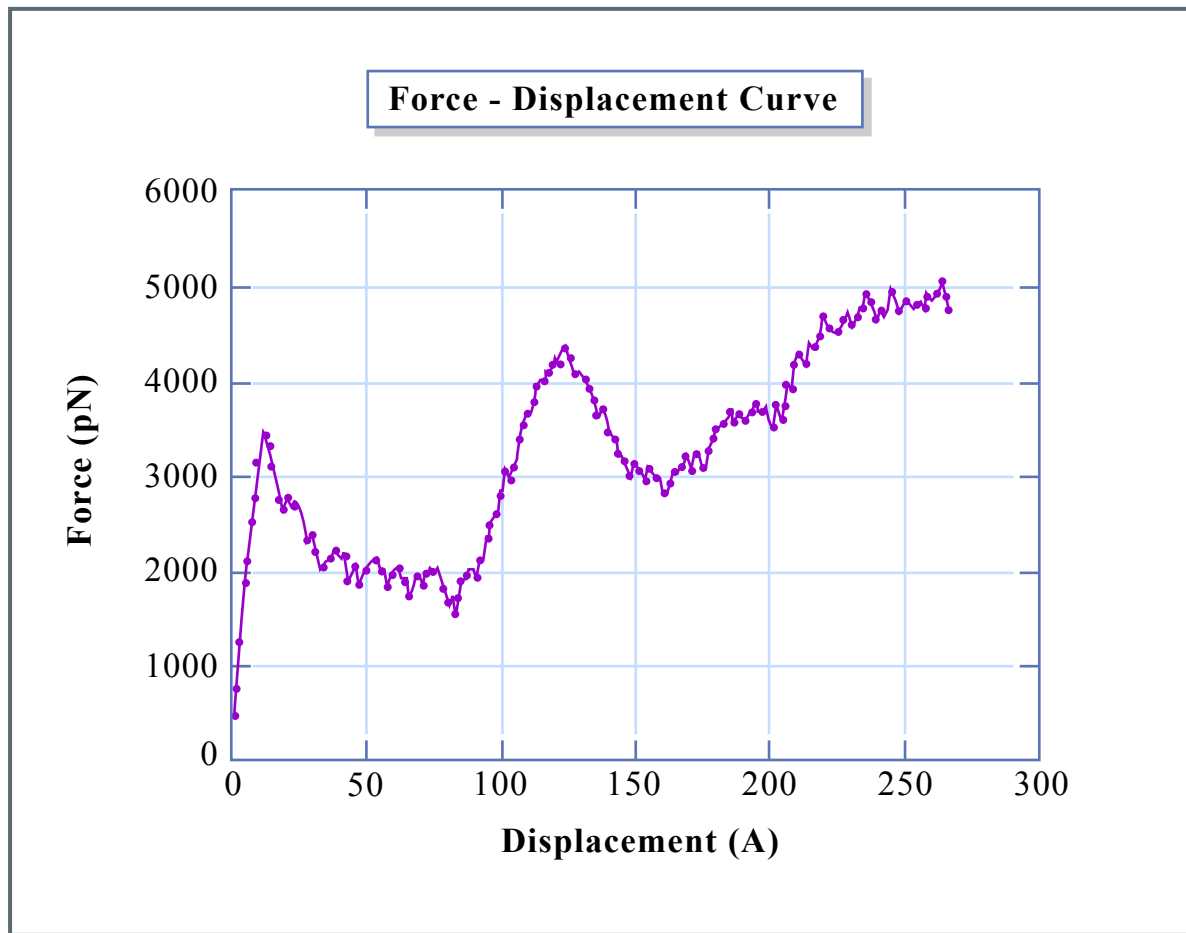
Titin I27 domain: Very resistant to unfolding due to parallel H-bonded strands

Figure by MIT OCW.

Image removed due to copyright restrictions.

