Massachusetts Institute of Technology
Logistical and Transportation Planning Methods
1.203J/6.281J/15.073J/16.76J/ESD.216J

Quiz #2 OPEN BOOK
December 6, 2004

Please do Problems 1 and 2 in one booklet and Problem 3 in a separate one. Remember to put your name on each booklet! And please explain all of your work! Good luck!

Problem 1 (30 points): Queuing in Pairs

[Note: This problem is a variation of Problem # 4 in Quiz 1. Several things have, however, changed in this problem: Type 2 customers are described differently; the queueing capacity of the system is 2 instead of 0; and some of the questions are different, as well.]

Consider a queueing system with two parallel servers and two spaces for queueing (in addition to the two servers).

This facility serves two types of customers. Type 1 customers are of the conventional type. They arrive in a Poisson manner at a rate of λ_1 per minute. The service time to these customers has a negative exponential pdf with a rate of μ_1 per minute for each server. Any arriving Type 1 customers who find the system full (i.e., 4 customers in the system) are lost.

Type 2 customers are unusual. They, too, arrive in a Poisson manner at a rate of λ_2 per minute, but they arrive IN PAIRS. [Think of a restaurant, where some of the customers (Type 1) arrive individually, and others (Type 2) arrive in pairs.] Moreover, when service to each one of these pairs begins, the pair occupies simultaneously TWO servers (i.e., both of the servers). The servers work together on each of these Type 2 pairs. The service time to the pair has a negative exponential pdf with a rate of μ_2 per minute. (Note that this means that the two servers begin and end service to each Type 2 pair simultaneously and, together, can serve μ_2 Type 2 pairs per minute if working continuously on Type 2 pairs.) Type 2 customers who do not find at least TWO available spaces upon arrival are lost.

[Please note: A Type 2 pair cannot occupy the servers, unless both servers are available. Thus, in the case where one Type 1 customer is in service and the only customers waiting are one Type 2 pair, the Type 2 pair must still wait in queue and the second server remains idle.]

(a) (20 points) Please draw carefully a state transition diagram that describes this queueing system. Please make sure to define clearly the states of the system.

[Note: You can answer part (b) without answering part (a); but, it will be easier to answer (b), if you have answered (a).]

(b) (10 points) Suppose that there are currently four Type 1 customers in the system. (Obviously, two of them are receiving service and the other two are in queue.) Write an expression for the probability that *exactly 3* state transitions later, there will be exactly two Type 1 customers *and* one Type 2 pair in the system. Your expression should be solely in terms of λ_1, μ_1, λ_2 and μ_2. [A state transition takes place whenever there is a new actual arrival (not a lost arrival) to the system or whenever there is a service completion. The arrival of a new Type 2 pair counts as *one* transition and so does the completion of a service to a Type 2 pair.]

Problem 2 (28 points): Dial-a-Ride

[Hint: This problem's two parts can be answered independently. The questions are not difficult.]

The single-vehicle dial-a-ride problem (DARP) in a Euclidean metric can be described as follows: Consider a vehicle, located at point 0. The vehicle has to serve (pick up and deliver) n customers and return to point 0 by covering as little distance as possible. Customer i (i = 1, 2, …, n) goes from a known origin (labeled node $+i$) to a known destination (node $-i$). The distances between any pair (i, j) of the ($2n$ + 1) points of this problem (i, j = $-n$, …, -1, 0, +1, +2, …, $+n$) are known. There are no vehicle capacity constraints.

Consider then the following simple heuristic for solving DARP:

<u>Step 1:</u> *Without distinguishing origins from destinations*, use the **Christofides heuristic** to construct a Traveling Salesman tour T_0 through the ($2n$ + 1) points of the problem. In completing the heuristic, we make sure not to visit any point twice. (Please note: At the conclusion of this step, we have a tour, T_0, that visits each of the $2n$ customer-related points exactly once before returning to point 0. However, some customer destinations may be visited before the corresponding origins. An example of T_0 for a 3-customer problem is shown in Figure 2a.)

