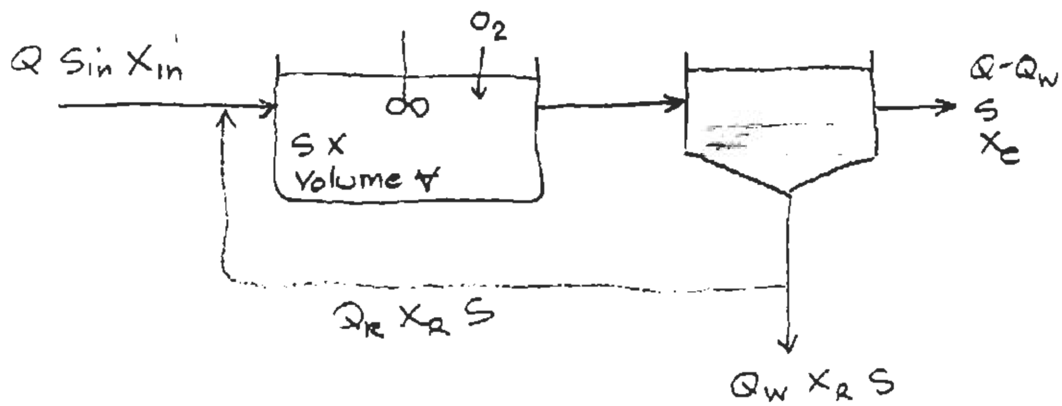
Lecture 17 - Reactor Modeling and Activated Sludge Treatment

Why model ASTs?

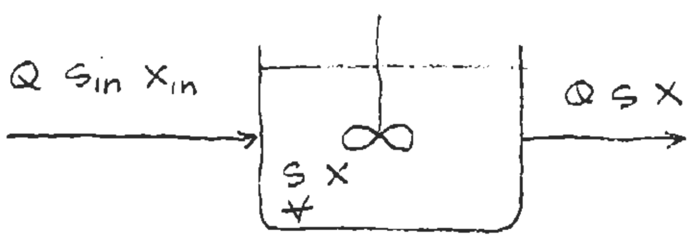
To construct good mass balances over treatment systems for design and operations

Example: activated sludge treatment:



Reference: Alonzo W. Lawrence and Perry L. McCarty, 1970. Unified basis for biological treatment design and operation. Journal of the Sanitary Engineering Division, ASCE. Vol. 96, No. SA3, pp. 757-778. June 1970.

First consider simpler system: FMT with Monod Kinetics



$x_{in} = 0$ $\qquad$ $t_R = \frac{\forall}{Q}$

Cell growth

$$\forall \frac{dX}{dt} = \forall \left(\mu_g X - k_e X \right) - QX \qquad (1)$$

change in biomass — growth — death (endogenous respiration) — outflow

(Note $\mu_g = \mu$ in Reynolds & Richards)

For steady state : $\frac{dX}{dt} = 0$

$$\mu_g X - k_e X = \frac{Q}{\forall} X = \frac{X}{t_R} \qquad (2)$$

Note : $$\mu_g X = - Y r_{su} \qquad (3)$$

r_{su} = substrate utilization rate [M substrate / T] < 0

Y = cell yield [M cells / M substrate]

$$\frac{1}{t_R} = \mu_g - k_e = - \frac{Y r_{su}}{X} - k_e \qquad (4)$$

Also interested in solids retention time or sludge age - time solids (sludge) spend in the system

$$\Theta_c = \frac{\text{total mass of solids in system}}{\text{rate of solids removal from system}}$$

$$\Theta_c = \frac{\forall X}{QX} = \frac{\forall}{Q} = t_R \qquad (5)$$

From above, we have $t_R = \theta_c$

and $$\frac{1}{t_R} = \mu_g - k_e$$

$$\therefore \quad \frac{1}{t_R} = \mu_{max} \frac{S}{S+K_s} - k_e \qquad (6)$$

This is solved for S to yield

$$S = \frac{K_s\,(1 + t_R k_e)}{t_R(\mu_{max} - k_e) - 1} \qquad (7)$$

k_e, K_s, μ_{max} are characteristics of biological population and therefore fixed

Equation 7 therefore makes effluent concentration (the design goal) a function of the retention time (the design variable) (but not of S_{in})

We can also look at another design variable, the specific substrate utilization rate

$$U = -\frac{r_{su}}{X} = \frac{\text{mass of substrate used / unit time}}{\text{mass of cells}} \qquad (8)$$

(U is called food-to-microbe ratio $\frac{F}{M}$ in book)

