Functions of Random Variables

Logistical and Transportation Planning Methods

Massachusetts Institute of Technology

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4 Steps:

1. Define the Random Variables
2. Identify the joint sample space
3. Determine the probability law over the sample space
4. Carefully work in the sample space to answer any question of interest

4 Steps: Functions of R.V.s

1. Define the Random Variables
2. Identify the joint sample space
3. Determine the probability law over the sample space
4. Carefully work in the sample space to answer any question of interest
   - **4a. Derive the CDF of the R.V. of interest, working in the original sample space whose probability law you know**
   - **4b Take the derivative to obtain the desired PDF**

Response Distance of an Ambulance

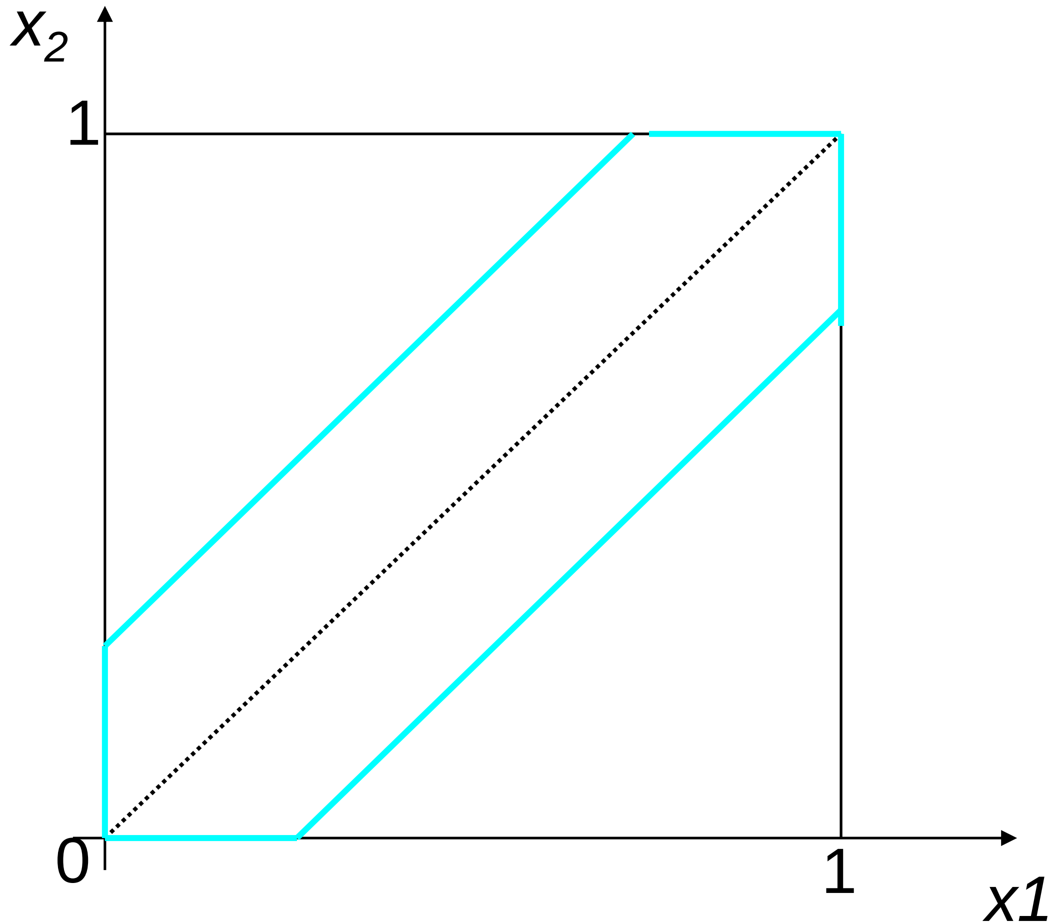
- 1. R.V.;s

  X_1 = location of the accident

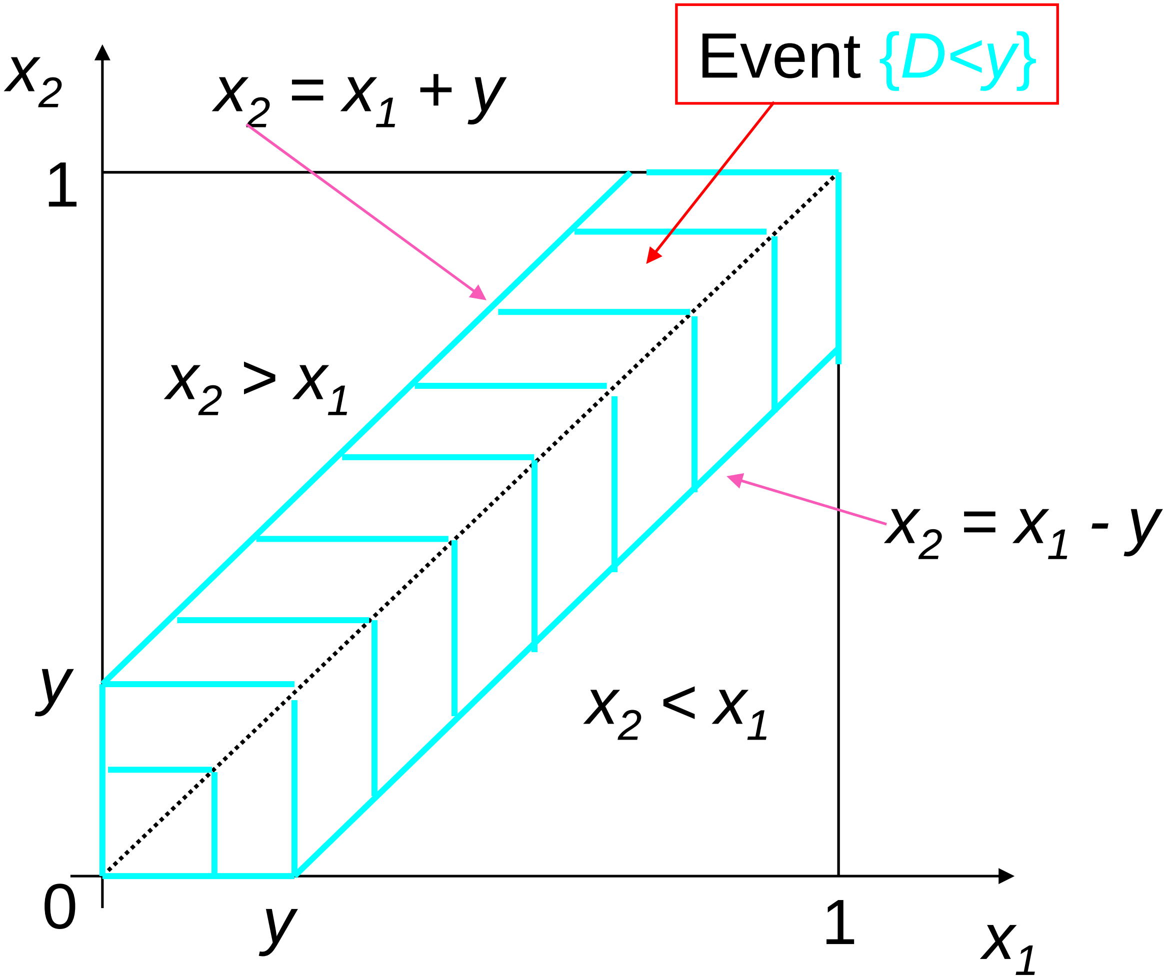
  X_2 = location of the ambulance

  D = response distance = $| X_1 - X_2 |$

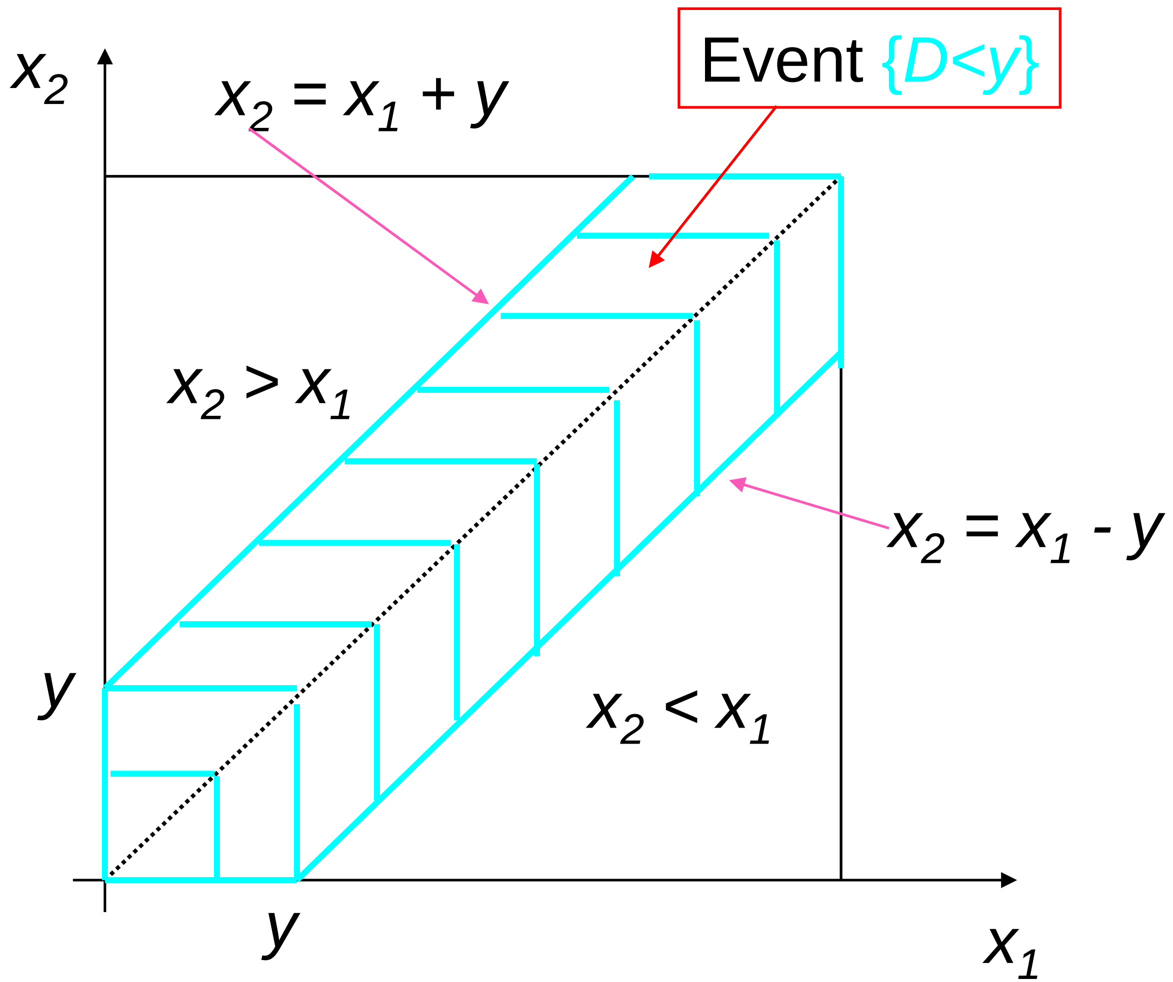
- 2. Joint sample space is unit square in X_1 X_2 plane
- 3. PDF over square is uniform



