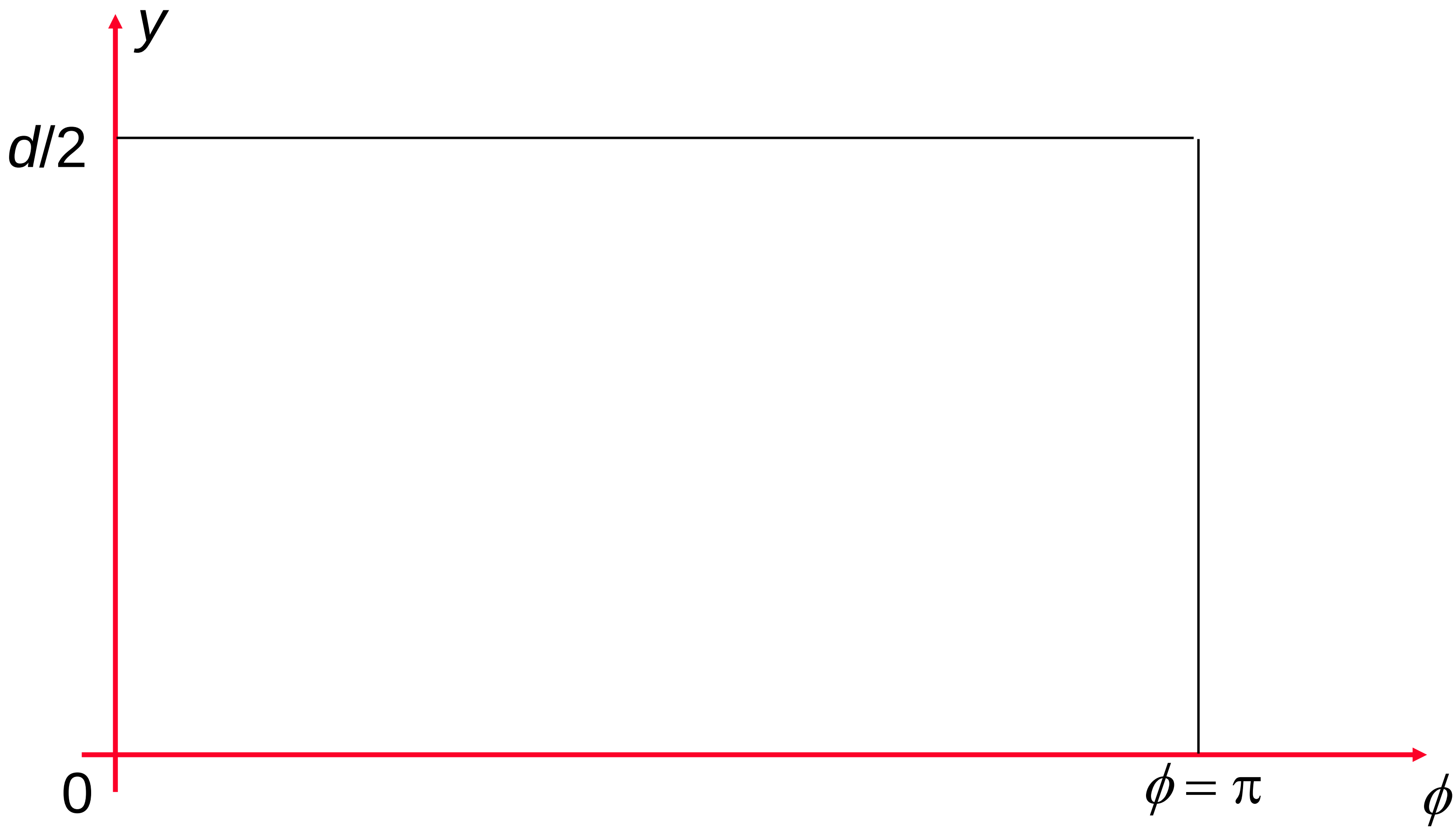
1. The R.V.'s

- Y = distance from the center of the needle to closest of equidistant parallel lines $0 < y < d/2$
- Φ = angle of needle wrt horizontal $0 < \phi < \pi$

2. Joint Sample Space



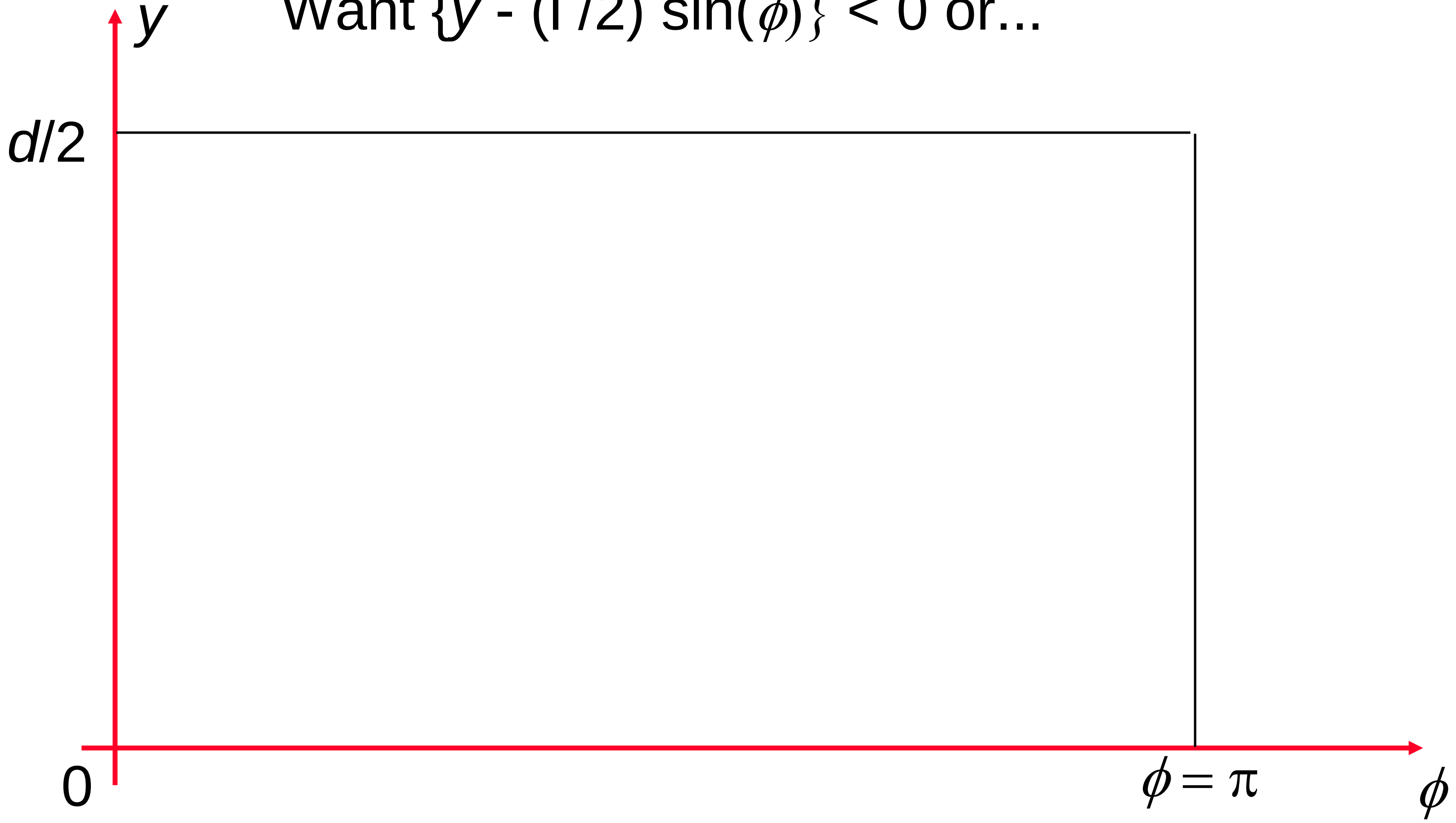
3. Joint Probability Distribution

- Want $f_{Y,\Phi}(y,\phi)$
- Think about that tricky phrase, "At random"
- $f_{\Phi}(\phi) = 1/\pi \quad 0 < \phi < \pi$
- $f_{Y}(y) = 2/d \quad 0 < y < d/2$
- Independence implies

