Making the Trade-Off Between Optimality and Tractability When Solving Large Transportation Models: A Case Study Based On the Airline Industry

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Abstract

Many transportation models share a common problem: they are too large to solve in a fully integrated fashion but the decisions are too interconnected to fully de-couple. Therefore, the challenge is how to break the problem into separate, tractible pieces that yield a feasible and high quality solution. If the sub-problems are too large, the model will not be solvable. If they are too small, solution quality will be poor and infeasibilities may result.

In this case study, we look at a specific example of how this trade-off is assessed. We consider the problems faced by a domestic passenger airline. Four different approaches are considered for solving two of their critical problems – aircraft routing and crew scheduling – and we examine the pros and cons of each of these approaches. We then pose questions for the reader which consider how technological advances can change the assessment of these different approaches.

1 The Case Study

Transportation systems are often complex networks with many interacting components. For example, an airline has to "manage" aircraft, crews, passengers, and many other resources. A less-than-truckload motor carrier routes drivers, tractors, trailers, and freight, balancing equipment utilization against transport time. Public transit agencies must address competing cost, convenience, and safety issues. Ideally, we would like to solve such transportation problems globally – making all decisions at once, fully accounting for the impact that each decision has on all other elements of the problem. In reality, this is rarely possible. In fact, it is often not even possible to fully *state* the problem in terms of quantifiable objectives and explicit constraints. Even when we can write a complete model of the problem, this model is often intractable.

What alternatives do we have, then, when developing such transportation systems? We can break the problem down into smaller sub-problems, with some connection between them, in hopes of finding a *feasible* solution with *reasonable objective value*. Such problem decomposition is very much an art, rather than a science – there is no recipe for breaking down a large problem into the "right" sub-problems. There are some useful rules of thumb, however. We want to keep closely inter-related decisions in the same sub-problem whenever possible. When this results in intractability, we alternatively try to develop models for the separate sub-problems that are in some way linked – for example, allowing elements of one sub-problem to have influence on the objective value of another. Another key issue is to identify those decisions which have the most significant impact on the system, either in terms of cost or feasibility, and take this into account in the development of the master model.

Fundamentally, though, solving such problems is a matter of trial-and-error, of intuition, of "looking outside the box" to view the problem in less obvious ways, and of pushing the envelope of tractability. In this case study we use a real-world problem faced by the airline industry to examine how an intractable problem might be decomposed. Four alternatives for solving the problem will be presented, with a discussion of their relative merits. You will then have the opportunity to look at this problem in a new light by reconsidering these four approaches when changes occur, both in the definition of the problem and in the availability of tools for solving it.

2 East Cost Air: Staying Profitable in Competitive Times

East Cost Air is a successful domestic passenger airline that primarily services New England and the mid-Atlantic states. CEO Brendan Loughlin began the airline in the prosperous mid 1980's and saw it grow in under two decades from a few propeller planes offering service between small regional airports such as Manchester NH and larger airport such as Boston, Hartford, and New York City into a major player in the domestic market, servicing 35 cities with hundreds of daily flights.

Today, East Coast Air is faced with great challenges. Fuel costs are sky-rocketing and Middle East instability suggests that costs will continue to rise. Labor unions are strengthening, with ground crews, pilots, and flight attendants all pushing for increases in pay and benefits. Competition is fierce as airlines form mergers, alliances, and partnerships. All of these factors have caused Brendan many sleepless nights, as he worries over how to keep his company profitable.

Fortunately, one thing working in his favor is an excellent managerial staff, with a good blend of long-term industry experts and younger staffers with technological savvy and an eagerness to try new ideas. Brendan has gathered his staff together for a major strategic retreat, in hopes of finding significant ways to reduce operating costs.

"We can no longer rely on the old way of doing business," he tells his staff in the retreat's kick-off meeting. "Our competitors are facing challenges similar to the ones we face. Those of us who find new ways of tackling our problems will succeed and grow stronger; those who maintain the status quo seemed destined to fail. The purpose of this retreat is to step out of our familiar surroundings, to look past our set ways of doing things, and consider how we might take new approaches as to how we operate our airline."

Brendan concludes his brief opening remarks and turns control of the morning's working session over to his Director of Operations, Olivia Daring. Olivia is a hands-on director, with a vast knowledge of the inner workings of East Cost Air and a willingness to consider new ideas. She begins with the following observation.