Note that $$r_{su} = -\frac{\mu_g X}{Y}$$

$$= -\mu_{max} \frac{S}{S+K_s} \frac{X}{Y}$$

$$\therefore \quad U = \frac{\mu_{max}}{Y} \frac{S}{S+K_s} \tag{9}$$

This can be solved for S:

$$S = \frac{K_s U Y}{\mu_{max} - UY} \tag{10}$$

As with Eq 7, Eq 10 gives effluent conc as a function of biological population characteristics (μ_{max} and K_s) and a design variable, U the food-to-microorganism ratio

Whether to use θ_c or U as the design variable is a matter of preference. Most designers use θ_c. since it is easier to measure and control

Another important parameter is the treatment efficiency:

$$E \equiv \frac{S_{in} - S}{S_{in}} \tag{11}$$

= fraction of influent conc. removed by treatment (usually expressed in %)

Using Eq. 7 for S:

$$E = \left(1 - \frac{K_s\,(1 + t_R k_e)/S_{in}}{t_R\,(\mu_{max} - k_e) - 1} \right) \tag{12}$$

We might also want to know the biomass concentration X. We can derive an expression from the mass balance for substrate S:

$$\forall \frac{dS}{dt} = Q S_{in} - QS + Y r_{su} \quad (13)$$

change in substrate mass — inflow — outflow — substrate converted to cells ($r_{su} < 0$)

$$\frac{dS}{dt} = \frac{Q}{\forall}(S_{in} - S) - \frac{\mu_g}{Y} X \quad (14)$$

$$= \frac{1}{t_R}(S_{in} - S) - \frac{\mu_g}{Y} X \quad (15)$$

consider steady state, $\frac{dS}{dt} = 0$

$$\frac{S_{in} - S}{t_R} = \frac{\mu_g}{Y} X \quad (16)$$

From Eq. 4, $\mu_g = \frac{1}{t_R} + k_e$

Also, Eq. 7, gives an expression for S

Substitute these into Eq. 16 and solve for X:

$$X = Y \left[\frac{S_{in}}{1 + K_e t_R} - \frac{K_s}{\mu_{max} t_R - (1 + K_e t_R)} \right] \quad (17)$$

Note that concentration of "bugs" depends on influent substrate conc.

Simple example: $K_s = 100$ mg/L
$S_{in} = 300$ mg BOD/L
$\mu_{max} = 5 \text{ day}^{-1}$
$K_e = 0$ (neglect death, endogenous resp.)

For $t_R = \theta_c = 8$ hr. , $E = 50\%$

$t_R = \theta_c = 16$ hr. , $E = 86\%$

→ Efficiency is highly dependent on residence time for wastewater (t_R) and solids (θ_c)

Note the special case in which the reactor achieves zero treatment and $S = S_{in}$

From Eq. 6 $$\frac{1}{t_R} = \frac{1}{\theta_c} = \mu_{max} \frac{S}{S+K_s} - K_e$$

With $S = S_{in}$
$$\frac{1}{t_W} = \frac{1}{\theta_{c,W}} = \mu_{max} \frac{S_{in}}{S_{in}+K_s} - K_e \qquad (18)$$

If this is substituted into Eq. 17, it predicts:

$X = 0$ no "bugs"

This is known as the "wash-out" condition – bacteria are washed out of the reactor before they can grow