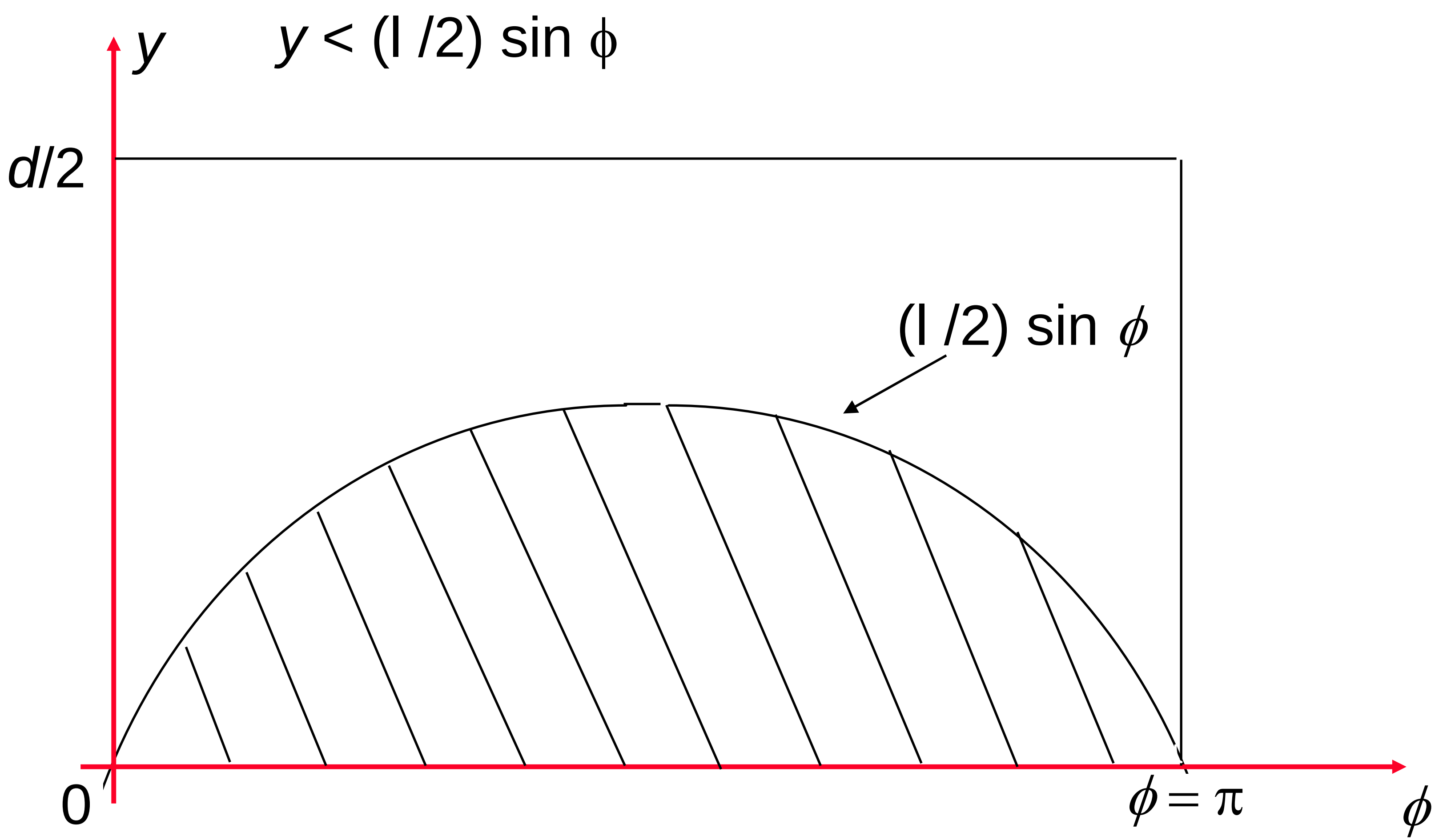
$f_{Y,\Phi}(y,\phi) = f_{\Phi}(\phi)\, f_{Y}(y) = \text{constant} = 2/(\pi d)$

4. Working in the Joint Sample Space

Want $\{y - (l/2)\sin(\phi)\} < 0$ or...