"There are many decisions that go into how we run our airline. These range all the way from major decisions about purchasing aircraft to decisions as small as which gate to assign an aircraft that has just landed at a crowded airport. And yet all of these decisions, big and small, have impact on the rest of our network. In the past, I think we have divided our decision-making too much – different groups responsible for different areas don't communicate enough with each other. Everyone is working towards optimizing their own set of decisions – instead, we need to work together to make our operations better across the board. We cannot be making decisions in isolation."

Olivia continues, displaying a poster illustrating their planning process. "We currently divide our tactical planning process into four steps. We first determine the schedule for a given quarter – this tells us what flights we're going to offer and when. Then, the scheduling group passes this information to the fleet assignment group – they decide what type of aircraft should be assigned to each flight, trying to balance operating costs against revenue potential. For each fleet type, the flights assigned to that fleet type are then sent to the maintenance routing group. The MR people assign aircraft to flights to ensure that we can meet FAA requirements governing how often we must maintain our planes. Finally, the crew schedulers assign cockpit and cabin crews to each flight. It's easy to see that all of these problems are very closely inter-related. By solving them in this sequential fashion, I'm sure we're missing out on cost-saving opportunities. But I don't think it's possible to solve all four problems at once – it's just too big. What I'd like to do this morning is break into groups and have you all brainstorm about other options – is there something we can do that's better than solving the problems sequentially but still remains tractable?"

The staffers break up into groups and each group goes into a meeting room to brainstorm. Fortunately, Brendan has cultivated a positive and open working environment, and the ideas flow – some good, some bad, but all given respectful consideration. In one of the groups, an idea is presented early on, and this idea takes up the rest of the session.

J.S. Fiedler first broaches the subject. "As you all know, I'm relatively new here. I've been learning a lot about the airline industry, and about East Coast Air, and I think I understand most of what I've seen. But there's one thing I've always wondered about," he says. "Crew costs are one of the biggest operating expenses that we face. The only thing more costly is fuel, and there's not much we can do about fuel costs – we have to use fuel to fly the planes. And yet the crew decisions are made last – why is that?"

Julia Young, VP of Crew Scheduling, provides J.S. with a brief tutorial. "Pilots are really the critical element in crew scheduling. We solve the crew scheduling problem for pilots by first finding a minimum cost set of *crew pairings*; we then use these pairings to build full month-long schedules. A crew pairing is just a sequence of flights that can be assigned to an individual cockpit crew, which starts and ends at their home base, spanning up to three or four days. There are lots of complicated rules determining whether or not a crew pairing is feasible – these are based on FAA regulations and also on collective bargaining agreements. They include things like limits on the number of hours a pilot can fly before having an extensive rest period; limits on the amount of time before a crew gets to return home; the maximum amount of time that a crew can be idle between two flights; and so forth. One of these rules I'll call *minimum connect time*. This says that if a crew is going to be assigned to fly flight A and then flight B, there must be enough time between these two flights for the crew to travel through the airport from the arrival gate of flight A to the departure gate of flight B. Let's say that the time between A's arrival and B's departure is only 35 minutes. That's not enough time for the crew to connect – our minimum connect time is 45 minutes for most airports. So when we construct the network that's used for generating crew pairings, we leave out this connection arc – two different crews have to cover flights A and B. But what if A and B are flown by the same aircraft? Then the crew doesn't need 45 minutes to walk through the terminal – they just stay with the plane! In this case, we *would* put the connection arc between A and B into the crew pairing network. How do we know whether or not A and B will be flown by the same flight? That gets decided in the maintenance routing phase, when aircraft are assigned to flights. That's why we have to solve maintenance routing before crew scheduling – maintenance routing determines the *forced turn* connections (connections that are valid for a crew only if both flights are assigned to the same aircraft) which in term determine the crew pairing network."

J.S. nods in understanding. "That makes a lot of sense to me," he says, "but at the same time it's frustrating. In maintenance routing, we're really just looking for a feasible solution – there must be a huge number of different solutions which are all feasible. And yet which one we choose can affect what opportunities we have for our crews, which can have significant impact on how much our schedule costs."

Julia agrees with him. "Yes, I've often wondered how much better we could do if we could somehow better choose the set of forced turns. Maybe this is something that merits further investigation."