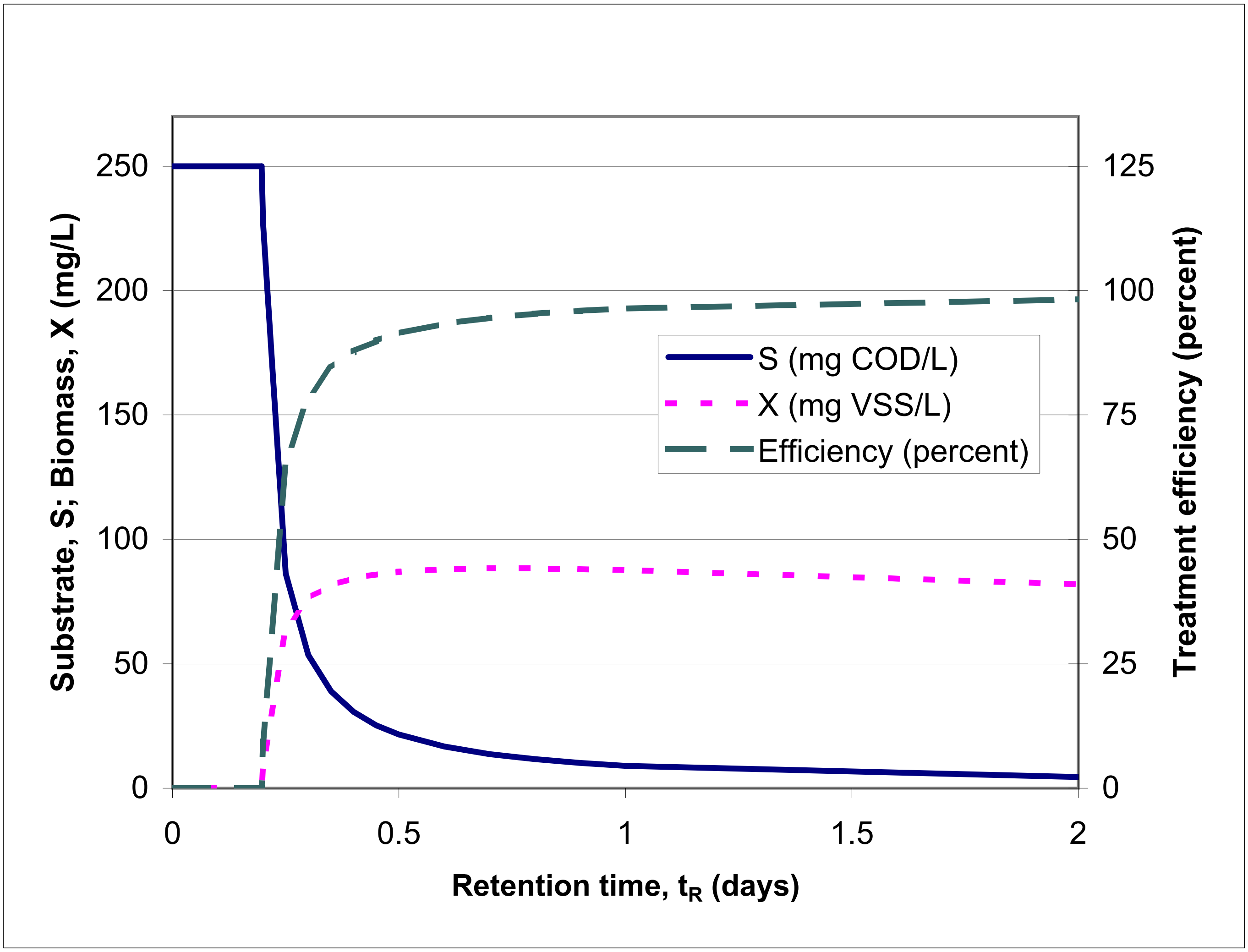
Graph on pg 8 shows S, X, and E vs. $t_R = \theta_c$ for typical parameter values:

K_s = 40 mg COD/L
K_e = 0.1 day^{-1}
μ_{max} = 6 day^{-1}
S_{in} = 250 mg COD/L
Y = 0.4 mg VSS/mg COD

Observations:

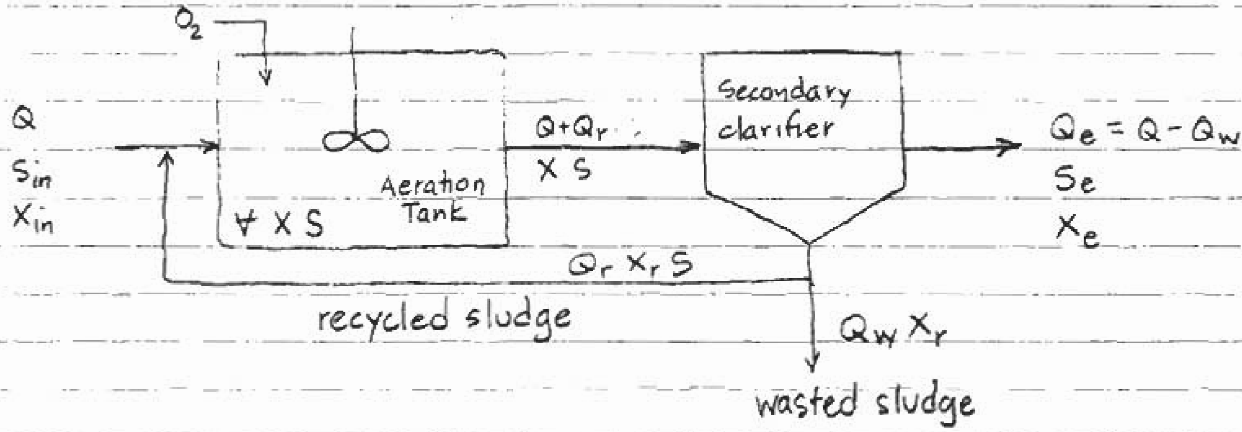
t_W = 0.19 days – no treatment, no biomass for $t_R < t_W$

Efficiency continues to improve with t_R, but improvement is marginal after $t_R \sim 1$ day



250
200
150
100
50
0
125
100
75
50
25
0
0
0.5
1
1.5
2
Substrate, S; Biomass, X (mg/L)
Treatment efficiency (percent)
Retention time, t_R (days)
S (mg COD/L)
X (mg VSS/L)
Efficiency (percent)

What if cells get recycled?



