Homework Set #8

A plain concrete column is subjected to a random axial load W with lognormal distribution, mean value $m_W = 3000\,\text{kN}$ [kN = Kilo Newton, a unit of force] and coefficient of variation $V_W = 0.2$. The resulting compressive stress σ is given by $\sigma = \frac{W}{A}$, where A is the cross-sectional area of the column. The crushing strength of concrete f_c also has lognormal distribution, with mean value $m_{f_c} = 35{,}000\,\text{kN/m}^2$ (m = meter) and coefficient of variation $V_{f_c} = 0.2$.

(a) Obtain the probability density function (PDF) of the applied stress σ (which depends on A).

(b) If the column has a 0.40 m x 0.40 m square cross-section, what is the probability P_F that it fails i.e. $P_F = P[\sigma > f_c]$? Assume W and f_c are independent.

Hint: $f_C < \sigma \;\Leftrightarrow\; \frac{f_C}{\sigma} < 1 \;\Leftrightarrow\; \ln\left(\frac{f_C}{\sigma}\right) < 0$

(c) Determine the required cross-sectional area of the column for a target failure probability of 10^{-3}.

(d) Find the mean stress $m_\sigma = \frac{m_W}{A}$ as a function of the failure probability P_F. Plot $m_\sigma(P_F)$ against $\log(P_F)$ for P_F between 10^{-4} and 10^{-1}.