Later that day, the staff all comes together again to discuss their morning brainstorming sessions. J.S. and Julia speak on behalf of their group, discussing their interest in pursuing the issue of maintenance routing and its impact on crew scheduling. Olivia seems excited by their presentation. "I think this is the type of idea that can lead to real improvements for us," she says. "I'd like to spend tomorrow in small groups again, seeing whether we can come up with a way to leverage J.S. and Julia's observations." The meeting then ends in time for an animated dinner, in which everyone continues to discuss new ways to improve the company's tactical planning process.

The next morning, they re-convene bright and early, and after some inspiring remarks by Brendan, Olivia divides the staffers into four groups, all charged with a similar task. Group one will look at the current process of routing and then scheduling, identifying how the process works today, and what the pros and cons are. The remaining three groups will try to come up with a better approach that will result in a decrease in crew costs.

3 Group One: Reviewing the Existing Process

Group one begins by reviewing the maintenance routing problem. They draft the following summary to present to the larger group:

The Federal Aviation Administration (FAA) requires aircraft to undergo a variety of maintenance checks varying in scope, frequency, and duration. The most routine of these, known as an A check, must occur every 65 flight hours. Should an aircraft exceed 65 hours of flight time without undergoing an A check, the FAA will ground the aircraft. Given how valuable and tightly utilized a resource aircraft are, East Coast Air, like most airlines, is very conservative in ensuring that this time limit is not exceeded. For most fleet types, standard policy is that each aircraft spend at least every third night at a station with facilities to perform an A check. For some fleet types, this is relaxed to every fourth night. Note that not all airports have maintenance facilities; furthermore, different fleet types might have different maintenance stations.

In order to ensure that this routine maintenance can occur, the tactical planners solve a maintenance routing problem after the fleet assignment process has been completed. The maintenance routing problem constructs strings of flights which begin and end at maintenance stations (not necessarily the same one) and span at most 3 (4, where appropriate) days. Each flight must be covered by exactly one string, and it must be possible to cover all of the strings without using more aircraft than there are in the fleet.

Although some airlines treat the fleet assignment problem as an optimization model, with the objective of maximizing through revenue *(the revenue associated with offering passengers itineraries that don't require a change of aircraft), East Cost Air is one of many airlines that views this revenue as inaccurate, hard to capture, and far less significant than costs such as fuel and crew costs. They instead focus on finding a feasible maintenance routing solution.*

It is difficult to pose maintenance routing as a network flow problem, because it is difficult to enforce the constraints on amount of time between maintenance checks. Therefore, the problem is often modelled as a set partitioning problem with side constraints. There is one variable for each feasible route string. The set partitioning constraints ensure that each flight is covered by exactly one chosen *route string. Additional side constraints ensure aircraft balance and enforce the restriction on the number of available aircraft. Because the number of feasible route strings is exponentially large, often in the hundreds of millions or more, approaches such as branch-and-price (branch-and-bound with column generation used to solve the LP relaxations) are often used.*

Based on this description, it is clear to see that the maintenance routing problem determines aircraft connections. These connections include those which are *forced turns* – connections permitted for a crew pairing only when they are covered by the same aircraft.

Group one next discusses the crew scheduling process. They identify key points in the following summary.

Of primary concern is scheduling pilots, or cockpit crews, *for recurring daily flights. This process occurs in two phases. First, a minimum cost collection of pairings are chosen – that is, strings of flights that represent a crew's work over a 1 - 4 day period. Then, these pairings are used to construct a month's worth of schedules, taking into account when crew members need training, vacation, or personal time, and ensuring that they get adequate rest over the course of the month.*

The critical problem is finding the optimal set of pairings. Instead of thinking of a pairing as a sequence of flights, it is sometimes easier to think of it as a sequence of duties. *A duty is a day's worth of work – a sequence of flights followed by a period of substantial rest. Duties are subject to an array of rules, determined both by the FAA and by collective bargaining agreements. These include restrictions on the minimum and maximum allowable time on the ground between two sequential flights; a maximum amount of flying time in a single duty; a maximum total elapsed time for a duty; and so forth. The "cost" (measured in minutes) associated with a duty is also quite complex. It is the maximum of three components – the total flying time in the duty, some fraction times the total elapsed time in the duty, and some specified duty minimum.*

Given a set of duties, a pairing can then be defined as a sequence of duties that begins and ends at the crew's home based, has adequate rest in between duties, satisfies some maximum time away from base (TAFB), and meets a number of other FAA and collective bargaining restrictions. The cost associated with a pairing is also the maximum of three quantities – the sum of the duty costs; some fraction times the total TAFB; and some specified pairing minimum.