X = cell concentration (e.g. mg VSS/L) ~ 2000-4000 $\frac{mg\ VSS}{L}$

S = substrate conc. (e.g. mg COD/L) ~ 220 $\frac{mg\ BOD5}{L}$

X_e ~ 15 mg VSS/L $\qquad X_r$ = 4000 - 12000 mg VSS/L

Above conceptual model assumes that only cells X are settled in secondary clarifier - whatever substrate S is left is soluble and does not settle

Implies all solids are cells

Overall mass balances:

$$\underset{}{\forall \frac{dX}{dt}} = \underbrace{QX_{in}}_{\text{inflow}} - \underbrace{Q_e X_e}_{\text{outflow}} - \underbrace{Q_w X_r}_{\text{wasted}} + \underbrace{\forall \mu_g X}_{\text{growth}} - \underbrace{\forall k_e X}_{\text{death}} \tag{19}$$

At steady state with $X_{in} = 0$

$$\forall(\mu_g - k_e)X = Q_e X_e + Q_w X_r = \underbrace{P}_{\text{sludge production}} \tag{20}$$

once again, we want an expression for sludge age

$$\theta_c = \frac{\text{mass of solids (cells) in system}}{\text{rate of solids removal from system}}$$

$$= \frac{\forall X}{Q_W X_r + Q_e X_e} \qquad (21)$$

$$= \frac{\forall X}{Q_W X_r + (Q - Q_W) X_e} \qquad (22)$$

Alternatively, recognize that solids production (cell growth) in system must equal cells removed during steady-state conditions

$$\theta_c = \frac{\forall X}{\forall \left(\mu_{max} \frac{S}{S + K_S} - k_e \right) X} \qquad (23)$$

$$\frac{1}{\theta_c} = \mu_{max} \frac{S}{S + K_S} - k_e \qquad (24)$$

Note that $\theta_c \neq t_R$ — sludge age has been decoupled from hydraulic residence time

Equation 24 can be rearranged to solve for S

$$S = \frac{K_S (1 + \theta_c k_e)}{\theta_c (\mu_{max} - k_e) - 1} \qquad (25)$$

Eq 25 is identical to Eq 7 for the fully-mixed tank <u>except</u> t_R is replaced by θ_c

Replacing t_R with Θ_c is a significant change

One can now design for a large Θ_c to get high efficiency of substrate removal (i.e. low S) independently of tank volume

Tank volume is expensive, so activated sludge recycle can save money

Typical designs: Θ_c = 4 to 10 days
t_R = 4 to 10 hours

→ 24-fold savings in tank size

Note as with FMT, S is not a function of S_{in}

Can again determine U - substrate utilization rate

$$U = \frac{\text{substrate used for cell growth / unit time}}{\text{unit mass of cells}}$$

$$= \frac{Q(S_{in}-S)}{\forall X} = \frac{S_{in}-S}{t_R X} \qquad (26)$$

Or alternatively,

$$U = \frac{\forall \; \frac{1}{Y} \; \mu_{max}\left(\frac{S}{K_s+S}\right) X}{\forall X}$$

$$= \frac{\mu_{max}}{Y}\left(\frac{S}{K_s+S}\right) = \frac{\mu_g}{Y} \qquad (27)$$

Units for U $\left[\frac{\text{M substrate}}{\text{M cells} \cdot \text{T}}\right]$ e.g. $\frac{\text{g COD}}{\text{g VSS - day}}$

Eq 27 can be used to find S as function of U:

$$S = \frac{U Y K_s}{\mu_{max} - UY} \qquad (28)$$

Can also determine Food : Microorganism (F/M) ratio, but note confusion in literature regarding $\frac{F}{M}$

Reynolds & Richards define

$$\left[\frac{F}{M}\right]_{R/R} = \frac{\text{substrate removal}}{\text{unit mass cells - time}}$$

$$= \frac{Q\,(S_{in} - S)}{\forall X} = U \qquad (29)$$

Metcalf & Eddy define

$$\left[\frac{F}{M}\right]_{M\&E} = \frac{\text{substrate inflow}}{\text{unit mass of cells - time}}$$

$$= \frac{Q\,S_{in}}{\forall X} = \frac{S_{in}}{t_R X} \qquad (30)$$

since $E = \left(\frac{S_{in} - S}{S_{in}}\right)$

$$\left[\frac{F}{M}\right]_{M\&E} = \frac{U}{E}$$

$$= \frac{1}{E}\left[\frac{F}{M}\right]_{R/R} \qquad (31)$$

Most references use the Metcalf & Eddy formula (Eq 30) for F:M

Some additional equations for θ_c are useful for design:

