Problem Set #7 3-30, 31, 33, 38, 41 Solutions

30. a) $C_{max} = \frac{M}{\sqrt{4\pi D_L t}}$

$area = \frac{Q}{v} = \frac{1 m^3}{min} \times \frac{sec}{0.2 m} \times \frac{min}{60 sec} = 0.083 m^2$ $M = \frac{moles}{area} = \frac{30 mol}{0.083 m^2} = 360 mol/m^2$

$t = \frac{x}{v} = \frac{500 m}{0.2 m/s} = 2500 s$

plugging it all in:

$D_L = \frac{M^2}{C_{max}^2 4\pi t} = \frac{(360 mol/m^2)^2}{(8 mol/m^3)^2 4\pi (2500 s)} = \boxed{0.064 m^2/s}$

68% of total mass is contained in 2σ (1σ on each side of C_{max})

$\sigma = \sqrt{2D_L t}$

$= \sqrt{2(0.064 m^2/s)(2500 s)} = 18 m$ width = $\boxed{36 m}$

b) find overall k for hydrolysis:

$k = k_a[H^+] + k'_n$

$= \frac{300}{M \cdot sec}(10^{-5.2} M) + \frac{10^{-8}}{sec} = 1.9 \times 10^{-3} s^{-1}$

$C = C_0 e^{-kt}$

$= (8 mol/m^3) e^{-1.9 \times 10^{-3} s^{-1} (2500 s)} = \boxed{0.07 mol/m^3}$

↑ this is peak conc. of sodium at the bridge (i.e. what the peak conc. of metham would have been if hydrolysis wasn't occurring)

c) photolysis: $\frac{C}{C_0} = e^{-0.0001 s^{-1} (2500 s)} = 0.77$

causes decrease of 23% ⇒ can account for the difference

error in amount of metham: not likely, because peak metham conc. is based on the measured sodium conc. and hydrolysis rates — also being off by 3 moles seems like a lot!

chemical analysis: a 10% error in measurement is quite possible, especially at such low concentrations

31. characteristics of aquifer + well.

$b = 4m$ $\quad$ $Q_w = 5000\ L/d$

$n = 0.3$ $\quad$ $q = 5cm/d$ (natural - w/o well)

a) remember that discharge in each streamtube is equal for a properly drawn flow net:

12 streamtubes total that terminate at the well; 4 of these from creek

$$Q_{creek} = \frac{4}{12}(5000\ L/d) \approx \boxed{1700\ L/d}$$

b) idea: calculate minimum travel time from creek to well (which will be for the most direct streamtube) and compare to 3 hours

for any streamtube, $Q_{st} = \frac{5000\ L/d}{12} = 417\ L/d$

① $q = \frac{Q}{A} = \frac{Q}{bw}$ $\quad$ ② $v = \frac{q}{n}$ $\quad$ ③ $t = \frac{x}{v}$

putting it all together:

$$t = \frac{x}{v} = \frac{x \cdot n}{q} = \frac{x \cdot n \cdot b \cdot w}{Q_{st}}$$

x: length of square (distance that the water travels)

w: width of square

for square nearest creek:

$$t_1 = \frac{(2.1m)(0.3)(4m)(1.9m)}{417\ L/d\ (m^3/1000L)} = 11.5\ d$$

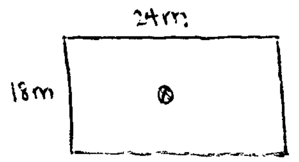
middle square: $x = 1.9m$, $w = 1.2m$, $t = 6.6d$ $\qquad$ $t_{total} = 19.3$ days

square nearest well: $x = 1.0m$, $w = 0.4m$, $t = 1.2d$

$\Rightarrow$ the guest could not have gotten ill due to the metham!

Is it legitimate to use a flow net to analyze a time-varying situation? As long as the hotel's well has been in use for a long time (and we have no reason to think otherwise), a steady-state drawdown has been reached. From a hydraulic point of view, this doesn't change when metham is introduced into the water. The chemical composition of the water may change, but the way the water flows does not change.

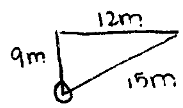
33. a)



drawdown of at least 2.5m throughout $\Rightarrow$

$s = 2.5\text{m}$ at corner

distance to corner:



(conveniently, a 3-4-5 right triangle)

$$s = \frac{Q_w}{2\pi K b} \ln \frac{R}{r}$$

$$Q_w = \frac{2\pi K b s}{\ln(R/r)} = \frac{2\pi (10^{-3}\ \text{m}^2/\text{s})(2.5\text{m})}{\ln(200\text{m}/15\text{m})} = \boxed{6.1 \times 10^{-3}\ \text{m}^3/\text{s}}$$

b) transient, not steady-state $\Rightarrow$ Theis equation

$$s(r,t) = \frac{Q_w}{4\pi T} W(u)$$

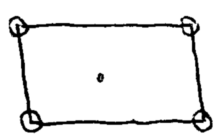
$$u = \frac{r^2 S}{4Tt} = \frac{(15\text{m})^2 (0.3)}{4(10^{-3}\ \text{m}^2/\text{sec})(172{,}800\ \text{sec})} = 0.098 \approx 10^{-1}$$

for $u = 1 \times 10^{-1}$, table 3-2 gives $W(u) = 1.82$

$$s = \frac{20\ \text{L/s}}{4\pi (10^{-3}\ \text{m}^2/\text{s})} \times \frac{\text{m}^3}{1000\text{L}} \times 1.82 = \boxed{2.9\text{m}}$$