<u>Step 2:</u> Starting from node 0, move on T_0 in some direction (say clockwise) until all nodes are visited. While doing this, skip any node that has been previously visited and any destination whose origin has not been previously visited. Call this DARP tour T_1. (An example of T_1 is shown in Figure 2b and it corresponds to the example for T_0 shown in Figure 2a.)

(a) (16 points) Argue in a few statements that

$$\frac{L(T_1)}{L(DARP)} < 3 \ ,$$

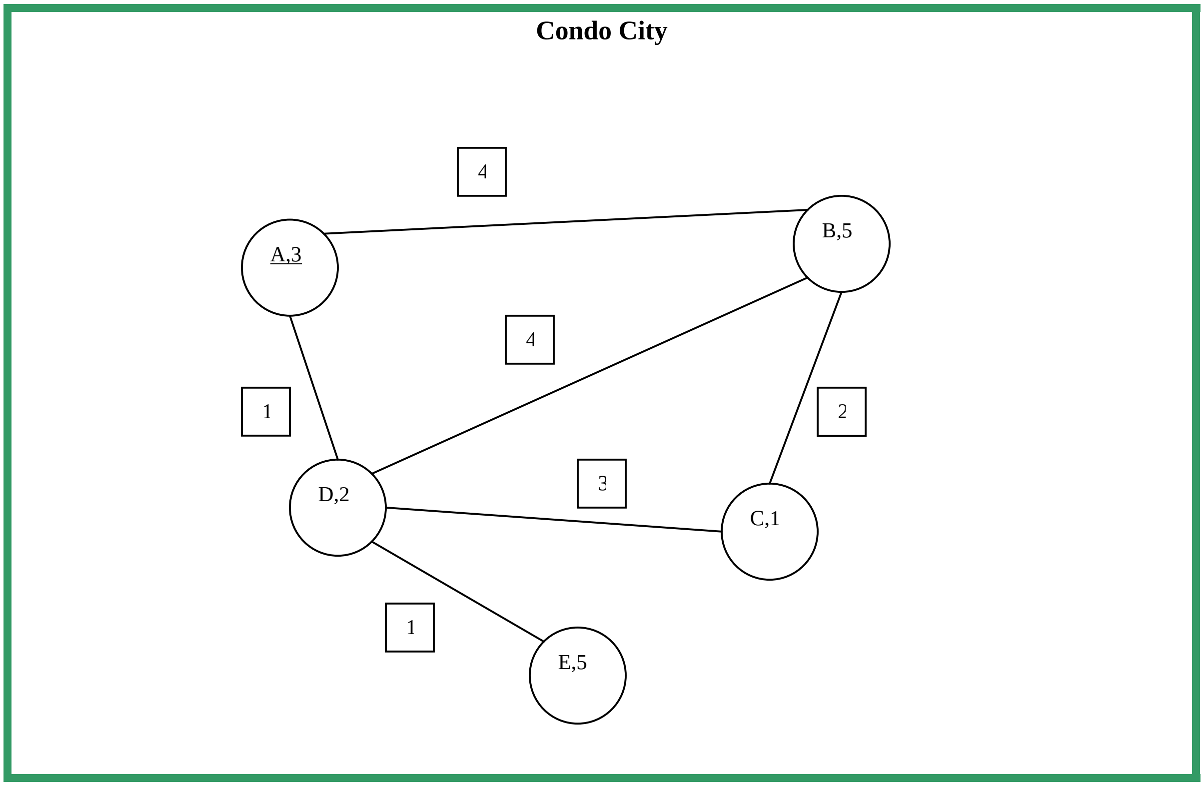
where $L(T_1)$ is the length of T_1 and $L(DARP)$ is the length of the optimal DARP tour. Please explain your reasoning briefly but clearly.

(b) (12 points) Suppose that after completing Step 2 of the above algorithm, one decided to use a 2-exchange heuristic in order to improve the solution. Specifically, assume that one takes T_1 and tries to replace repeatedly 2 edges of T_1 with 2 other edges which were not previously in T_1. [This process is illustrated in Figure 2c for a *traveling salesman problem* where edges (5, 6) and (1, 7) in the tour on the left are replaced by (5, 7) and (1, 6) in the tour on the right.] Please discuss briefly, possibly with a simple example, how the precedence constraints in the DARP complicate the application of the 2-exchange heuristic to the DARP. [Hint: Tour T_1 satisfies all precedence constraints, i.e., customer origins are visited before their destinations. Could this be a problem with the exchange?]