From prior equations, we have:

$$\frac{1}{\theta_c} = \mu_{max} \frac{S}{S+K_s} - k_e \qquad (24)$$

$$\mu_{max} \frac{S}{S+K_s} = UY \qquad (27)$$

$$\therefore \quad \frac{1}{\theta_c} = UY - k_e \qquad (32)$$

$$U = E \frac{F}{M} \qquad (31)$$

$$\therefore \quad \frac{1}{\theta_c} = E \frac{F}{M} Y - k_e \qquad (33)$$

Also, by definition of θ_c as sludge retention time, we can define sludge production rate P as:

$$P = \frac{X\forall}{\theta_c} = \frac{\text{mass of cells in system}}{\left(\dfrac{\text{mass of cells in system}}{\text{rate of solids removal from system}}\right)} \qquad (34)$$

The concentration of recycled sludge, X_r, is a property of the sludge determined by a lab test for the sludge density index (SDI) or sludge volume index (SVI)

$$X_r = SDI = \frac{1}{SVI}$$

One of the factors in design is how much sludge is recycled, Q_r

Consider the mass balance over the secondary clarifier only

$$(Q+Q_r)X = Q_e X_e + Q_w X_r + Q_r X_r \qquad (35)$$

inflow — sludge in effluent — wasted sludge — recycled sludge

Define recycle ratio $r = \frac{Q_r}{Q}$

$$Q_e X_e + Q_w X_r = (Q+Q_r)X - Q_r X_r$$

$$\frac{Q_e X_e + Q_w X_r}{Q} = (1+r)X - rX_r \qquad (36)$$

From Eq. 21 $\quad \theta_c = \frac{\forall X}{Q_w X_r + Q_e X_e} \qquad (21)$

$\therefore$

$$\theta_c = \frac{\forall X}{Q[(1+r)X - rX_r]}$$

$$= \frac{t_R}{1+r-r(X_r/X)} \qquad (37)$$

By managing r and X_r/X, we can achieve $\theta_c > t_R$ and improve treatment over simple FMT

Note: R/R use $\Theta = t_R$

t_R applies to aeration tank only

Equation for X comes from mass balance for S:

$$\forall \frac{dS}{dt} = QS_{in} - QS + \forall r_{su} = 0 \quad \text{for steady state}$$

(same as Eq 13 for FMT)

Since $r_{su} = \frac{\mu_g}{Y} X$: substrate utilization rate

$$\frac{1}{t_R}(S_{in} - S) = \frac{\mu_g}{Y} X$$

$$= \frac{\mu_{max}}{Y} \frac{S}{K_s + S} X \qquad (38)$$

From Eq. 24

$$\mu_{max} \frac{S}{K_s + S} = \frac{1}{\theta_c} + k_e$$

$$\therefore \quad \frac{1}{t_R}(S_{in} - S) = \frac{X}{Y}\left(\frac{1}{\theta_c} + k_e\right) \qquad (39)$$

or

$$X = \frac{\theta_c}{t_R}\left[\frac{Y(S_{in} - S)}{1 + \theta_c k_e}\right] = \frac{\theta_c}{t_R} Y_{obs} (S_{in} - S) \qquad (40)$$

Eq. 26 gives S for substitution in Eq 40:

$$X = \frac{\theta_c}{t_R} Y \left[\frac{S_{in}}{1 + \theta_c k_e} - \frac{K_s}{\mu_{max}\theta_c - (1 + \theta_c k_e)}\right] \qquad (41)$$

Eq. 40 and Eq. 34 give sludge production rate

$$P = \frac{QY(S_{in} - S)}{1 + \theta_c k_e} \qquad (42)$$

N

Note that Eq 41 helps explain how it is that S can be independent of S_{in}.