x_2
1
0
1
x1

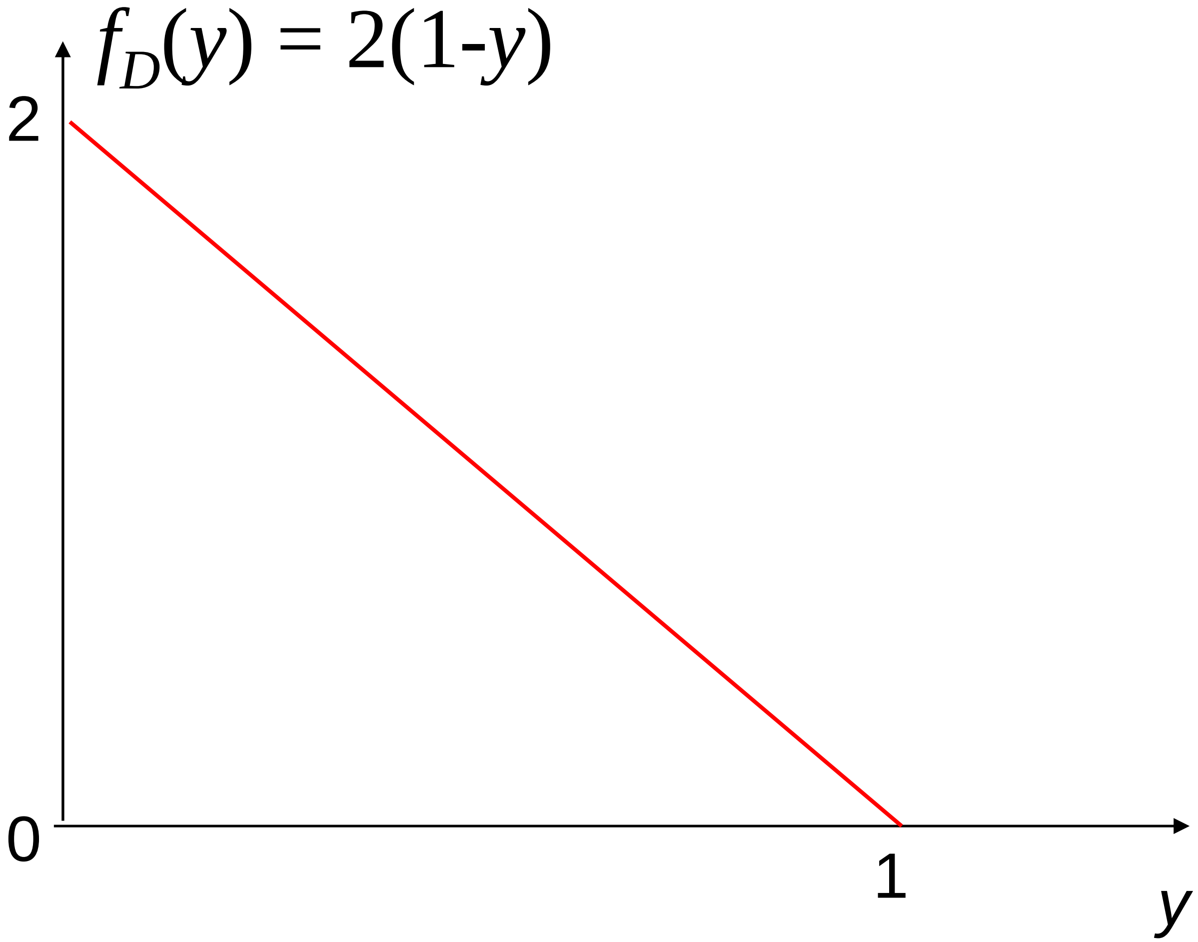


Event {D<y}
x_2
$x_2 = x_1 + y$
1
$x_2 > x_1$
$x_2 = x_1 - y$
y
$x_2 < x_1$
0
y
1
x_1



