Lecture Packet #2: Aquifers, Porosity, and Darcy's Law
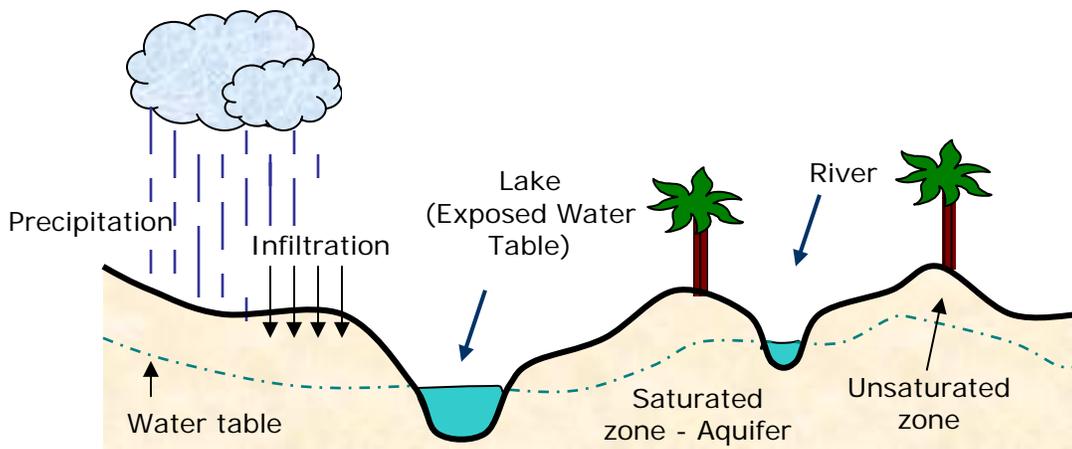

Unsaturated Zone, Vadose Zone, Soil Moisture Zone, Zone of Aeration – rock, water and air

Capillary Fringe – region above water table where water rises due to capillary forces in the porous medium.
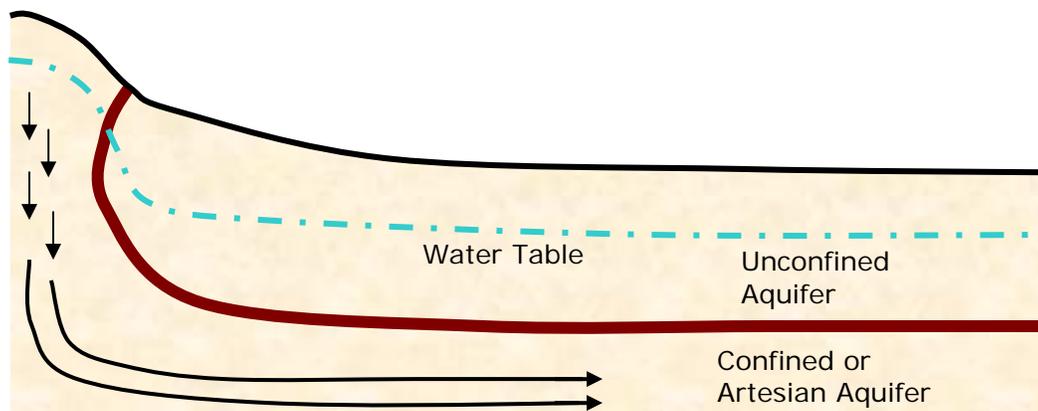
Saturated Zone, Phreatic Zone – rock and water

Water Table
- Top of saturated zone
- Depressed version of topography
- Surface waters are manifestations of the water table – exposed water table

**Aquifer** - a geologic unit that stores and transmits water

Recharge Area

Unconfined Aquifer – water is in contact with atmospheric pressure – drill and well hit the water table

Confined Aquifer – recharge upgradient forces water to flow down and get trapped under an aquiclude. Water is under pressure due to the weight of the upgradient water and the confinement of the water between "impermeable" layers. Water flows to surface under artesian pressure in an Artesian Well.