Problem 3 (42 points): Locating Facilities for Condo Complexes in a small town

Consider the 5 node network shown bellow. The nodes in condo city represent condo complexes, and the weights on each node represent the number of families living in each complex (thus the node weights must all be integral), while the arcs represent the distance between the complexes.

Thus A,3: Node A with weight 3



1. (5 Points) We are trying to locate a mail facility for the town. Find an optimal location of the mail facility such that the total weighted travel distance from the complexes to the facility is minimized.

2. (5 Points) What is the minimum number of additional families that must move into complex E in order to make E an optimal location for the mail facility? Please explain your answer briefly.

3. (9 Points) Assume we never built the mail facility in part 1. Assume also that we now wish to house 4 additional families somewhere on this network, all at one point. All 4 of these families are physically challenged and need to be located at an optimal point on the network, i.e., a point that would coincide with the mail facility. The obvious answer is, of course, to locate these 4 families at the location you identified in response to Part 1 of this problem. Unfortunately, this location happens to be the only point on the entire network where no additional families (beyond the ones originally in the complexes) can be located. With the exception of this restriction, there are two types of places where these four families can be located: either at an existing complex (e.g., if we located the four families at node A, the weight of A would be increased to 7), or at a point on some arc of the network (i.e., by building a new complex F on some arc with weight 4). Identify on the network ALL locations (nodes and points along arcs) where we can locate this complex; such that an optimal location for the mail facility will indeed co-incide with the complex where the four additional families will be housed.

4. (5 Points) Return to the original network of Part 1. Now 50 additional families would like to move into the town. However, currently the complex where you located the mail station in Part 1 is filled to capacity. However, the rest of the complexes have infinite capacity. Is it possible to allocate these additional 50 families in the remaining 4 complexes in such a way that the optimal location of the mail center is not changed? If so state one way this can be done (i.e., how many additional families should move into each complex); if not, give a short proof of why not.

Now let us only consider Complexes A, D and E, and the two arcs that connect them (each with length equal to 1 mile). In other words the "network" consists only of nodes A, D and E and of the arcs (A, D) and (D,E). Let the number of families living in each complex be as follows: A = a, D = d and E = e. We are interested in locating a private warehouse with one delivery truck somewhere on this route. Assume the following:

- The weights are proportional to the number of families at each node (i.e.,. the fraction of calls for deliveries arising from node A is a/(a+d+e).

- Once the facility is located, it serves all three complexes and calls are handled in a FIFO order, one at a time.

- The delivery truck travels to each delivery point at speed v = 1 mile per hour and returns to the facility at speed v=1 mile per hour.

- The delivery truck stays at the delivery point for a length of time which is exponentially distributed with a mean of 1 hour, i.e., $f_T(t) = 1 \cdot e^{-1 \cdot t}$ for $t \geq 0$.

Our goal in each part is to find the location that minimizes the expected total response time. Total response time is defined as the sum of (i) the waiting time until the truck is dispatched and (ii) the service time, i.e. the time needed for the truck to travel to the delivery point, spend the required time at the delivery point and then travel back to the facility.

- For Part 5, let the total rate of calls for deliveries be very small, i.e., $\lambda = 0^+$ calls per hour.

  5. (5 Points) If a = e and d = 0, is it true that every point on the network is an optimal location for the facility?

- For parts 6 and 7, let the total rate of calls for deliveries is $\lambda = .04$ calls per hour.

  6. (5 Points) .If a = e = 4 and d = 5, where is the optimal location for the facility to minimize expected response time?

  7. (8 Points) If a = 3, e = 0 and d = 8 and we have located the facility at 0.5 miles from A, what is the expected total response time? Please state your answer in terms of units of distance away from complex A (for example, the facility is located at x = 0.5.)