The "trick" is that X is not independent of S_{in}, but increases linearly with S_{in}

In essence, X self adjusts to changes in S_{in} such that, for a given θ_c, S is unchanged

Washout condition for FMT with recycle

At washout $\quad S = S_{in}$

Substitute $S = S_{in}$ into Eq. 24

$$\frac{1}{\theta_{cw}} = \mu_{max} \frac{S_{in}}{S_{in} - K_s} - k_e \tag{43}$$

At limit, $S_{in} >> K_s$

$$\left(\frac{1}{\theta_{cw}}\right)_{lim} = \mu_{max} - k_e \tag{44}$$

Another limit is very long θ_c

Take limit of Eq 25 as $\theta_c \to \infty$

$$S = \frac{k_e K_s}{\mu_{max} - k_e} \quad \text{for } \theta_c >> 0 \tag{45}$$

Effluent conc is function only of bugs

Comparison of FMT and AST:

	FMT	AST (FMT with recycle)
t_R	$t_R = \forall / Q$	$t_R = \forall / Q$
θ_c	$\theta_c = \forall / Q = t_R$	$\theta_c = \dfrac{\forall X}{Q_w X_r + (Q - Q_w) X_e} = \dfrac{t_R}{1 + r - r\,(X_r / X)}$
S	$S = \dfrac{K_s\,(1 + t_R k_e)}{t_R\,(\mu_{max} - k_e) - 1}$	$S = \dfrac{K_s\,(1 + \theta_c k_e)}{\theta_c\,(\mu_{max} - k_e) - 1}$
U	$U = \dfrac{\mu_{max}}{Y}\,\dfrac{S}{S + K_s}$	$U = \dfrac{\mu_{max}}{Y}\,\dfrac{S}{S + K_s}$
X	$X = Y\left[\dfrac{S_{in}}{1 + k_e t_R} - \dfrac{K_s}{\mu_{max} t_R - (1 + k_e t_R)}\right]$	$X = \dfrac{\theta_c}{t_R}\,Y\left[\dfrac{S_{in}}{1 + k_e \theta_c} - \dfrac{K_s}{\mu_{max} \theta_c - (1 + k_e \theta_c)}\right]$

N

Evaluation of assumptions:

No biological reactions in secondary clarifier

In other words, should $\forall = \forall_{reactor}$ or
$\forall = \forall_{reactor} + \forall_{clarifier}$?

Answer – if significant reactions were occurring, larger $\forall$ should be used

In fact, biological activity in clarifier is limited and only a fraction of clarifier volume contributes

Can adjust by using $\forall = \forall_{reactor}$

$$K_{e,eff} = K_e \frac{\forall_{reactor} + \forall_{clarifier}}{\forall_{reactor}}$$

assumes endogenous respiration but not growth continues in clarifier

$X_{in} = 0$

Bacteria are present in wastewater, but enteric bacteria, not the bacteria that degrade wastes, so this is a valid assumption

No substrate settles in clarifier

OK if θ_c is long enough to hydrolyze suspended organic matter in waste

$\theta_c \gtrsim 2$ days

Design

Usually have regulatory standard for either S or E
(e.g. $S \le 30$ mg BOD5/L
$E \ge 85\%$)

Design needs to provide safety factor (SF) to ensure regulations are met

$$\frac{\theta_{c,\,design}}{(\theta_{cw})_{lim}} = SF$$

Typical values in practice

Conventional AST -	SF = 10 - 80
High-rate AST	3 - 10
Low-rate AST	> 80

High-rate - closely monitored by skilled operators
Low-rate - "package plants" for small installations with limited operator attention

Characteristics of system that are specified as part of the design are:

Solids retention time, θ_c

Mixed-liquor suspended solids conc., X

Solids retention time affects properties of MLSS (Figure 6.3 from WEF, 2003 - pg 21)

If θ_c is too short, sludge "bulking" occurs either due to too much slime (viscous bulking) or growth of filamentous bacteria (filamentous bulking)
Sludge does not settle well and sludge can be carried to clarifier effluent (i.e. X_e is high)

If θ_c is too long, sludge is "dispersed" or "pin" floc
Sludge particles are too small and do not settle well

Typical SRTs:
(Note: plants typically operate well over range of θ_c)
Typical 3-5 days
Warmer climates (18 to 25°C) 1-3 days
Colder climates (10°C) 5-6 days

MLSS concentration affects treatment efficiency, oxygen transfer efficiency, solids settling, solids recycle ratio