Aquifer contamination
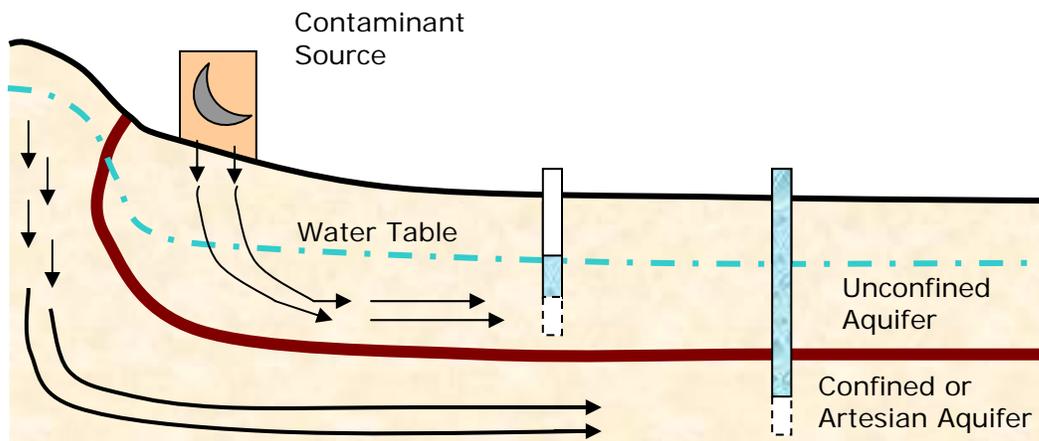
In confined vs. unconfined aquifers

- Although unconfined aquifers are used for water supply, they are often contaminated by wastes and chemicals at the surface.
- Confined aquifers are less likely to be contaminated and thereby provide supplies of good quality.
- Mechanisms of transport are advection and dispersion.
- There can be chemical interactions in aqueous phase or between the water and solid media
- Covered in Contaminant Hydrology course (1.72 is a prerequisite)

In A.D. 1126, in the province of Artois:
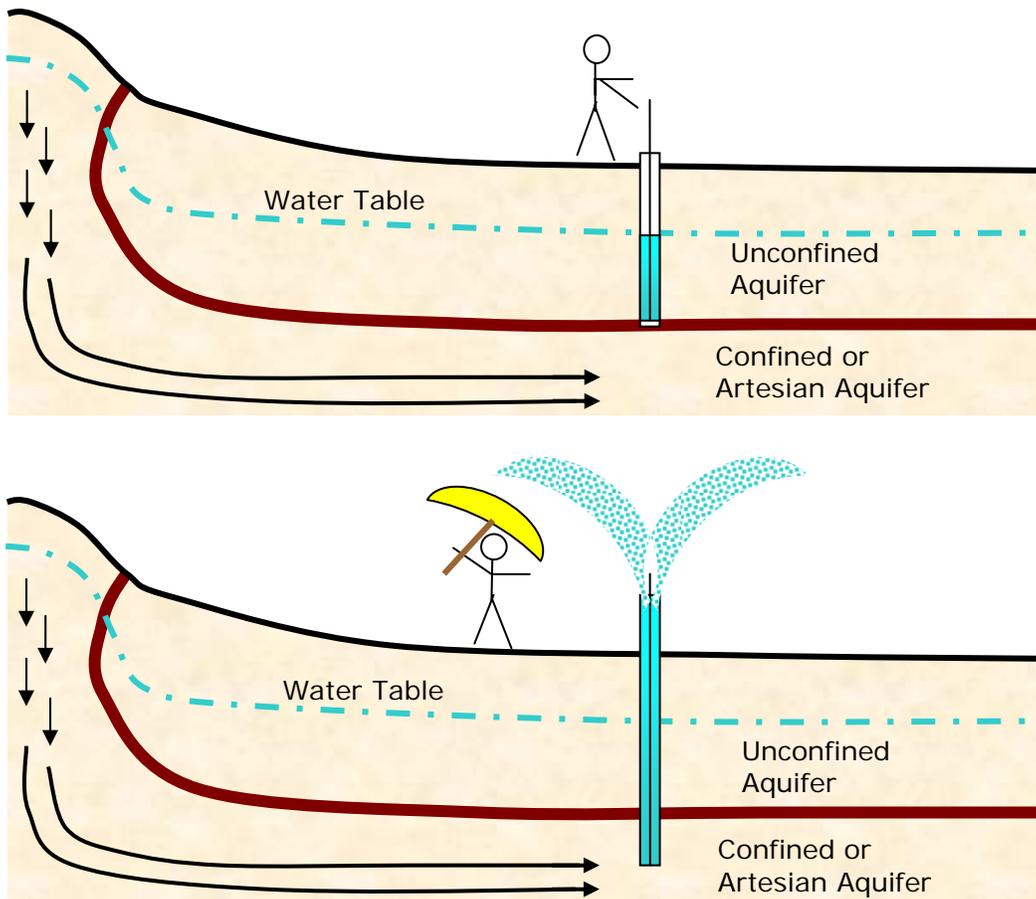
Key Aquifer Properties