4. Working in the Joint Sample Space

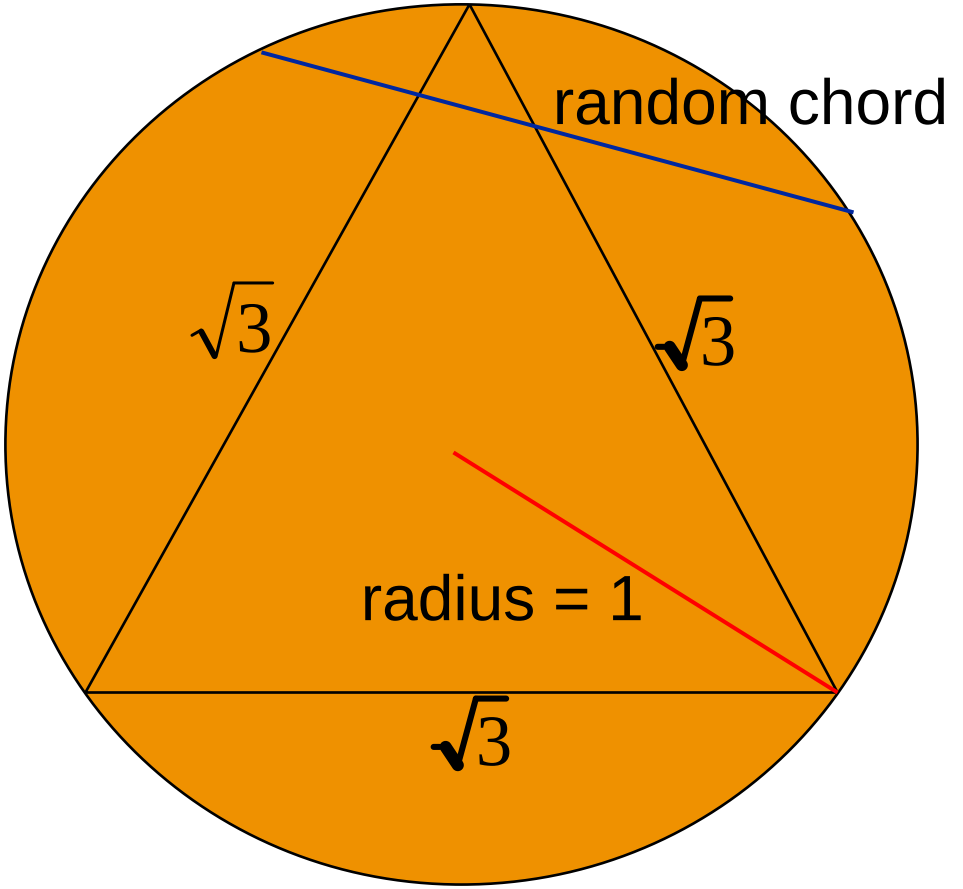


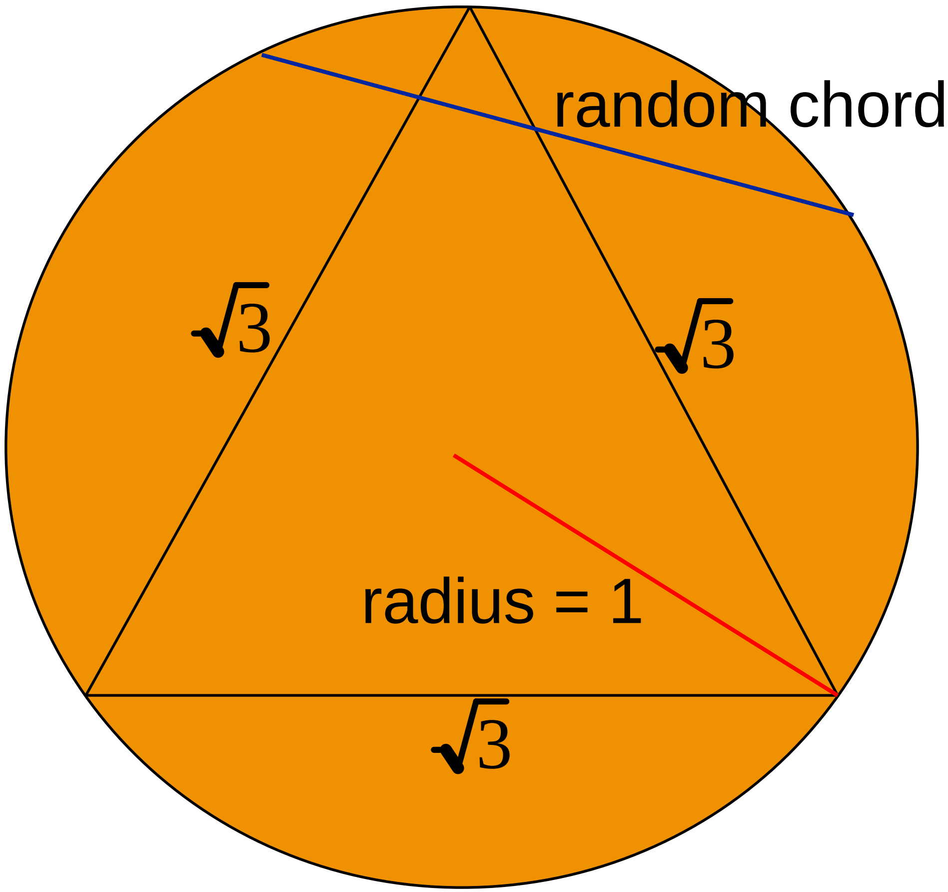
$$P = \int_0^{\pi} d\phi \int_0^{(l/2)\sin\phi} dy (2/[\pi d])$$

$$P = (l/[\pi d]) \int_0^{\pi} d\phi \sin\phi = -(l/[\pi d])(-1-1)$$

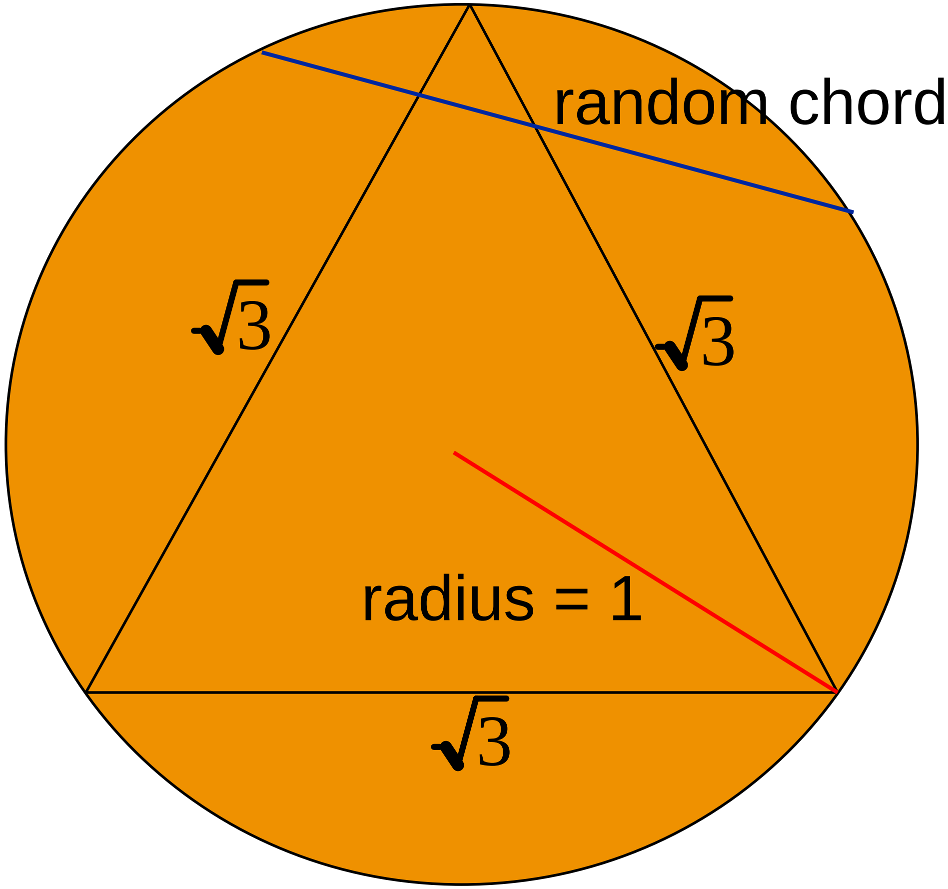
$$P = 2l/[\pi d]$$

Bertrand's Paradox


random chord
$\sqrt{3}$
$\sqrt{3}$
radius = 1
$\sqrt{3}$



Bertrand's Paradox: What is the probability that a random chord on this circle has length greater than $\sqrt{3}$?



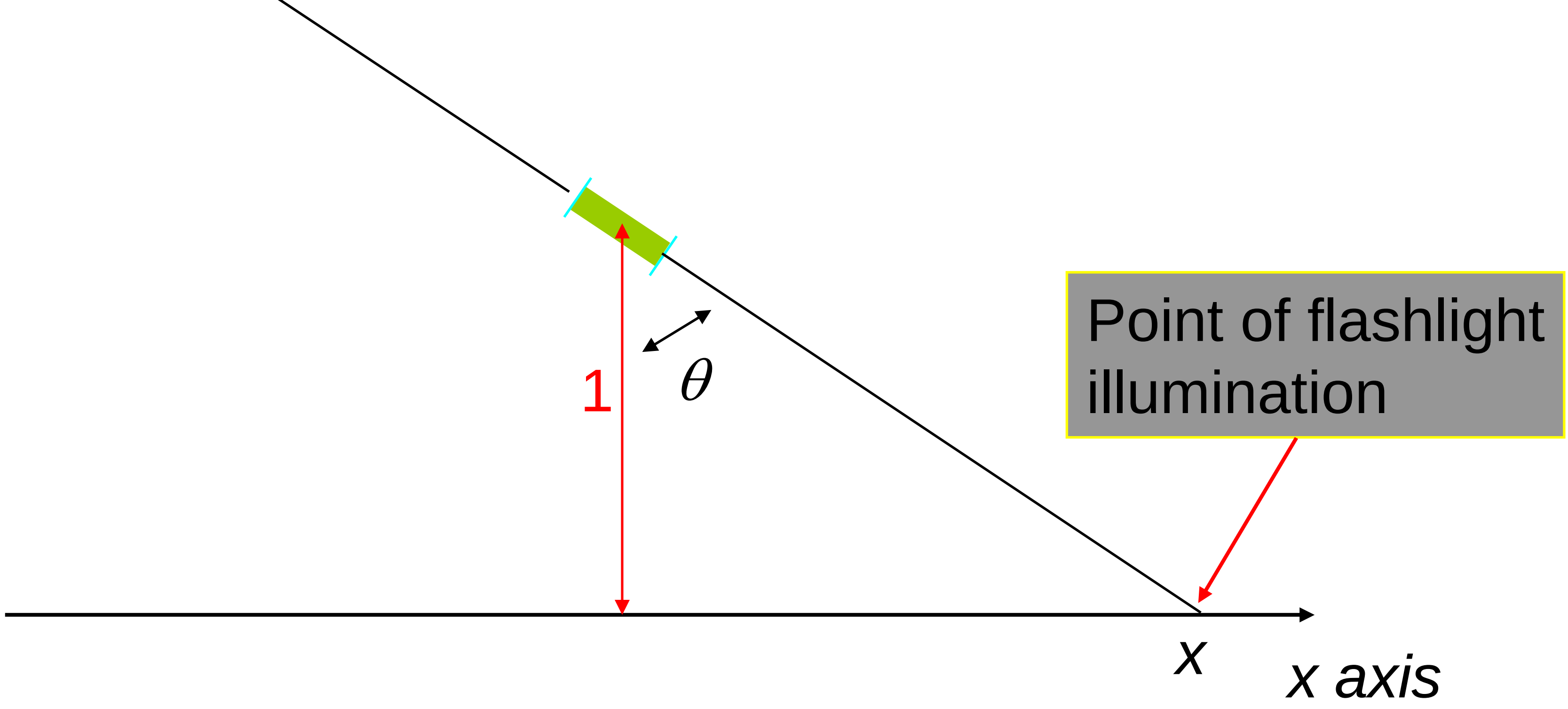
Bertrand's Parodox: What is the probability that a random chord on this circle has length greater than $\sqrt{3}$?