As with the maintenance routing problem, it is difficult to formulate the crew pairing problem as a network flow problem, both because the cost is nonlinear and the constraints are difficult to enforce. Instead, the crew pairing problem can be posed as a pure set partitioning problem. One binary variable is used to represent each feasible pairing; the objective coefficient of this variable corresponds to the cost of the pairing. There is one constraint for each flight, stating that the number of chosen pairings containing that flight must be exactly one. This model, although simple to describe, is difficult to solve because the number of potential pairings is exponentially large – often in the hundreds of millions or more. Therefore, the problem typically is solved using an approach such as branch-and-price.

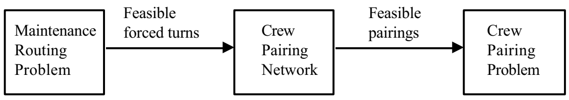

Figure 1: Sequential Process

Figure 1 depicts the sequential process. First, the maintenance routing problem is solved, yielding a set of forced turns. These forced turns are used to augment the crew pairing network. This network is used to generate feasible pairings, which are then input into the crew scheduling problem. Thus, there is a clear link between the two problems. Different maintenance routing solutions will result in different sets of forced turns. This impacts the structure of the crew pairing network, which in turn influences the quality of the crew pairing solution. Group one concludes their review of the sequential process with an assessment of the pros and cons of the currently used sequential method.

The benefits of the current approach are clear – it allows us to solve the problem in a reasonable time period, ensures feasible solutions to both problems, and has resulted in reasonable crew costs. The cons are more subtle – by allowing the maintenance routing problem to determine the set of feasible forced turns, we are limiting our crew pairing opportunities, and yet our maintenance routing approach has no way of distinguishing between two feasible routing solutions, one of which has a "good" set of forced turns and one of which has a "bad" set of forced turns. In fact, we don't even know how far we are from the optimal crew solution!

Group one wraps up their working sessions with a good understanding of the current sequential approach. They recognize that the current way leads to sub-optimality, and are curious if the other groups will be able to come up with some good new ideas.

4 Group Two: Reversing the Order

Group two sets a specific objective for their brainstorming session. Their goal is to improve over the existing sequential approach to solving maintenance routing and then crew scheduling. They focus, however, on finding improvements that can be used in the very near future, without a significant amount of start-up work.

Michael Thomas quickly proposes a straightforward but potentially important idea. "If the quality of the crew pairing solution is so much more important than that of the maintenance routing solution, why don't we just solve crew pairing first? We can include all of the forced turns in the crew pairing network. Then, when we solve the maintenance routing problem, the forced turns used in the crew pairing solution will be maintenance routing *input* rather than output."

His idea is met with interest. It has a number of potential benefits. First, it is immediately implementable – they can use the existing solvers for the maintenance routing and crew pairing problems, just in reverse order. Second, it will take roughly the same amount of computational time as the current approach. Third, and perhaps most important, if it yields a feasible solution to the maintenance routing problem, then it will result in the best possible crew pairing solution.

"Hmmm," worries Michael. "That's a pretty big IF. Maybe it's not such a good idea after all."

Olivia is quick to jump in. "I don't think we should rule it out yet. First, let's try it out for a number of our fleets and see how often it results in an infeasible solution. If it works a large percent of the time, we might want to implement your idea, with the option of re-solving the original way in those cases where the maintenance solution is infeasible. If it doesn't work out, we haven't lost anything in trying, because we won't need any new coding to test it. I think it's an excellent starting approach!"

5 Group Three: A Basic Integrated Approach

In a nearby meeting room, group three takes a more aggressive approach. "Computers are getting faster and faster; techniques for solving these problems are becoming more and more sophisticated – maybe we can solve both problems simultaneously," suggests Sam Sharfstein. The group begins to look at a fully integrated model.