Keten and Buehler, 2007

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Three-point bending test: Tropocollagen molecule

$$EI = \frac{F_{\text{appl}}L^3}{48d}$$

Displacement *d*
Force F_{appl}

Figure by MIT OCW.

y = 0.8068x
y = 0.4478x
y = 0.3171x
y = 0.1485x

Force (pN)

Displacement (Angstrom)

Source: Buehler, M. J., and S. Y. Wong. "Entropic Elasticity Controls Nanomechanics of Single Tropocollagen Molecules." *Biophys J* 93 (2007): 37-43.
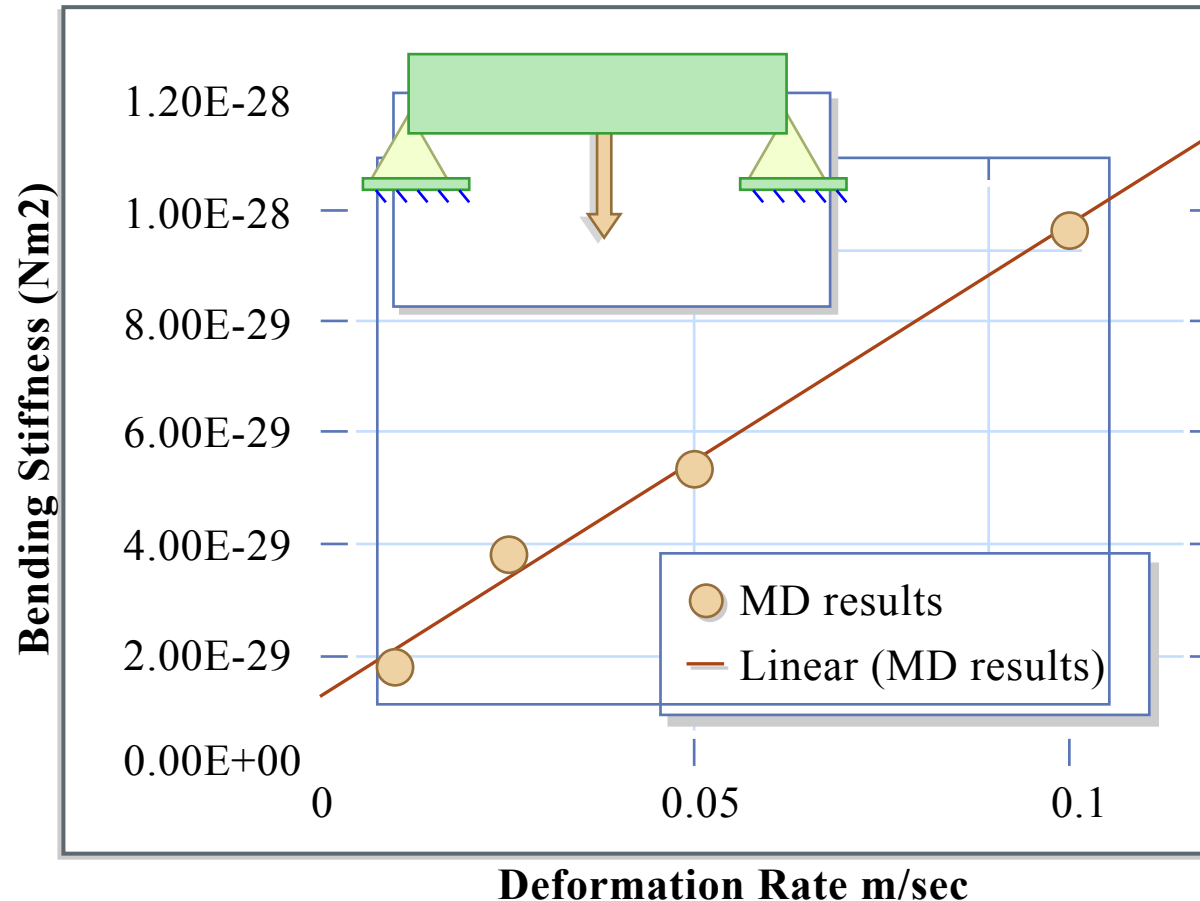
Three-point bending test: Tropocollagen molecule

Figure by MIT OCW.

MD: Calculate bending stiffness; consider different deformation rates

Result: Bending stiffness at zero deformation rate (extrapolation)

Yields: Persistence length – between **3 nm** and **25 nm** (experiment: **7 nm**)

Source: Buehler, M. J., and S. Y. Wong. "Entropic Elasticity Controls Nanomechanics of Single Tropocollagen Molecules." *Biophys J* 93 (2007): 37-43.