Three correct answers: 1/3, 1/4 and 1/2 !!!!

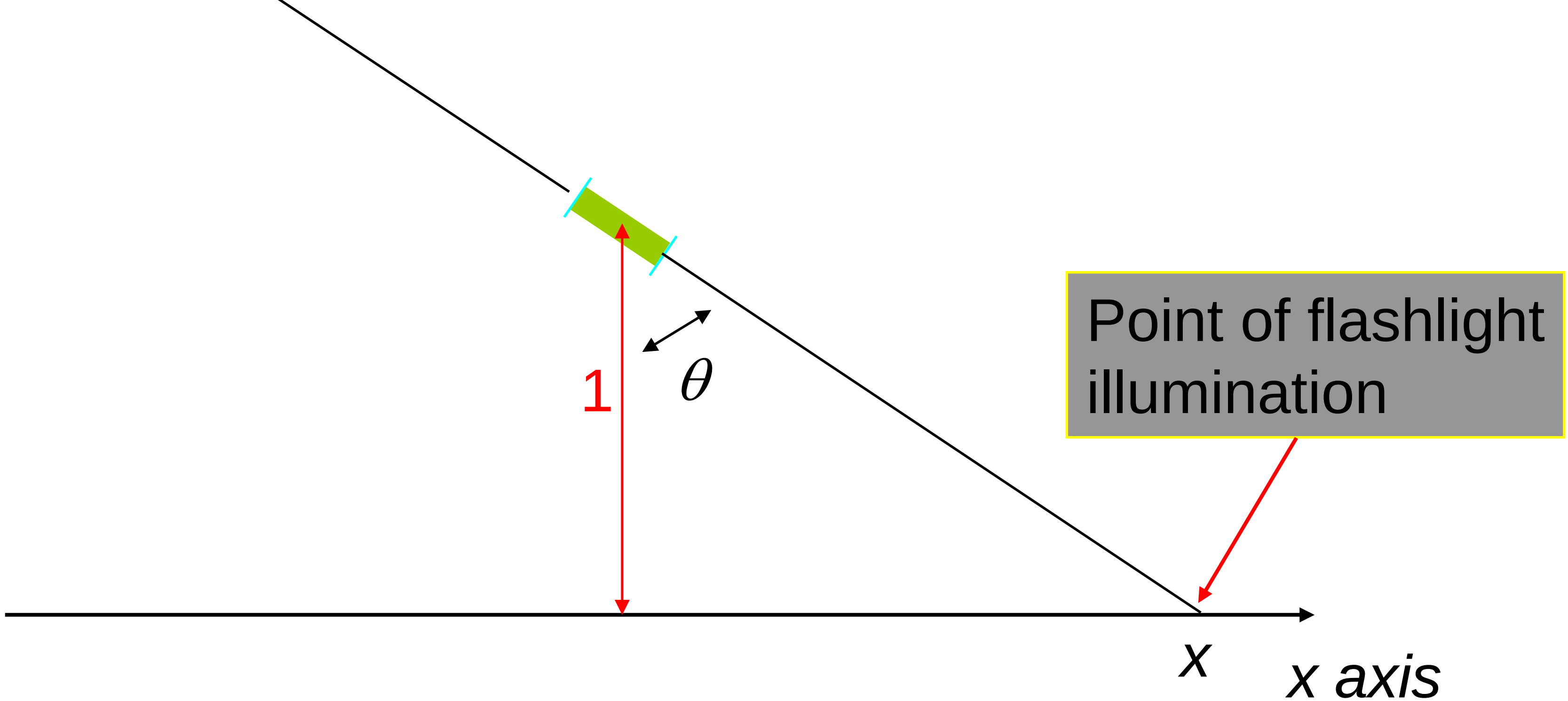
Be very careful about that ambiguous word, "random".

See text for a sample space, probability assignment argument.