Porosity – Percentage volume occupied by voids.   Porosity is independent of scale. For example, a pile of marbles and a pile of beach balls have spherical shape and differing sizes; the porosities are identical due to the similar shaping.
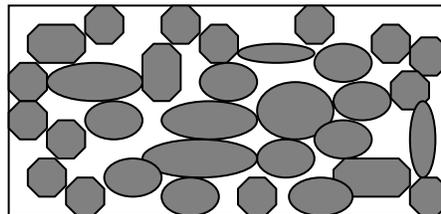
Permeability – Measures the transmission property of the media and the interconnection of the pores. Related to hydraulic conductivity and transmissivity. (more later)
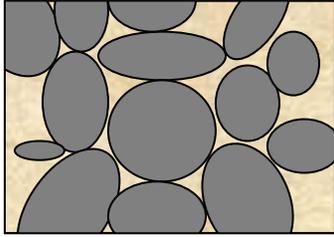
Good aquifer – High porosity + High permeability
- Sand and gravel, sandstone, limestone, fractured rock, basalt
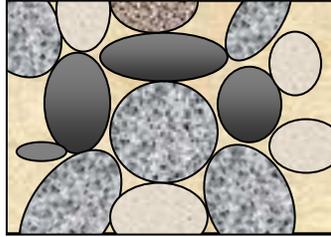
Aqiuiclude, Confining bed, Aquitard – "impermeable" unit forming a barrier to groundwater flow.
- Granite, shale, clay

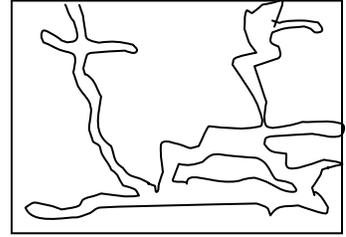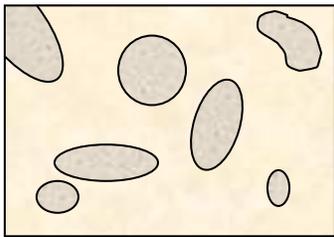**Porosity**

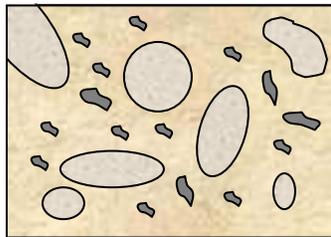|		
Well Sorted	Intragranular	Dissolution
Poorly Sorted	Decreased Porosity by Diogenesis	Fracture
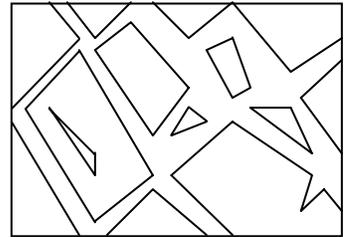
Diogenesis – The formation of rock; pores fill up with precipitations of mineral and reduce porosity

<u>Two Origins of Porosity</u>

Primary
- A function of grain size distribution, also packing
- Decreases with depth – compaction and pressure solution

Porosity vs. Depth Curves

Shales (Athy, 1930)

Sandstones (Blatt, 1979)