They begin by writing down the basic maintenance routing model:

$$\min \sum_{r \in R} c_r x_r$$

$$st$$

$$\sum_{r \in R} \delta_{fr} x_r = 1 \qquad \forall \text{ flight } f$$

$$\sum_{\substack{r \text{ into} \\ \text{node } n}} x_r + z_n^- - \sum_{\substack{r \text{ out of} \\ \text{node } n}} x_r - z_n^+ = 0 \qquad \forall \text{ node } n$$

$$\sum_{r \in R} \phi_r x_r + \sum_{n \in N} \phi_n z_n^+ \leq K$$

$$x_r \in \{0, 1\}$$

$$z_n^{+/-} \geq 0.$$

x_r is the binary decision variables representing whether or not to include route string r in the solution. $z_n^{+/-}$ represents the number of aircraft on the ground arc out of / into node n. The objective function minimizes the cost of the chosen route strings; at East Coast Air, these coefficients are all zero, as the problem is solved simply for a feasible solution. The first set of constraints are cover constraints. δ_{fr} is a notational device used to simplify the writing of the model – it has value one if flight f is covered by route string r and zero otherwise. Thus, these constraints state that each flight must be included in exactly one chosen route string. The second set of constraints are balance constraints. Route strings begin and end at specific maintenance stations and specific times. We create ground arcs to connect these nodes in order to keep track of aircraft. The balance constraints state that the number of aircraft coming into a node, either covering a string or on the ground as indicated by the ground arc variables, must equal the number coming out of the node. In the final constraint, we use ϕ_r to indicate the number of times route string r crosses the countline, and similarly for ϕ_n. This enables us to ensure that the total number of aircraft used to cover the route strings while maintaining balance does not exceed the number of aircraft in the fleet, namely K.

Group three next writes the crew pairing model:

$$\min \sum_{p \in P} c_p y_p$$

$$st$$

$$\sum_{p \in P} \delta_{fp} y_p = 1 \qquad \forall \text{ flight } f$$

$$y_p \in \{0, 1\}.$$

The objective is to find a minimum cost set of crew pairings. The constraints specify that each flight be included in exactly one pairing. Here, δ_{fp} is one if flight f is contained in pairing p and zero otherwise.

They decide to simply combine the two models, with additional constraints to link them together. The new model becomes:

$$\min \sum_{p \in P} c_p y_p$$

$$st$$

$$\sum_{r \in R} \delta_{fr} x_r = 1 \qquad \forall \text{ flight } f$$

$$\sum_{\substack{r \text{ into} \\ \text{node } n}} x_r + z_n^- - \sum_{\substack{r \text{ out of} \\ \text{node } n}} x_r - z_n^+ = 0 \qquad \forall \text{ node } n$$

$$\sum_{r \in R} \phi_r x_r + \sum_{n \in N} \phi_n z_n^+ \leq K$$

$$\sum_{p \in P} \delta_{fp} y_p = 1 \qquad \forall \text{ flight } f$$

$$\sum_{r \in R} \delta_{tr} x_r - \sum_{p \in P} \delta_{tp} y_p \geq 0 \qquad \forall \text{ forced turn } t$$

$$x_r \in \{0, 1\}$$

$$z_n^{+/-} \geq 0$$

$$y_p \in \{0, 1\},$$

where δ_{tp} is one if forced turn t is included in pairing p and zero otherwise, and similarly for δ_{tr}.

Everyone is in agreement that this new model solves the integrated problem. There is dissent, however, over whether it is a good approach.

"Each of the individual problems has an exponential number of constraints. When we combine them, the new model will be enormous." someone argues. "Futhermore, it seems like the linking constraints might encourage fractional solutions – we could spend an enormous amount of time seeking an integer solution. Solving the combined model could be significantly slower than solving the two basic models sequentially. And worst of all, we won't have a feasible solution until the very end!"

Many nod in agreement, but others are still optimistic. "Sure the problem is big, but we aren't going to state it explicitly – we'll use column generation for both the crew and maintenance variables. The structure of the pricing problems doesn't have to change. We can also build in some heuristics to speed up the branching – and most importantly, we might see a real decrease in our crew costs!"

As the session time nears an end, they all agree that a direct integrated approach still needs further study, but might yield a real improvement in their planning process if it's tractable.

6 Group Four: An Extended Crew Pairing Model

Group four begins their session by observing that each maintenance routing solution results in a different crew pairing solution; given that the maintenance objective is far less important, the one resulting in the best crew pairing solution should be chosen. "It's too bad we don't have "black boxes" for solving the maintenance routing and crew pairing problems instantaneously," J.S. observes. "Then we could just generate each of the maintenance solutions, find the corresponding crew pairing solution, and pick the best."

"Unfortunately, the crew and maintenance solvers are a lot slower than that," someone replies. "And just think how many maintenance solutions there must be!"