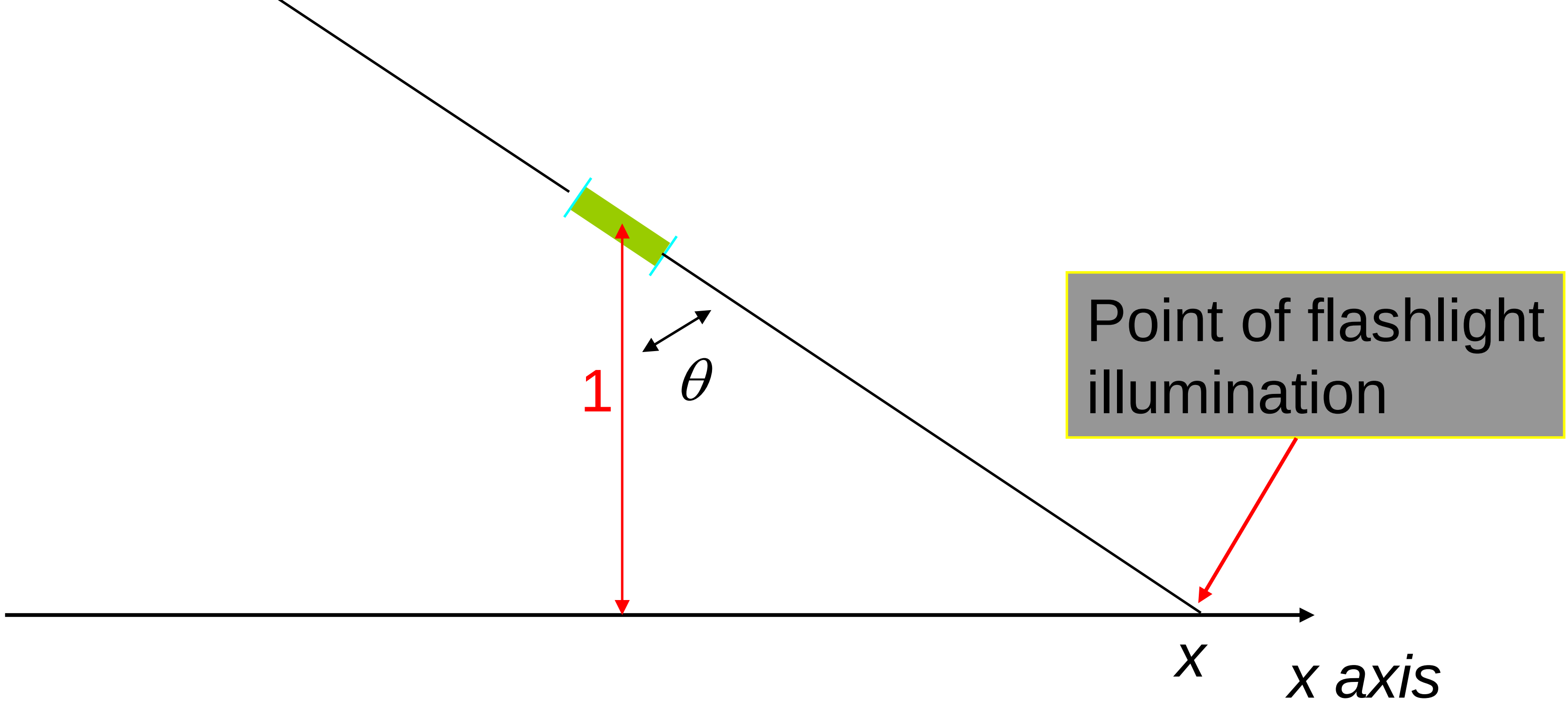
Spin the Flashlight



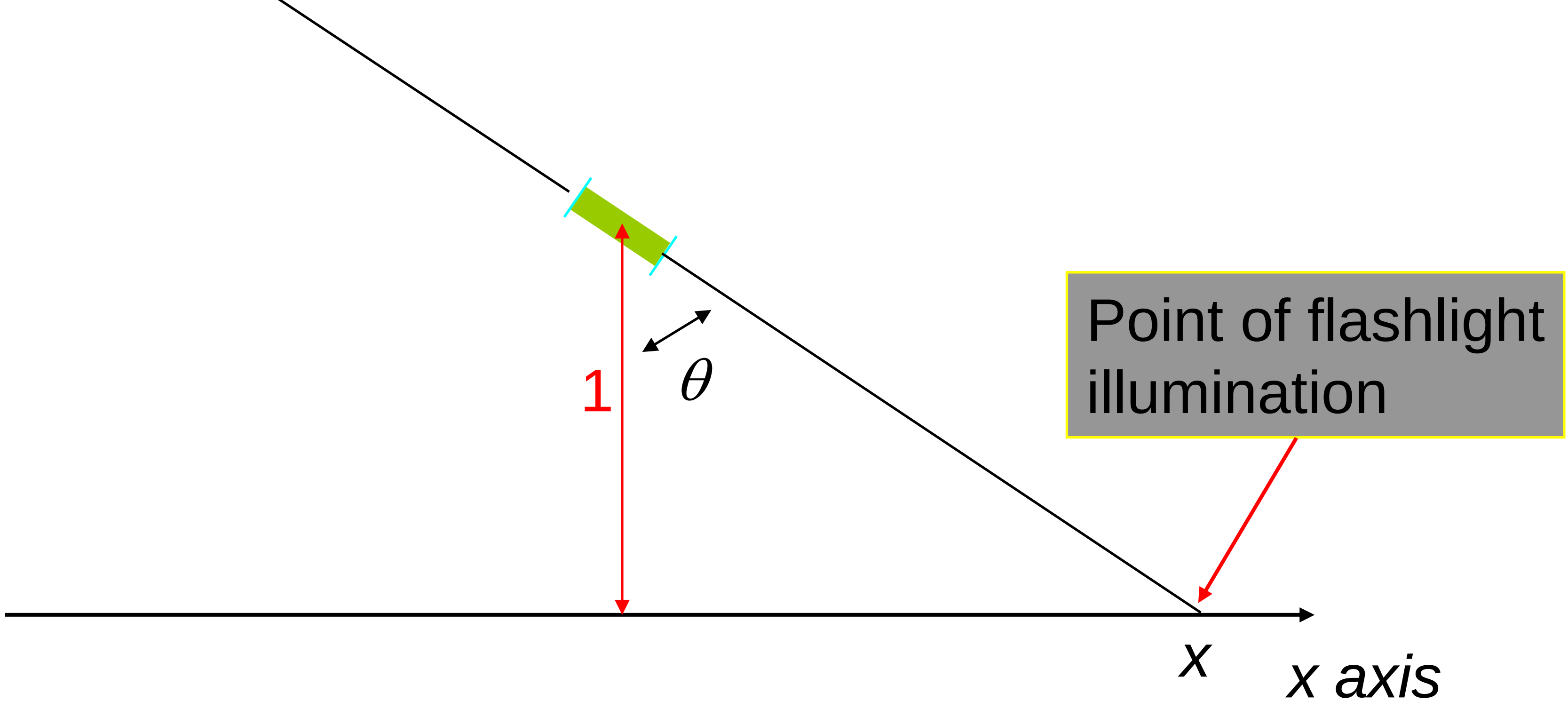
Point of flashlight illumination
1
θ
x
x axis