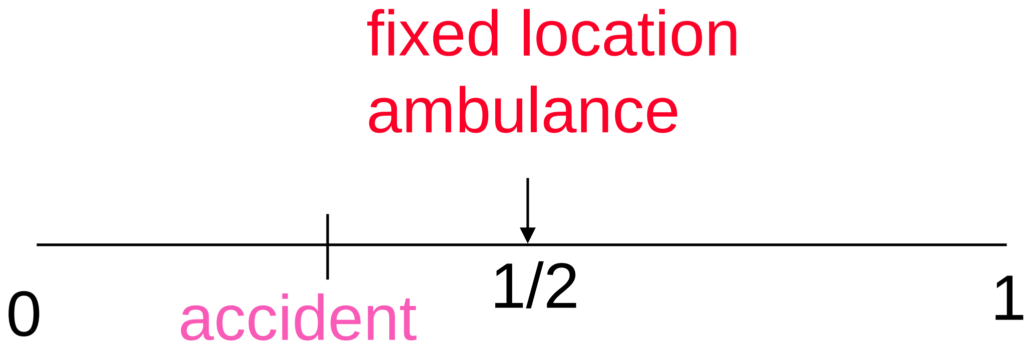
4.a $F_D(y) = P\{D<y\} = 1 - (1-y)^2,\ 0<y<1$