Depth (kilometers) — vertical axis: 0, 1, 2, 3, 4, 5, 6, 7

Porosity — horizontal axis: 10, 20, 30, 40, 50
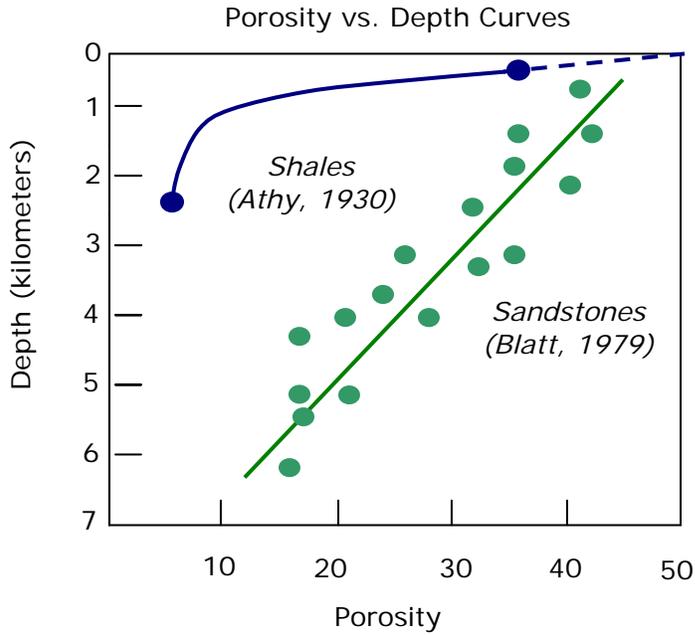
Porosity increases as depth decreases. This is on account of the weight on top of the deeper materials. Porosity also tends to increase with grainsize. Why?

Secondary
- Dissolution
- Fracture

Fracture Number — vertical axis: 0, 100, 200, 300, 400, 500, 600, 700

Quartz SS   **Calcite Cemented SS**   **Limestone**

Lithology
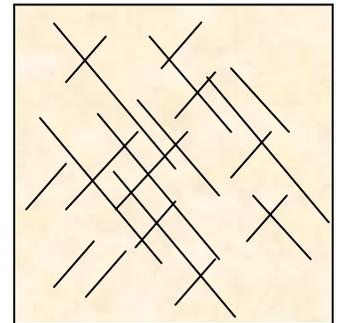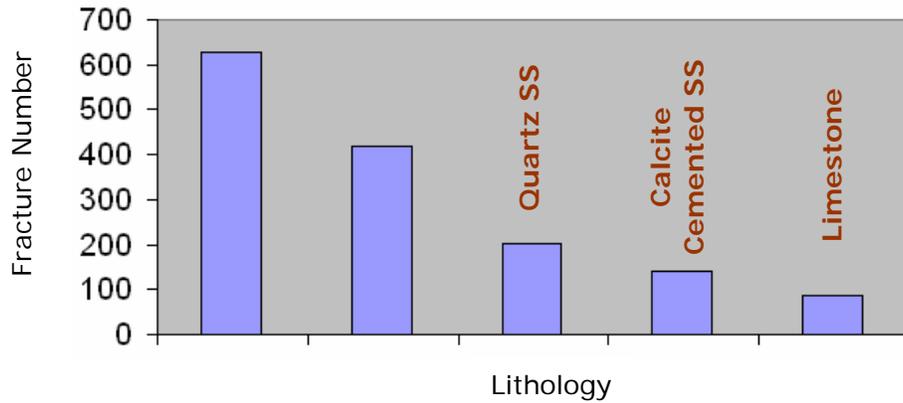
Two types of Porosity
- Intergranular
    - Between grains, mostly part of the effect of porosity, but also dead-end pores
- Intragranular
    - Within grains
    - Usually not considered part of the effective porosity
    - Incredible wide range of widths and length scales
- Simple dichotomous model - dual porosity

Darcy's Law

In 1856 in Dijon, France, Henry Darcy conducted his now famous experiment of pouring water through sediment-packed pipes to see how much would flow through them in a given amount of time [volume of flow per unit time].