Typical values are shown in Figure 6.4 from WEF, 2003 on page 22

$$\text{Typical value of } X = 2000 \ \frac{\text{mg VSS}}{\text{L}}$$

Note that actual sludge also includes inert (non-biomass) solids which are considered in more sophisticated models

$$X_{Total} \approx 2500 \ \frac{\text{mg TSS}}{\text{L}}$$

21/

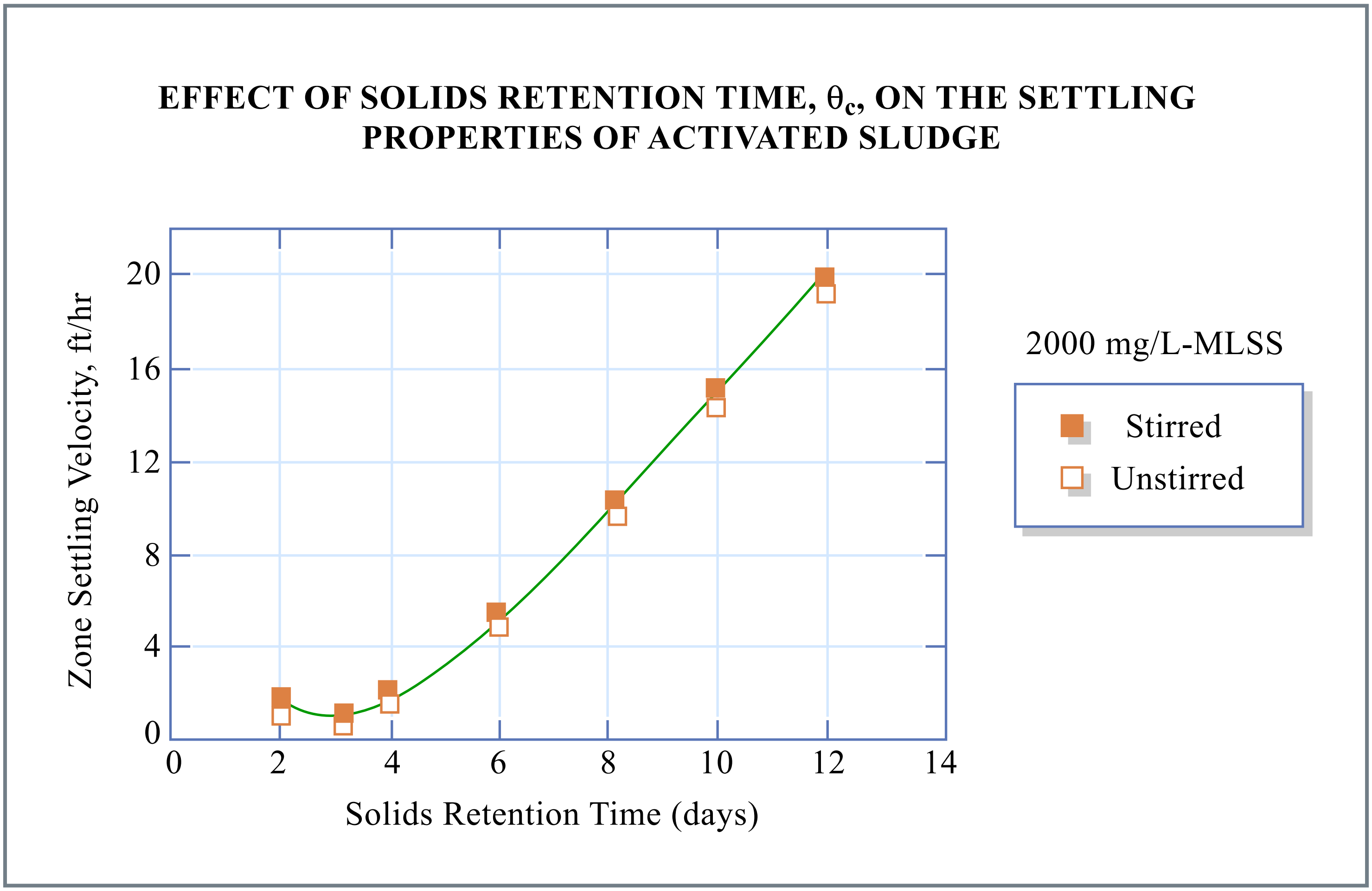


Figure by MIT OCW.

Adapted from: WEF. *Wastewater Treatment Plant Design*. Alexandria, Virginia: Water Environment Federation, 2003.