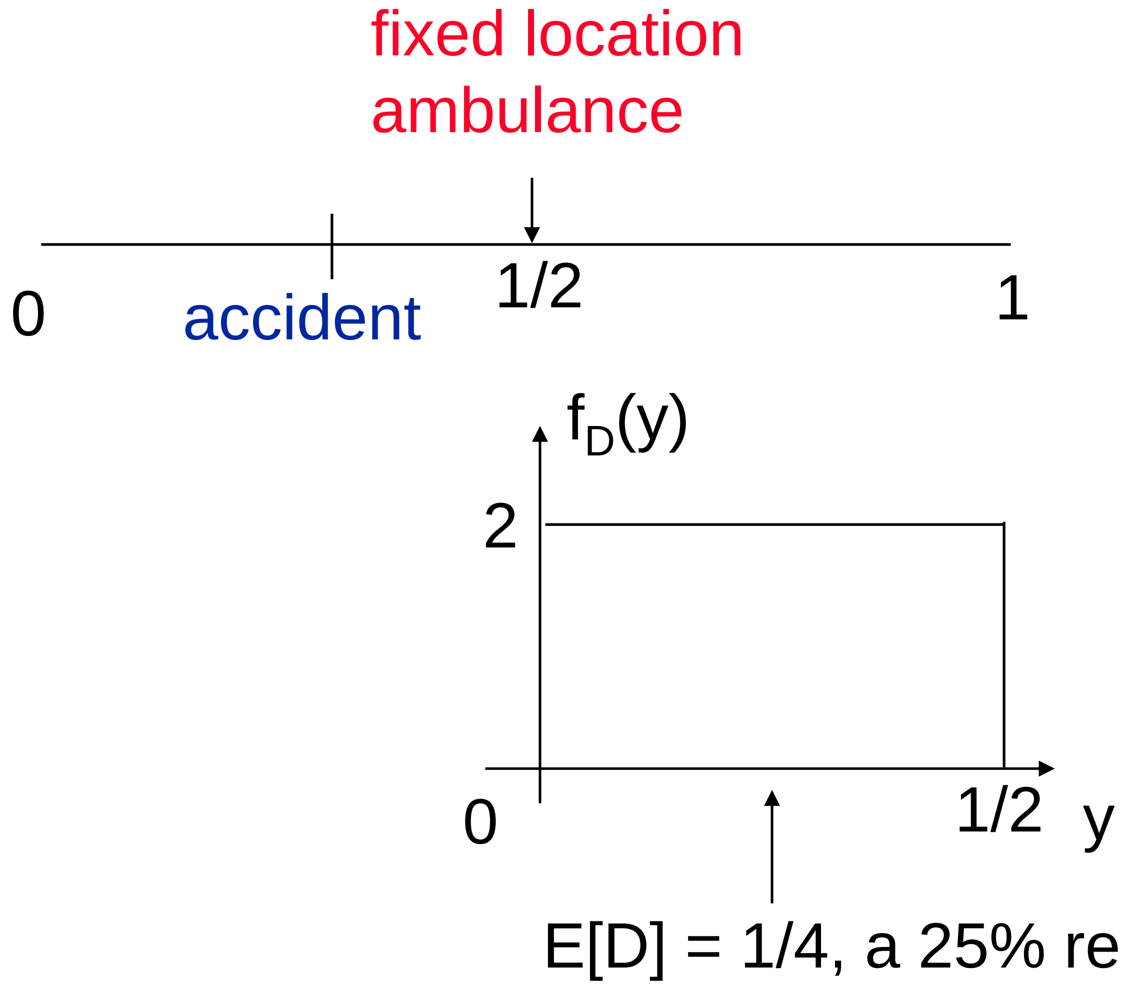
4.b $f_D(y) = 2(1-y),\ 0<y<1.$


$f_D(y) = 2(1-y)$
2
0
1
y

In previous problem, E[D] = 1/3

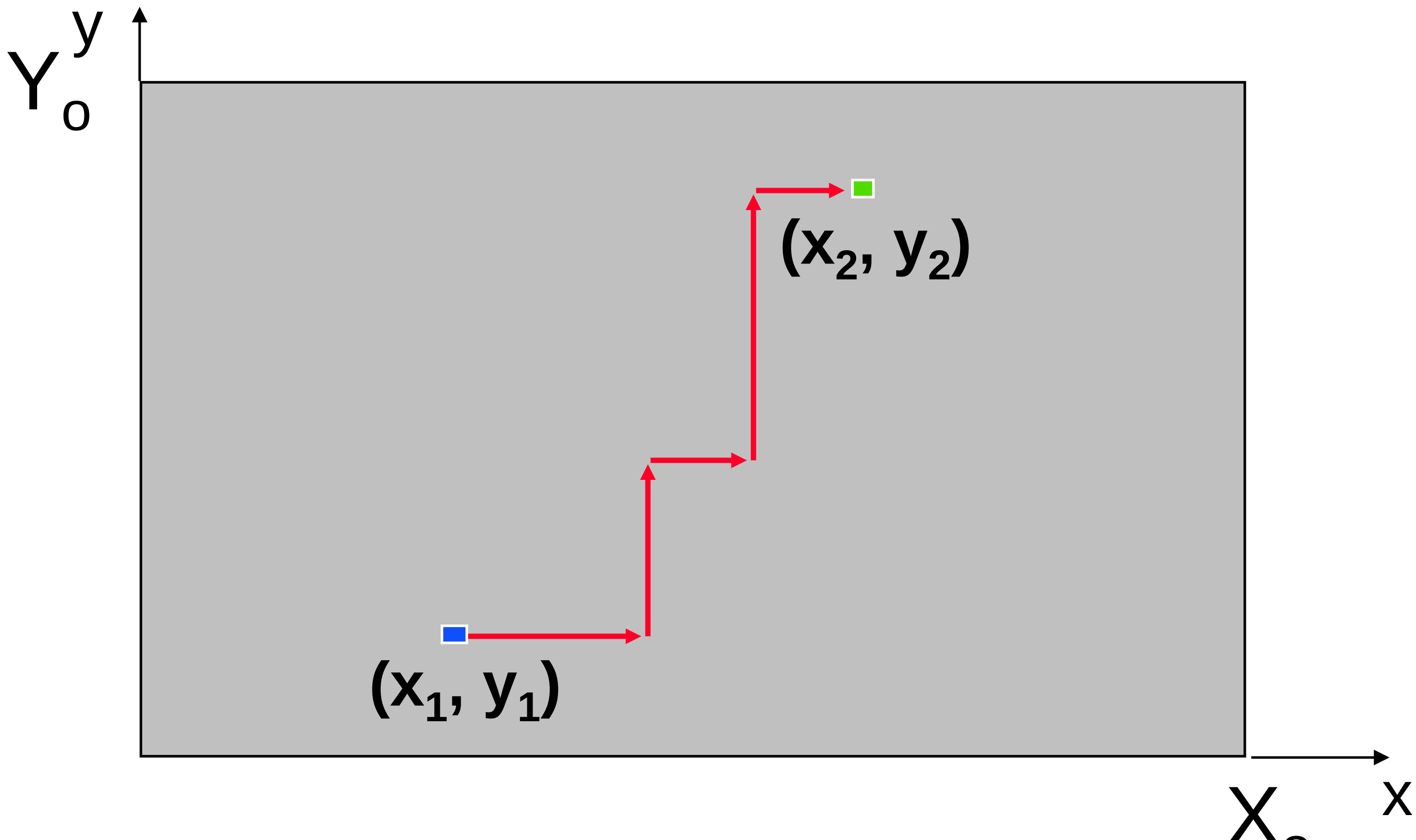
What if we fix the location of the ambulance at $X_2 = 1/2$?