Flow through column is Q in L^3/T ⟵ most important quantity

The flow per unit area is **specific discharge**

$$q = \frac{Q}{A}$$ with units of velocity L/T – called **Darcian velocity** or **Darcian flux**, but not actual velocity of the fluid
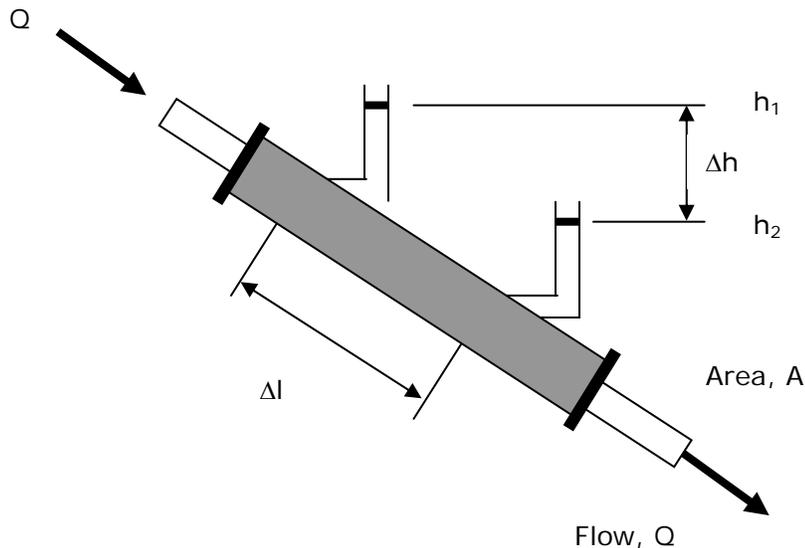
Darcy showed that:

Q is in direction of decreasing head

q is proportional to $h_2 - h_1 = \Delta h$, given Δl fixed, $q \propto (-\Delta h)$
q is inversely proportional to Δl, given Δh fixed, $q \propto (1/\Delta l)$

The proportionality constant is K, and flow is from higher to lower hydraulic head.

$$q = -K\frac{\Delta h}{\Delta l} = -K\frac{dh}{dl} \qquad \longleftarrow \qquad \text{Hydraulic gradient}$$

K is hydraulic conductivity and has units of velocity (L/T). It is a function of both media and fluid.

Q is a flow per unit cross section and is **not** the actual velocity of groundwater flow.

Δh represents the frictional energy loss due to flow through media.

Darcy's law is a macroscopic law. It doesn't tell you about the flow through individual pores.

The discharge is Q in L^3/T

$$q = -K\frac{\Delta h}{\Delta l} = -KiA$$

i is commonly used for gradient.

Note the difference between Q and q!

<u>What is hydraulic conductivity?</u>

K is a property of both media and fluid.

Experiments show: $K = \dfrac{k\rho g}{\mu}$

- $\boldsymbol{K}$ is the intrinsic permeability (L^2), a property of media only

- ρ is the mass density (M/L^3)

- μ is the dynamic viscosity (M/LT) and measures the resistance of fluid to shearing that is necessary for flow

<u>Range of Applicability of Darcy's Law</u>

At extreme gradients some have questioned the applicability of Darcy's Law (controversial)

Low Gradients
- Compacted clays and low gradients
- Threshold gradient to get flow
- Below a certain gradient – nonlinear
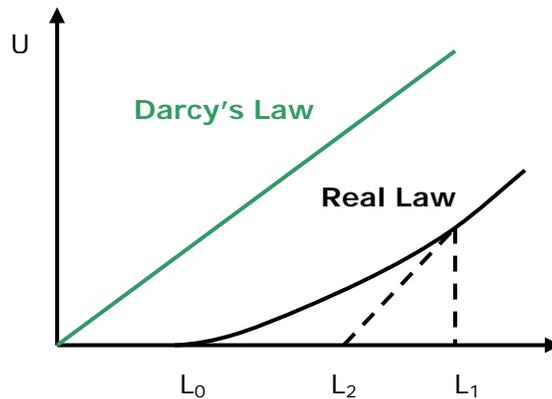
High Gradients

$$\frac{dh}{dl} = c_1 q + c_2 q^2$$

- Term 1 is loss – viscous friction against wall of solids and
- Term 2 is loss – dissipation of kinetic energy in pores – flow converges and diverges.