* or you can be really good and interpolate between 0.09 and 0.1 to get $W(u) = 1.84$

c)



$s = 2.5\text{m}$ at center

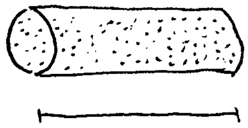
Due to superposition, $s = \frac{2.5\text{m}}{4} = 0.625\text{m}$ (drawdown due to one well)

$$Q_w = \frac{2\pi K b s}{\ln(R/r)} = \frac{2\pi (10^{-3}\ \text{m}^2/\text{s})(0.625\text{m})}{\ln(200/15)} = 1.52 \times 10^{-3}\ \text{m}^3/\text{s}$$

total pumping rate $= 4 \times 1.52 \times 10^{-3}\ \text{m}^3/\text{s} = 6.1 \times 10^{-3}\ \text{m}^3/\text{s}$, same as before

- this is the same as just recognizing that Q_w is proportional to s

38.



2m

$\rightarrow q = 15\ cm/hr$

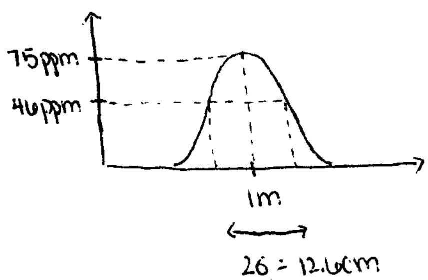
a) need C_{max}, x, σ

$C_{max} = 75\ ppm$

$V = \frac{q}{n} = \frac{15\ cm/hr}{0.3} = 50\ cm/hr$ $\qquad x = \frac{50\ cm}{hr} \times 2hr = 100\ cm$

$D = \alpha \cdot v = 0.2\ cm\left(\frac{50\ cm}{hr}\right) = 10\ cm^2/hr$

$\sigma = \sqrt{2Dt} = \sqrt{2(10\ cm^2/hr)(2hr)} = 6.3\ cm$



1σ away from C_{max},

$C = 0.61\ C_{max} = 46\ ppm$

b) $K_d = f_{oc} K_{oc}$

$= 0.006(25\ mL/g) = 0.15\ mL/g$

$R = 1 + \frac{K_d \rho_b}{n} = 1 + \frac{0.15\ mL/g\ (2\ g/cm^3)}{0.3} = 2$

$V_{eff} = \frac{V}{R} = \frac{50\ cm/hr}{2} = 25\ cm/hr$

This chemical moves slower than the tracer because it spends part of the time "stuck" to the organic carbon.

$x = \frac{25\ cm}{hr} \times 2hr = \boxed{50\ cm}$

c) without hydrolysis, $C_{max}(2hr) = 75\ ppm$ due to dispersion – the only difference between this chemical and the tracer in part a is the hydrolysis.

$k' = k_a[H^+] + k_n' + k_b[OH^-]$ $\qquad pH = 9$

$= 7.8 \times 10^{-2}\ M^{-1}s^{-1}(10^{-9}\ M) + 6.6 \times 10^{-6}\ s^{-1} + 14\ M^{-1}s^{-1}(10^{-5}\ M) = 1.47 \times 10^{-4}\ s^{-1}$

$C = C_o e^{-k't}$

$= (75\ ppm)\ e^{-1.47 \times 10^{-4} s^{-1}(7200s)} = \boxed{26\ ppm}$

41. a) $s = \frac{Q_w}{2\pi K b} \ln\left(\frac{R}{r}\right)$ $R = b\sqrt{K/2N}$ N: recharge

$$= 5m \sqrt{\frac{10^{-4}\ cm/s}{2(80\ cm/yr)} \times \frac{86400\ s}{d} \times \frac{365\ d}{yr}} = 70.2\ m$$

$$Q_w = \frac{2\pi K b s}{\ln(R/r)} = \frac{2\pi(10^{-4}\ cm/s)(5m)(0.3m)}{\ln(70.2m/1m)} \times \frac{m}{100\ cm} = \boxed{2.2 \times 10^{-6}\ m^3/s}$$

b) initial: $C_T = 10^{-3}\ M$, $pH = 4.5$ $\}$ Alk ≈ 0 (Deffeyes)

We want $[H_2CO_3] = 10^{-4}\ M$, because the ionic species will be removed by the filter but H_2CO_3 will not.

- neglecting CO_3^{2-}, so $[HCO_3^-] = C_T - [H_2CO_3]$

$$= 10^{-3}\ M - 10^{-4}\ M = 9 \times 10^{-4}\ M$$

$$\frac{[HCO_3^-][H^+]}{[H_2CO_3]} = 10^{-6.3} \qquad [H^+] = \frac{10^{-6.3}(10^{-4})}{9 \times 10^{-4}} = 5.6 \times 10^{-8} \Rightarrow pH = 7.25$$

final: $C_T = 10^{-3}\ M$, $pH = 7.25$ $\}$ Alk ≈ 9×10^{-4} eq/L

Alk = $C_B - C_A$ so $C_B = \Delta Alk = \boxed{9 \times 10^{-4}\ eq/L}$