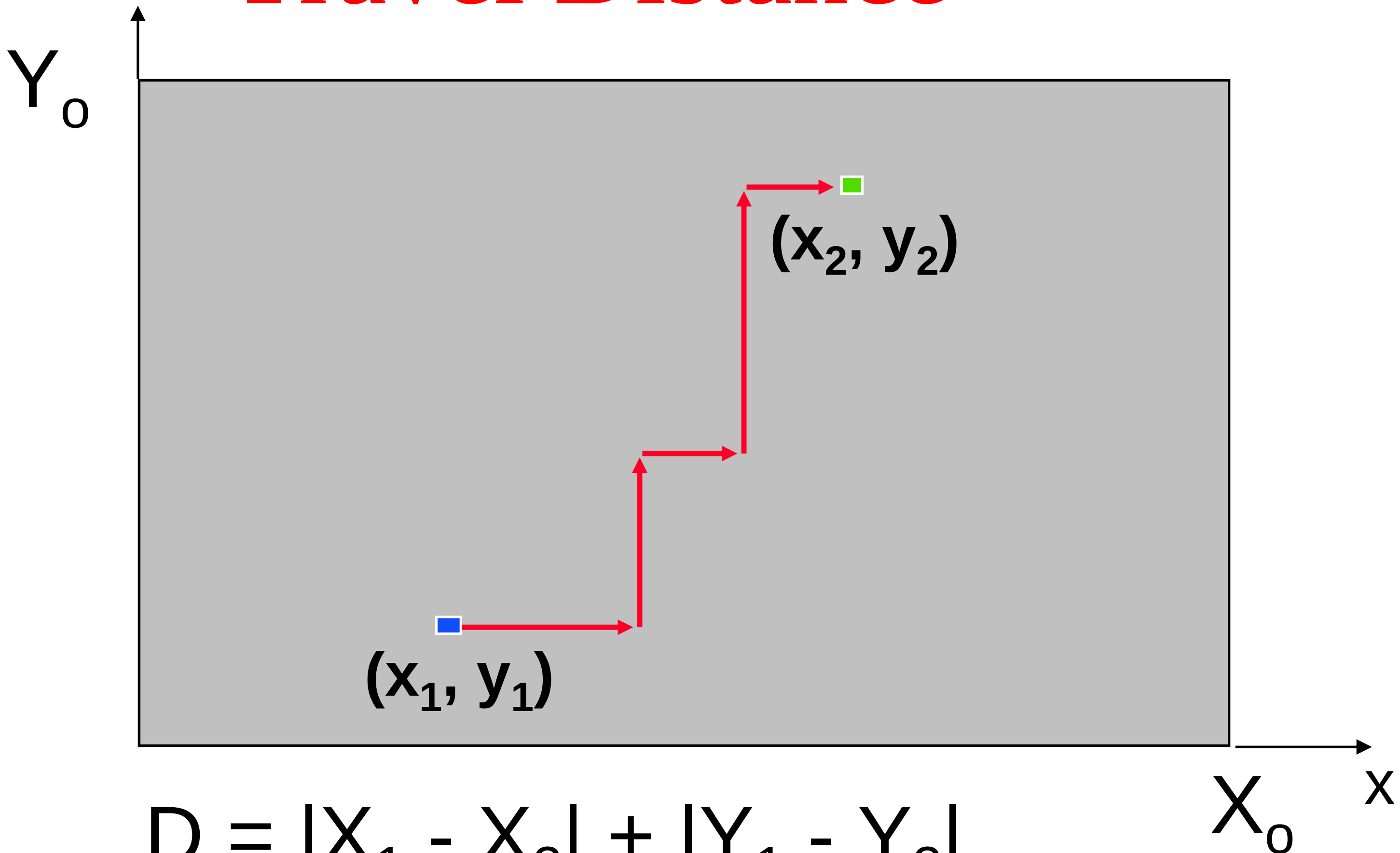
E[D] = 1/4, a 25% reduction

Rectangular Response Area



$$D = |X_1 - X_2| + |Y_1 - Y_2|$$

Scaling to Get Expected Travel Distance



$$D = |X_1 - X_2| + |Y_1 - Y_2|$$

$$E[D] = E[|X_1 - X_2| + |Y_1 - Y_2|]$$

$$E[D] = (1/3)[X_o + Y_o]$$