Must have laminar flow within pores.

$$R_e = \frac{\rho \bar{v} d}{\mu} \equiv \frac{Inertial\ forces}{Viscous\ forces}$$

Laminar in pipes < 2,000 in rocks < 10

Measures of hydraulic conductivity (L/T)
- Commonly cm/s, m/d, ft/d
- Older unit, gpd/ft^2, or meinzer

Measures of permeability, (L^2)

Often the **Darcy Unit** is used, recall

$$q = -\frac{k\rho g}{\mu}\frac{dh}{dl} \qquad\qquad Q = 1\ cm/s = -\frac{k\rho g}{cp}\frac{dh}{dl}$$

1 darcy is the permeability that gives a specific discharge of 1 cm/s for a fluid with a viscosity of 1 cp under a **hydraulic gradient times density times g of 1 atm/cm.**

- It equals about 10^{-8} cm^2
- About 0.831 m/d at 20°

Important – A typical aquifer measure of the transmission property of media for the flow of water is given over a thickness, b

Transmissivity – T = Kb [L^2/T] ⟵ 2D

- Very common quantity for site and regional studies
- Much more on this when we get to groundwater flow equation and well tests

Relations Between Grain Size and Hydraulic Conductivity

Equation	Reference
$K = C(d_{10})^2$	Hazen (1911)
$K = (9.66\,E-04\,)(760\,d_g{}^2\,)\,EXP\,(-1.31\,\sigma_g\,)$	Krumbein and Monk (1942)
$K = \left(\dfrac{\rho g}{\mu}\right)\dfrac{d^2\phi^3}{180(1-\phi)^2}$	Kozeny-Carman (in Bear, 1972)
$K = \left(\dfrac{\rho g}{\mu}\right)\dfrac{\phi^3}{C_0 S_{sa}{}^2(1-\phi)^2}$	Kozeny-Carman (in de Marsily, 1986)
$K = \left(\dfrac{\rho g}{\mu}\right)\dfrac{\phi^3}{C_T T(S_{sa})^2}$	Kozeny Equation, modified by Collins (1961)

C₀	= factor reflecting pore shape and packing in the Kozeny-Carmen eqn. [-]
C_T	= factor reflecting pore shape and packing in Kozeny eqn, mod. By Collins [-]
C	= factor in the Hazen equation [T/L]
d₁₀	= grain diameter for which 10% of particles are smaller [L]
d_g	= geometric mean grain diameter [L]
d	= representative grain diameter [L]
g	= gravitational acceleration [L/T^2]
K	= hydraulic conductivity [L/T]
φ	= total porosity, accounting for compaction [-]
μ	= dynamic viscosity [M/LT]
ρ	= density [M/L^3]
σ_g	= geometric mean standard deviation [L]
S_sa	= surface area exposed to fluid per unit volume of solid medium [1/L]
T	= tortuosity [-]

Remember Darcian Velocity is not an actual velocity; it is discharge per unit area (area is TOTAL cross section)

Darcian Velocity

Q

Total Area

Area of
Pore Space

Acts like
this
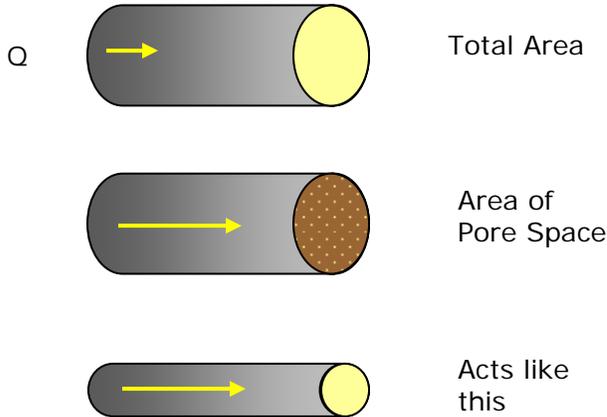
Average Linear Velocity

Primary porosity – the original interstices

Secondary porosity – secondary dissolution or structural openings (fractures, faults, and openings along bedding planes).

Computed porosity - n = 100[1-ρ_b/ρ_p]

ρ_b – bulk density, M/L^3 -> mass of dry sample/original volume

ρ_p – particle density, M/L^3 -> mass of dry sample/volume of mineral matter from water-displacement test (2.65 g/cc)

Effective porosity – porosity available for flow, n_e

> Can have isolated water as dead-end pores or trapped gas. IMPORTANT to transport!

$$\overline{V} = \frac{-K}{n_e}\frac{\Delta h}{\Delta l} = \frac{-K}{n_e}\frac{dh}{dl}$$

$\overline{V}$ is the average linear pore water velocity. Measure of the rate of advection of a slug of water. $\overline{V}$ is larger than the Darcian Velocity. -> $q = n_e \overline{V}$