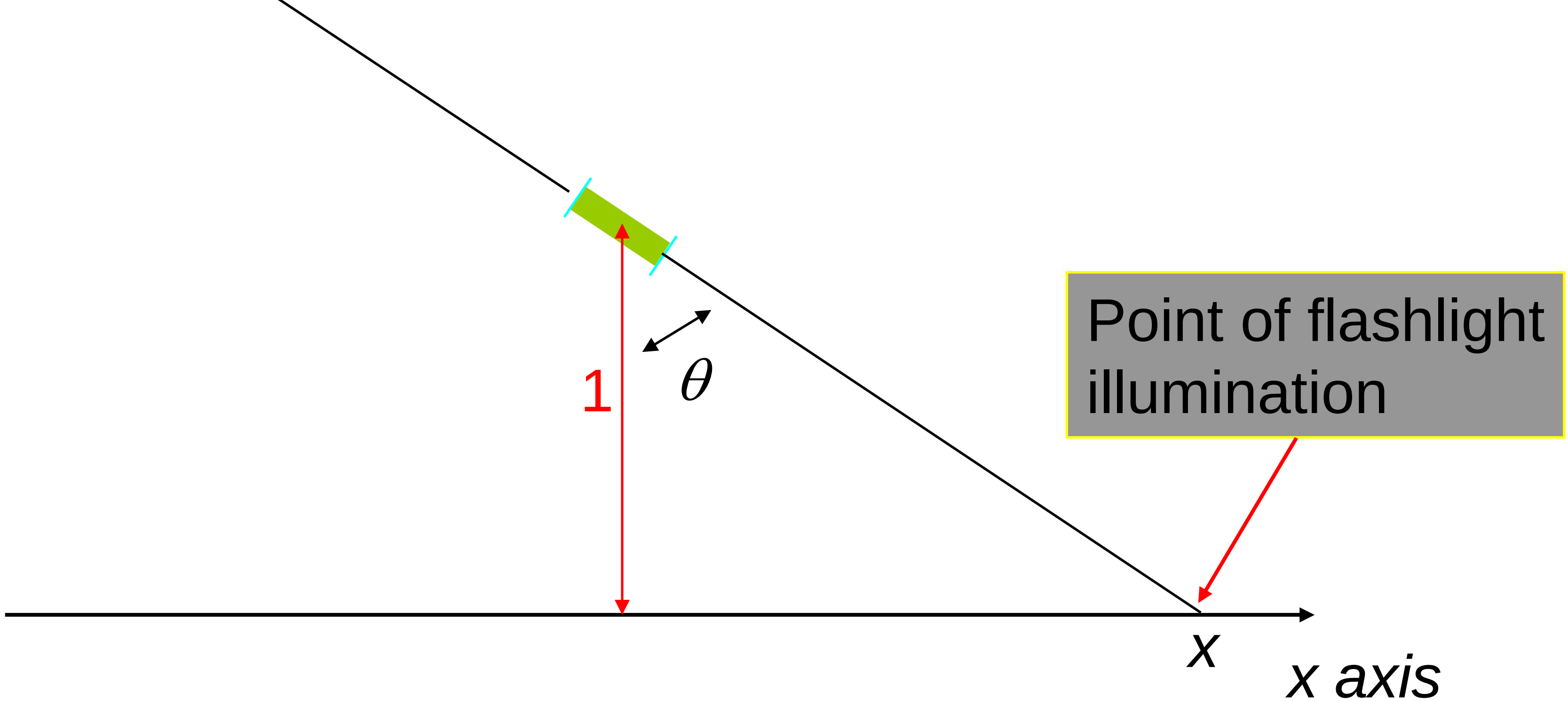
1. R.V.': X, Θ



1. R.V.': X, Θ
2. Sample space for Θ: $[-\pi/2, \pi/2]$



1. R.V.': *X*, *Θ*
2. Sample space for *Θ*: [-π/2, π/2]
3. *Θ* uniform over [-π/2, π/2]



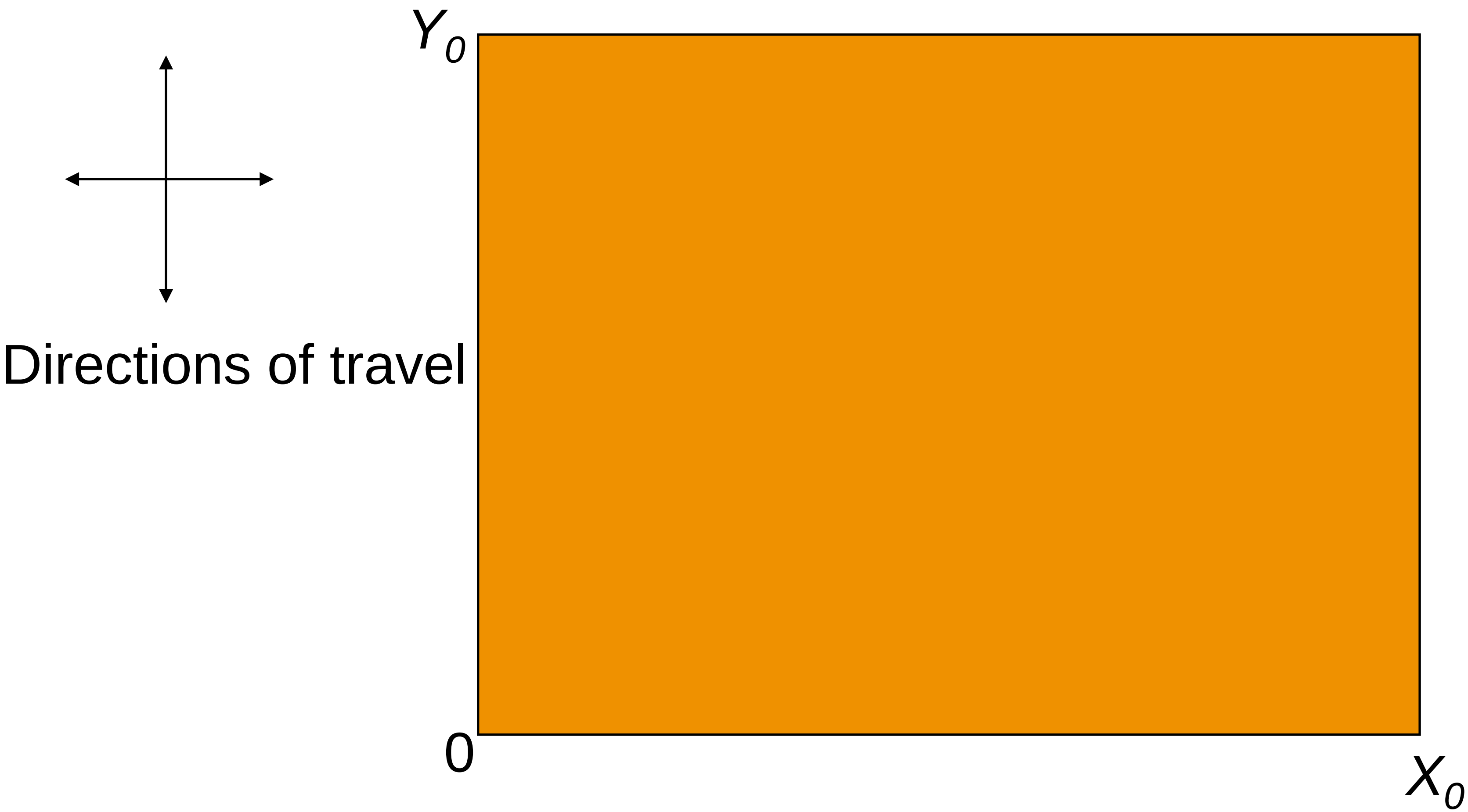
1. R.V.': X, Θ
2. Sample space for Θ: $[-\pi/2, \pi/2]$
3. Θ uniform over $[-\pi/2, \pi/2]$
4. (a) $F_X(x) = P\{X<x\} = P\{\tan\Theta < x\} = P\{\Theta < \tan^{-1}(x)\} = 1/2 + (1/\pi)\tan^{-1}(x)$