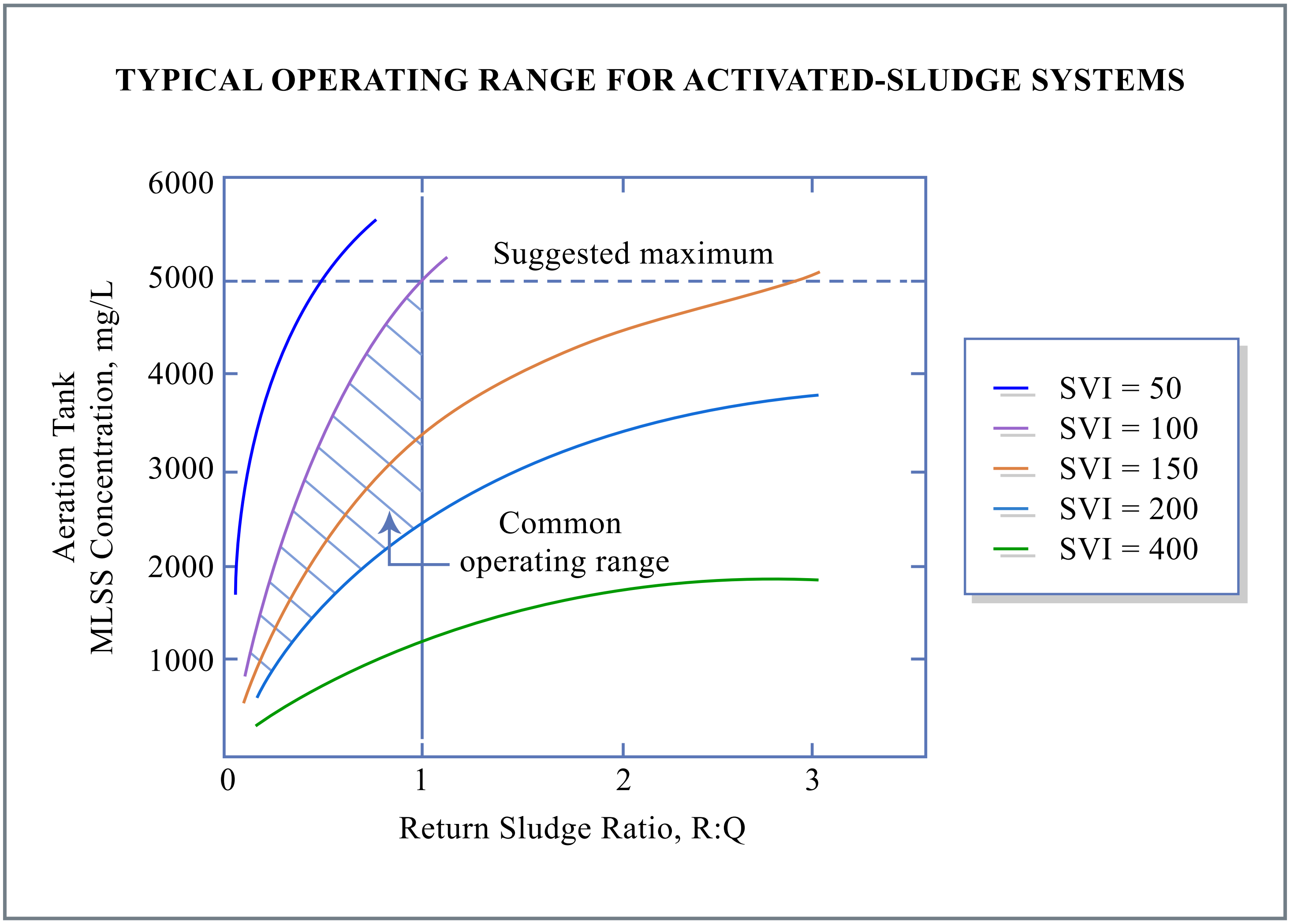


Figure by MIT OCW.

Adapted from: WEF. *Wastewater Treatment Plant Design*. Alexandria, Virginia: Water Environment Federation, 2003.

Use of models in design:

1. Q, S_{in} are given

2. Regulations prescribe S or E: $\quad E = \left(\frac{S_{in} - S}{S_{in}}\right)$

3. Bench-scale studies give Y, K_s, k_e, μ_{max}, SVI

4. SF (or θ_c) and X are design parameters chosen up front

5. $(\theta_{cw})_{lim} = 1/(\mu_{max} - k_e) \qquad \theta_c = SF \cdot (\theta_{cw})_{lim}$

6. Solve Eq 42 for P: $\quad P = \dfrac{QY(S_{in} - S)}{1 + \theta_c k_e}$

7. Solve Eq 34 for $\forall$: $\quad P = \dfrac{X\forall}{\theta_c}$

8. Solve Eq. 25 for S: $\quad S = \dfrac{K_s(1 + \theta_c k_e)}{\theta_c(\mu_{max} - k_e) - 1}$

9. $X_r = 1/SVI$ (discussed in next lecture)

10. Solve Eq. 36 for r: $\quad \theta_c = \dfrac{t_R}{1 + r - r(X_r/X)}$

11. Solve Eq. 20 for X_e: $\quad P = (Q - Q_w)X_e + Q_w X_r$

12. Solve for O_2 demand (discussed in next lecture):

$$R_{O_2} = Q(S_{in} - S) - 1.42P$$