J.S. agrees, but sticks with the idea. "Well, we wouldn't have to look at all of them," he muses. "Only the ones with unique sets of forced turns. For example, one solution might be flight strings A-B-C and D-E-F. Another might be D-A-B and C-E-F. If A-B is the only forced turn, then these two solutions would yield an equivalent crew pairing network, and therefore we would only have to look at one of these two maintenance solutions. I bet that would really cut down on the number of crew pairing problems that we'd have to solve."

Julia continues this line of thought. "We wouldn't just need the unique forced turn set," she added. "We would only need forced turn sets that are dominating. In other words, if one maintenance solution uses forced turns A-B and Q-R and another one only uses A-B, we would only need to consider the first maintenance solution."

J.S. returns to one of the original concerns. "Let's say that the number of maintenance solutions that we'd need to look at was pretty small, and that we could find them all. We'd still have to solve the crew pairing problem over and over again – I'm afraid that would take too long."

While he speaks, Julia has been scribbling on a note pad. She rises and walks to the wipe board in the front of the room. "Not necessarily, J.S." she smiled. "What if we could do it all at once?"

She formulates the following model on the wipe board:

$$\min \sum_{p \in P} c_p y_p$$

$$st$$

$$\sum_{p \in P} \delta_{fp} y_p = 1 \qquad \forall \text{ flight } f$$

$$\sum_{s \in S} \delta_{ts} x_s - \sum_{p \in P} \delta_{tp} y_p \geq 0 \qquad \forall \text{ forced turn } t$$

$$\sum_{s \in S} x_s = 1$$

$$x_s \in \{0, 1\}$$

$$y_p \in \{0, 1\}.$$

"This is just the basic crew pairing model with a few enhancements," she explains. "In addition the the crew pairing variables, which we generate from a network that permits us to use *all* of the forced turns, we have one variable representing each of the relevant maintenance routing solutions. The first set of constraints is just the basic flight cover constraints from the crew model. The last constraint is a *convexity constraint* – it forces the model to choose exactly one of the maintenance routing solutions. Then, the *forced turn constraints* ensure that the crew pairings chosen don't use any of the forced turns that aren't part of the selected maintenance routing solution. This way, we meet our objective of identifying the maintenance routing solution that yields the best crew pairing solution, but we only have to solve the crew pairing problem once!"

J.S. is very interested in her formulation. "You know, one of the things I really like about this approach is it's flexibility – if someone figured out a new way of solving the maintenance routing problem, or we needed to add some new constraints to it, we wouldn't have to change this integrated approach, given that the maintenance solutions are input to the model. And even more importantly, if it's too slow to generate all of the relevant maintenance solutions to get an optimal integrated solution, we can use this model heuristically – generate as many maintenance solutions as we have time for, and get the best crew pairing solution from these. If we are smart about how we generate the maintenance solutions, we might get close to optimal anyway."

Julia adds, "And don't forget that we can use dual information from this problem to generate even more maintenance solution columns, if we want. I think this is worth looking at some more!"

They end their session with three next steps planned. First, they'll try to estimate how many relevant maintenance routing solutions there are for real-world problem instances. Second, they'll look into how much slower the crew pairing problem is to solve, given these new variables and constraints. Third, they'll run some computational experiments to estimate how much the crew pairing objective improves as you consider heuristically a subset of the maintenance routing solutions.

7 Conclusions

As they all gather together to present the results of each group's session, Brendan sits in the back nodding and smiling. He concludes the day with a few positive remarks. "I am delighted by the work I see going on here," he says. "You've taken a variety of approaches to a complicated problem, and I think all of the ideas discussed here have potential. We have the reverse-order approach, which could be implemented immediately and for certain instances would lead us to the optimal crew pairing solution. We have the basic integrated approach, which – if it's tractable – would ensure us optimality. We have the extended crew pairing model, which would give us flexibility in how much time we spend on improving the crew pairing solution. I suspect that as we dig deeper with these approaches, we may also find that there are algorithmic ways to combine the best elements of each of them. I look forward to our follow-up meeting in a few months, when I'll get to hear what you've discovered as you start to test out your ideas!"

8 Further Study

Questions for further study will be provided in problem set 3.

9 References

The following is a partial list of references that may be of interest to the reader. This list is by no means complete, but merely serves as a starting point for additional reading.

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