   (b) $f_X(x) = (d/dx)\, F_X(x) = 1/(\pi)(1 + x^2)$ all x

Cauchy pdf

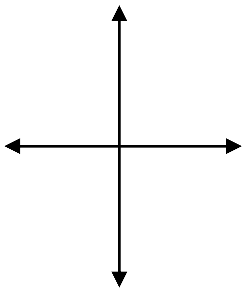
Barriers to Travel

Perturbation Random Variables

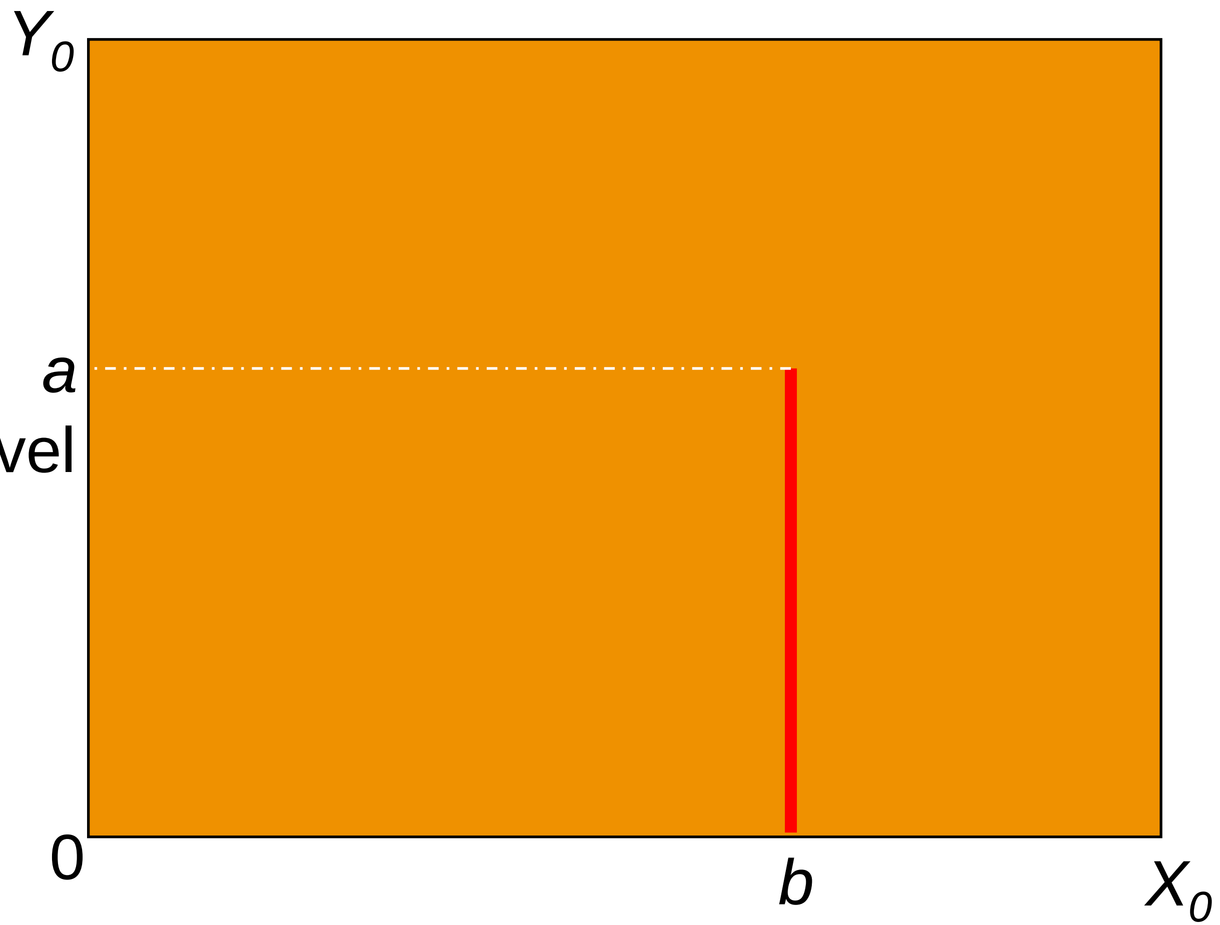


$$E[D] = (1/3)\,[X_0 + Y_0]$$

Add a barrier to travel



Directions of travel

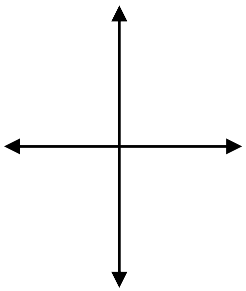


$$E[D] = (1/3)\,[X_0 + Y_0]$$

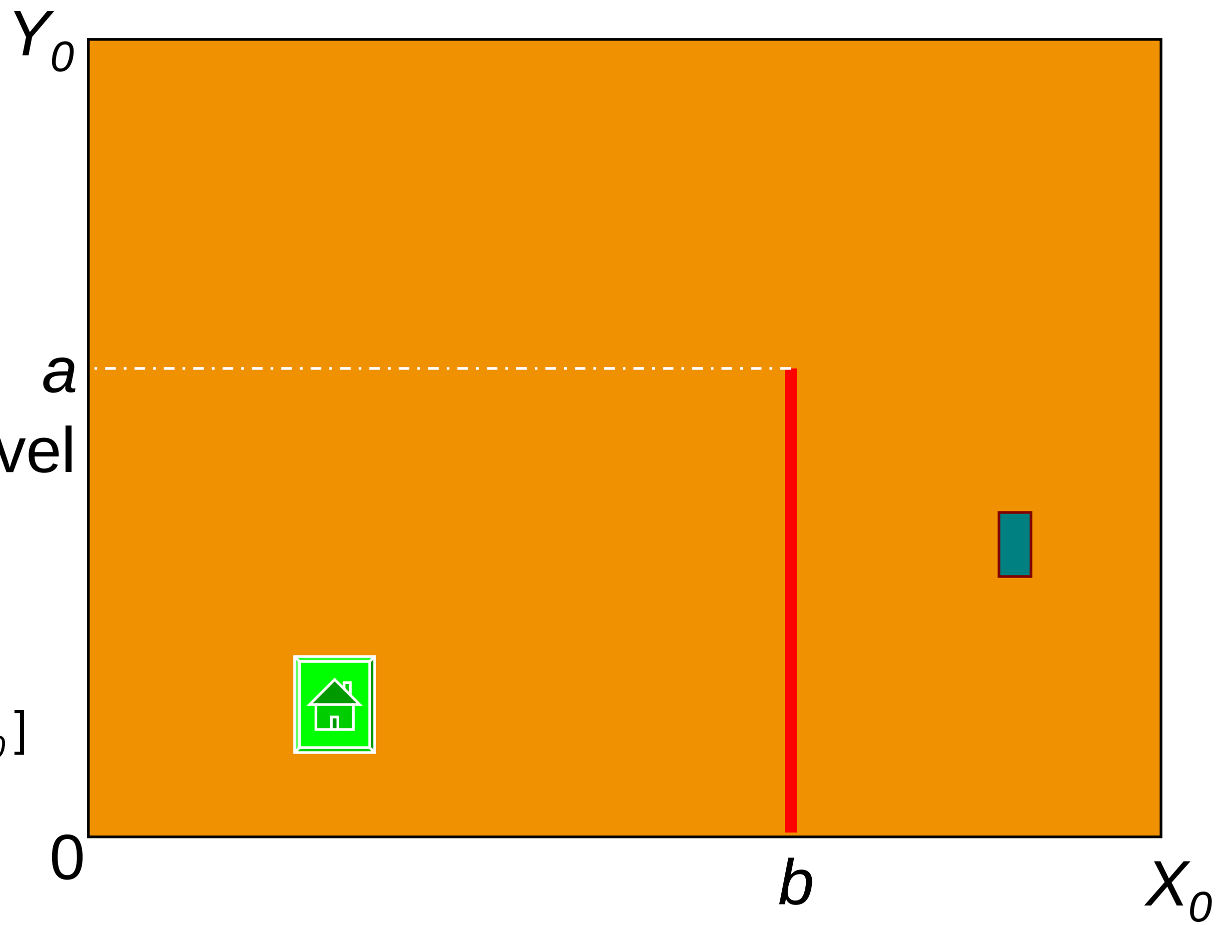
$$D' = D + D_e, \qquad E[D'] = E[D] + E[D_e]$$

$$E[D_e] = E[D_e \mid D_e > 0]\; P\{D_e > 0\}$$

Add a barrier to travel



Directions of travel



$E[D] = (1/3)\,[X_0 + Y_0]$

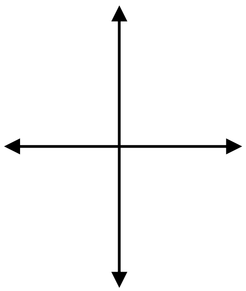
$D' = D + D_e$,

$E[D'] = E[D] + E[D_e]$

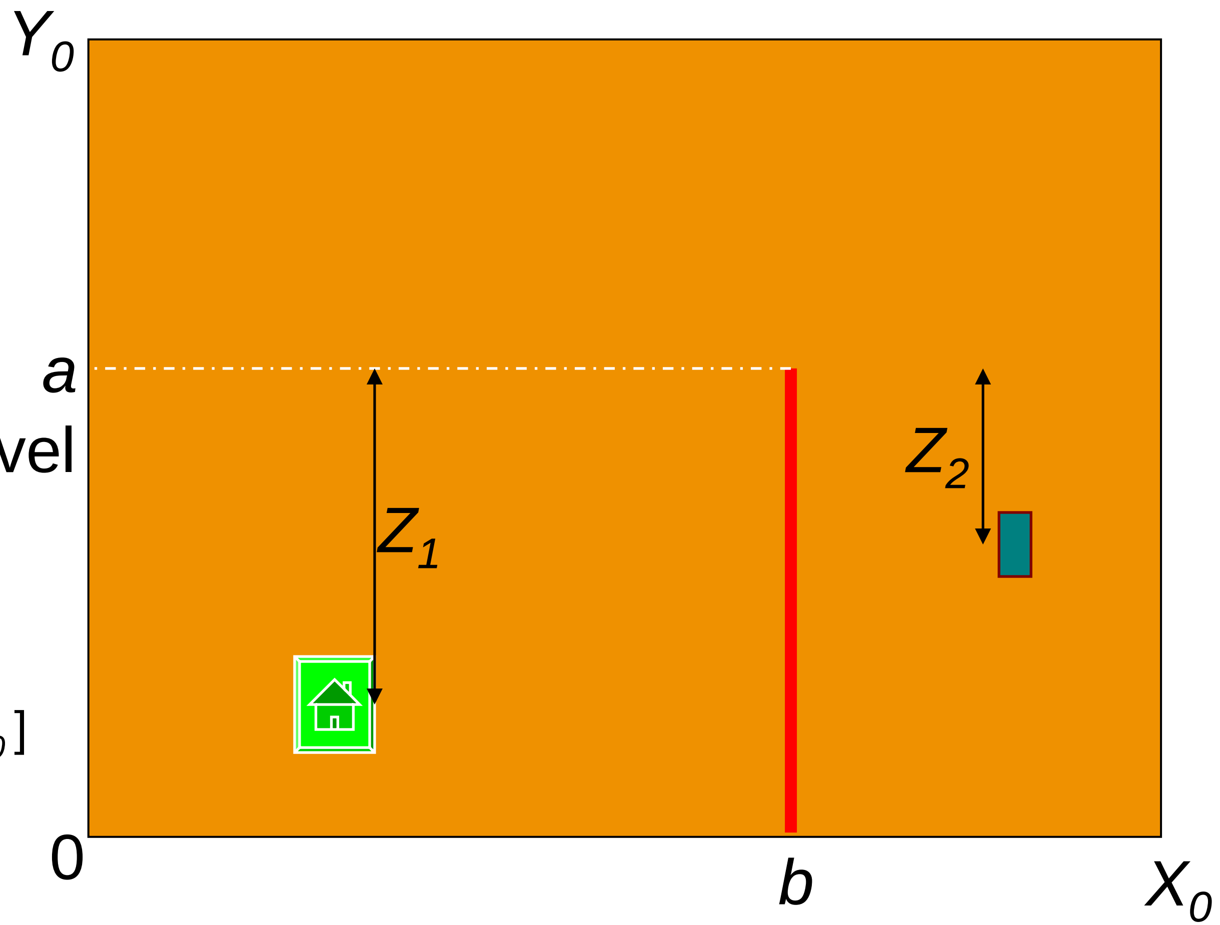
$E[D_e] = E[D_e \mid D_e > 0]\; P\{D_e > 0\}$