Ratio of Manhattan to Euclidean Distance Metrics

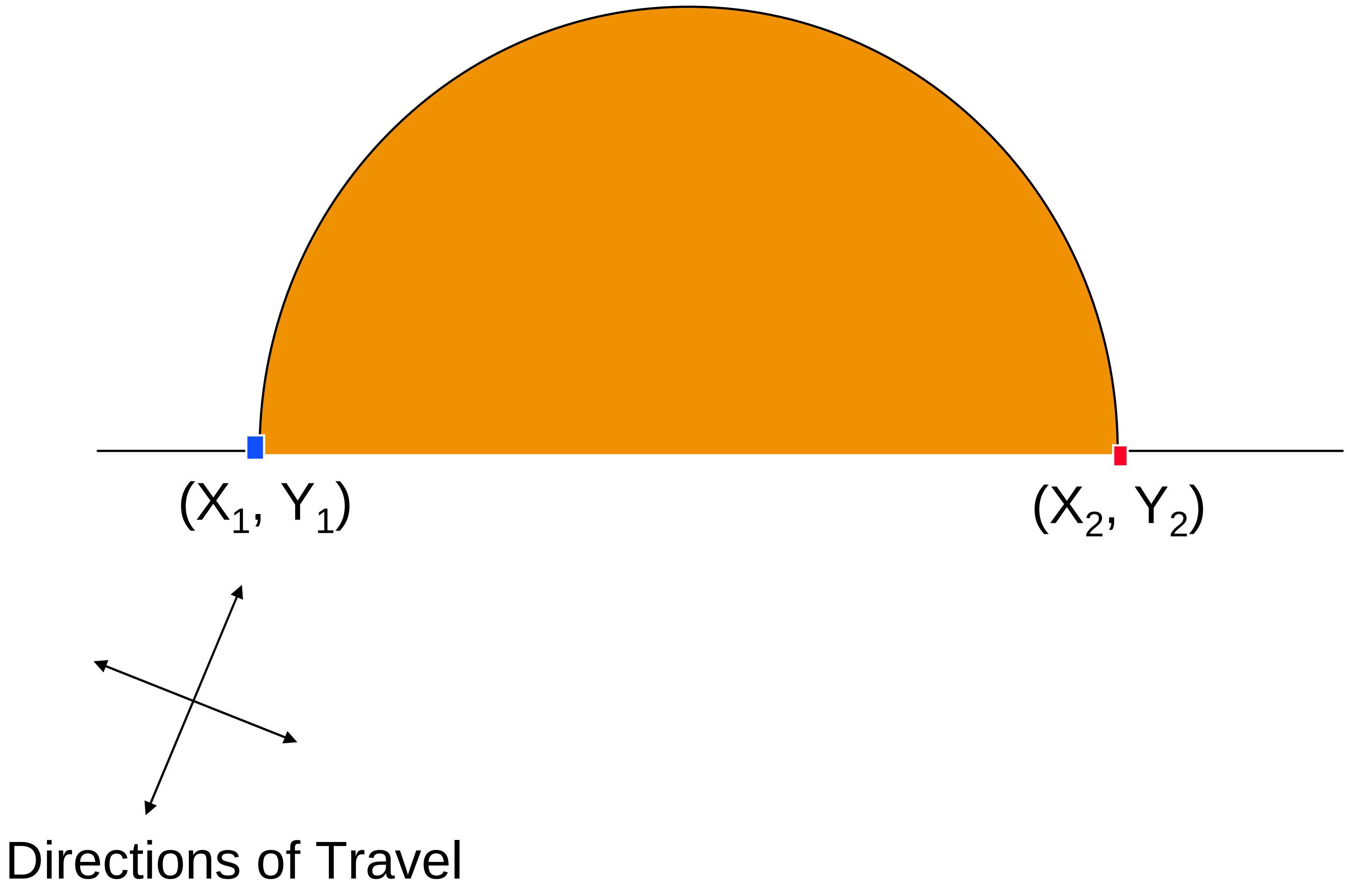
- 1. Define R.V.'s

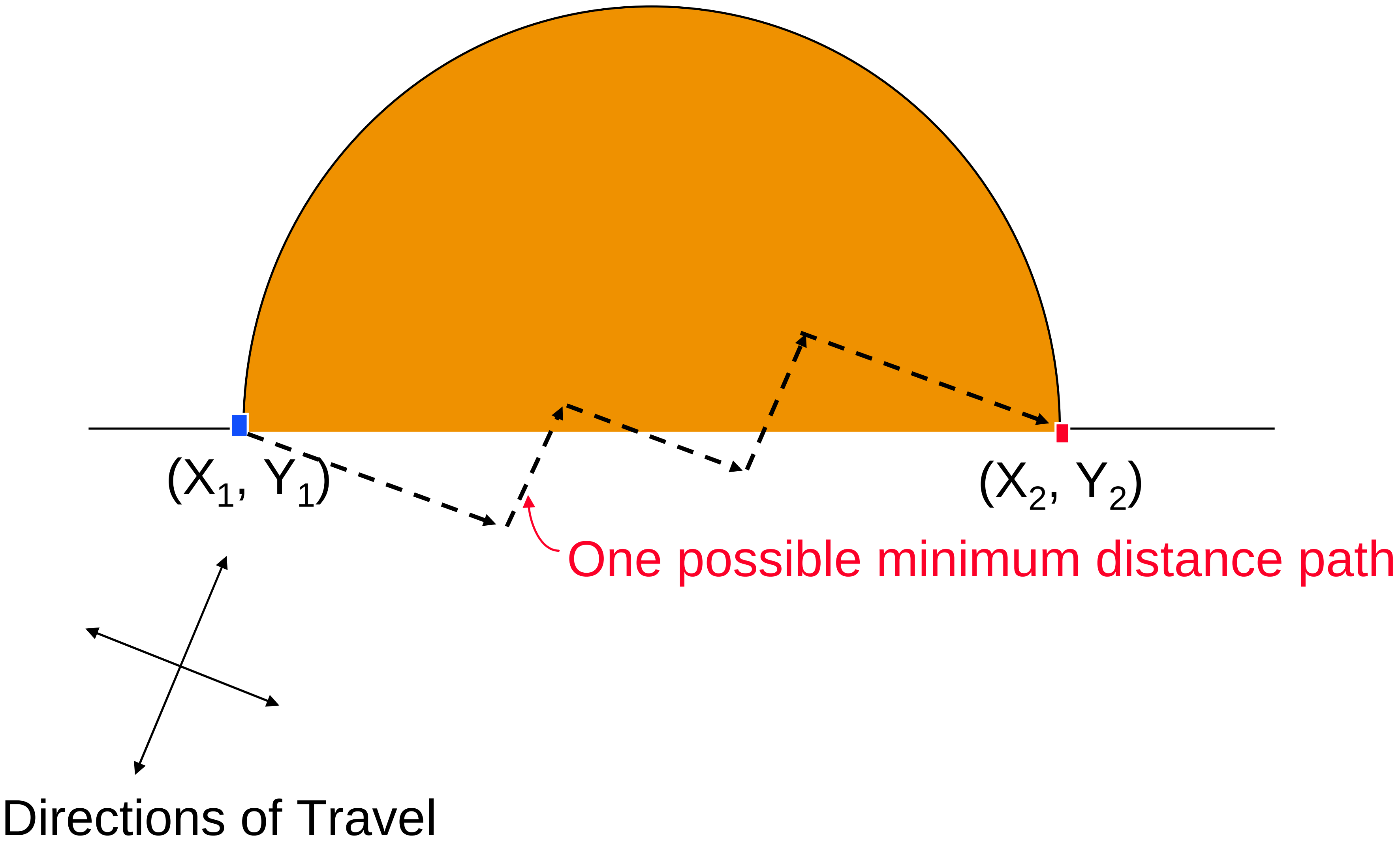
  $D_1 = |X_1 - X_2| + |Y_1 - Y_2|$

  $D_2 = \sqrt{(X_1 - X_2)^2 + (Y_1 - Y_2)^2}$

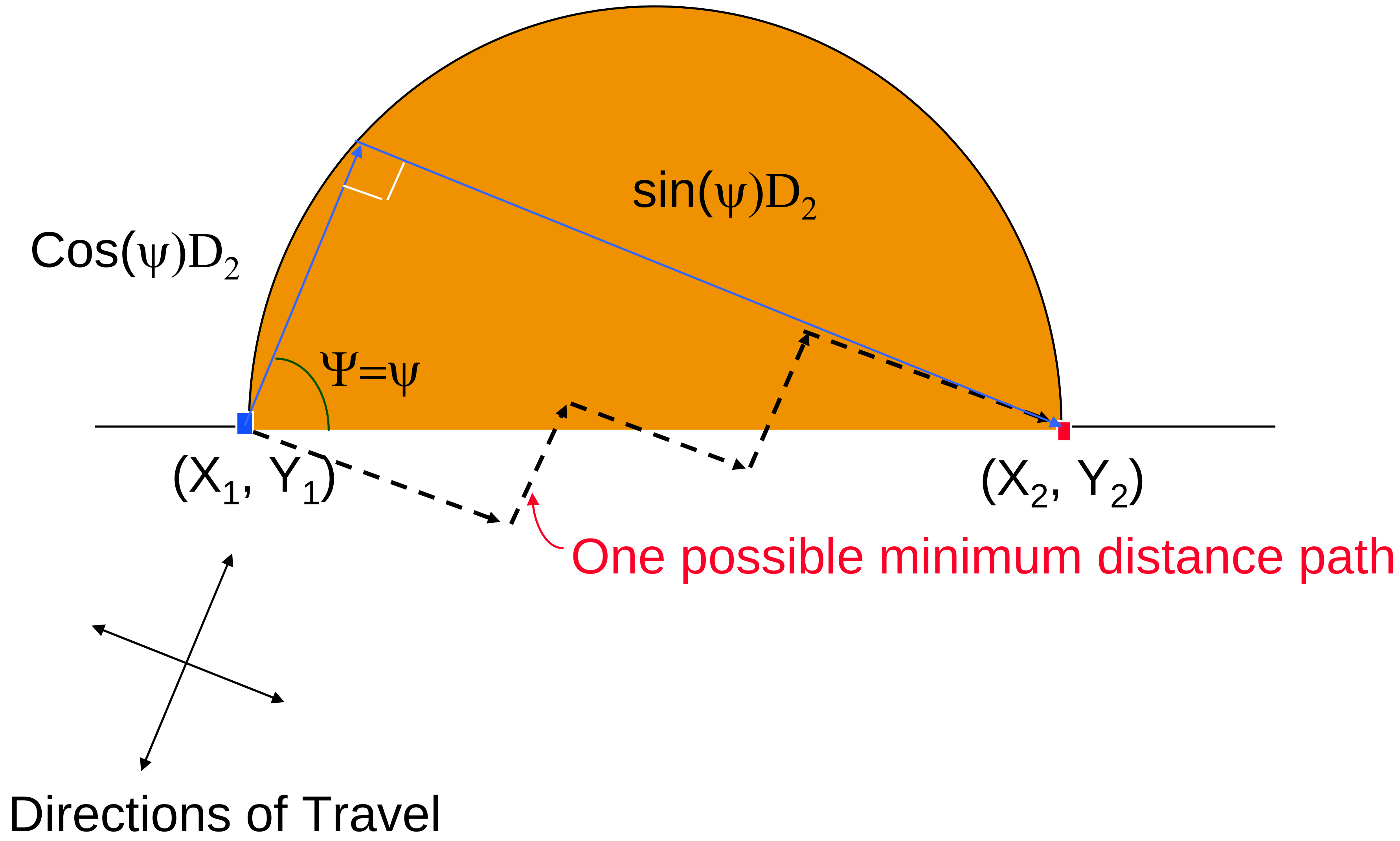
  Ratio = R = D_1 / D_2

  Ψ = angle of directions of travel wrt straight line connecting (X_1 , Y_1) & (X_2 , Y_2)


(X_1, Y_1)
(X_2, Y_2)
Directions of Travel



(X_1, Y_1)
(X_2, Y_2)
One possible minimum distance path
Directions of Travel



sin(ψ)D2
Cos(ψ)D2
Ψ=ψ
(X1, Y1)
(X2, Y2)
One possible minimum distance path
Directions of Travel