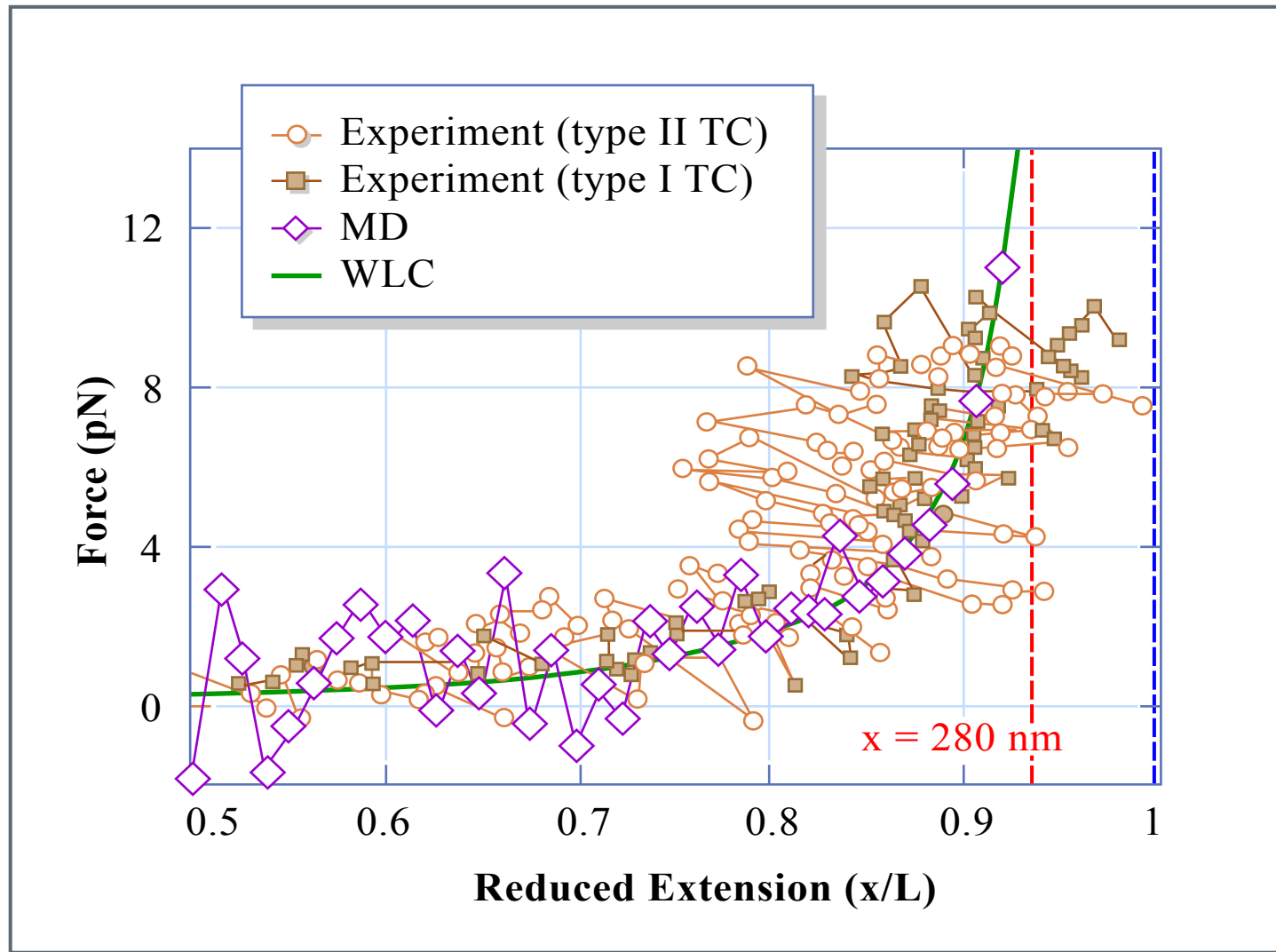# Stretching experiment: Tropocollagen molecule

Figure by MIT OCW. Source: Buehler, M. J., and S. Y. Wong. "Entropic Elasticity Controls Nanomechanics of Single Tropocollagen Molecules." *Biophys J* 93 (2007): 37-43.