$$P\{D_e > 0\} = 2\,(ab)(a[X_0 - b])/\{(X_0 Y_0)(X_0 Y_0)\}$$

Add a barrier to travel



Directions of travel



$E[D] = (1/3)\,[X_0 + Y_0]$

$D' = D + D_e$,

$E[D'] = E[D] + E[D_e]$

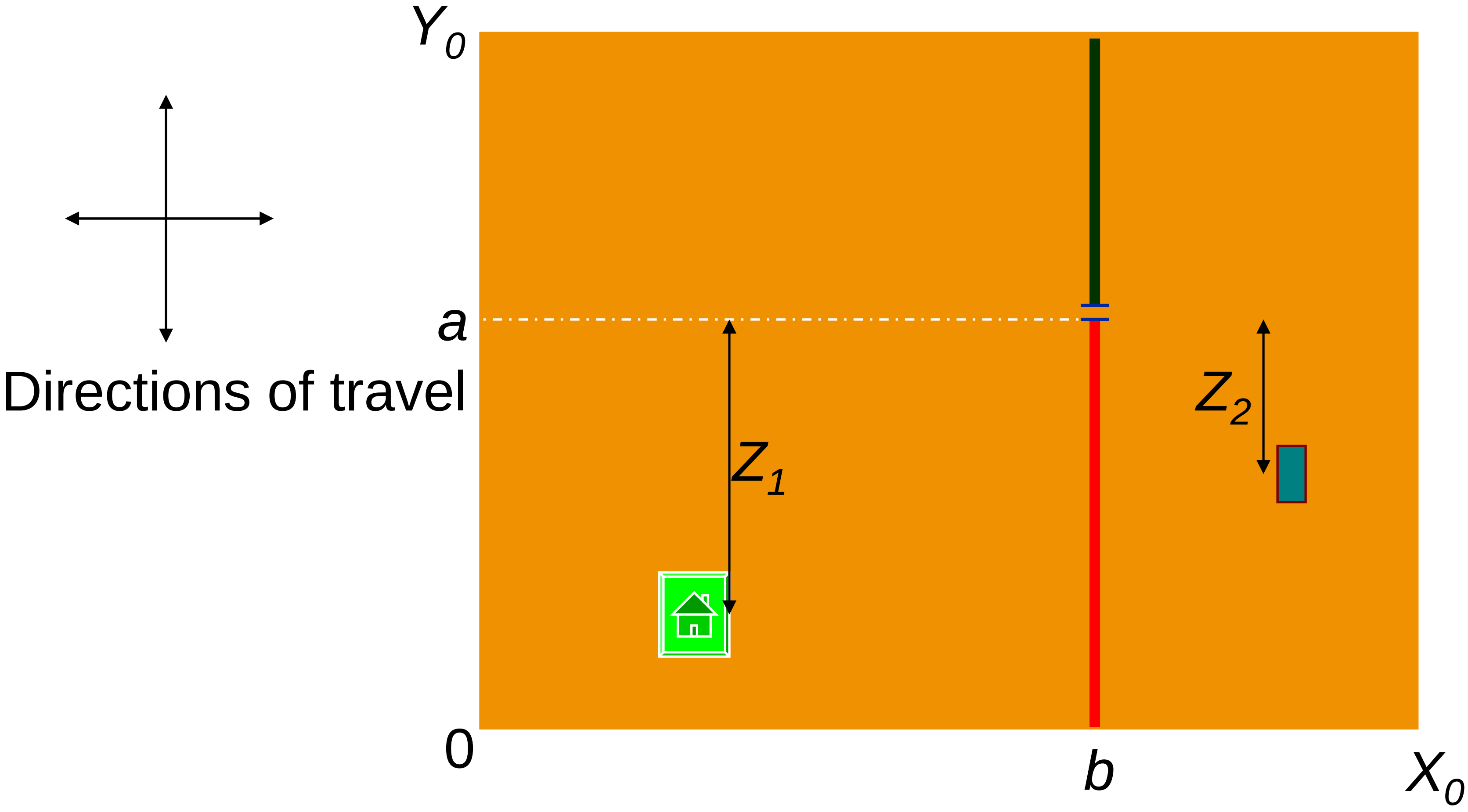
$E[D_e] = E[D_e \mid D_e > 0]\; P\{D_e > 0\}$

$P\{D_e > 0\} = 2\,(ab)(a[X_0 - b])/(X_0 Y_0)(X_0 Y_0)$

$\{D_e \mid D_e > 0\} = 2\ \mathrm{MIN}\,[Z_1, Z_2]$

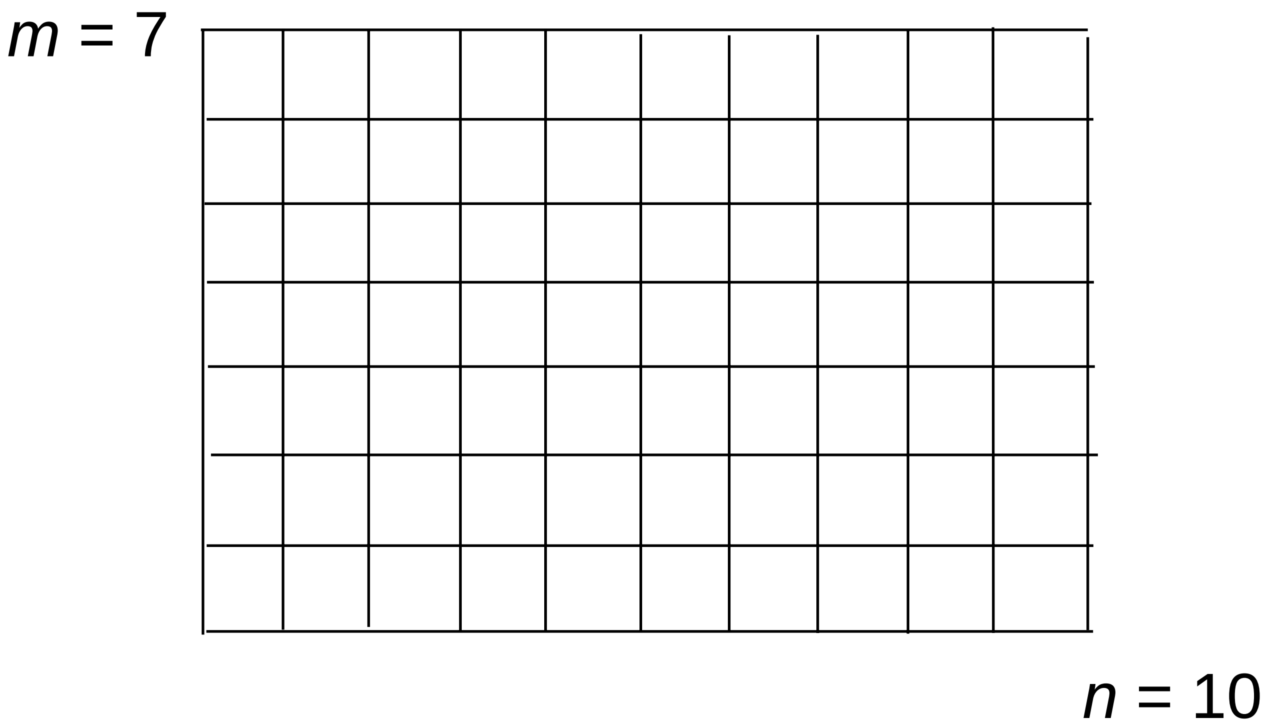
$E[D_e \mid D_e > 0] = (2/3)a$

Add 2 barriers to travel



And what about grids and one way streets?

Two-Way Streets



$$\frac{1}{3}(m+n) \le E[D] \le \frac{1}{3}(m+n+1)$$

Then, what about alternating one-way streets?