Recitation 10 - Problems

May 4th and 5th

Problem 1#

Figure 1 shows a rectangular channel of width $b = 10\ m$ and Manning's $n = 0.015$ (SI) that carries a discharge $Q = 50\ m^3/s$. The channel has four stretches with different slopes. In the upstream region of the first stretch, the channel encounters a gate with an opening $h_g = 1.2\ m$ and contraction coefficient $C_V = 0.6$. The last stretch discharges into a large lake, where water elevation with respect to the channel bottom is $h_D = 2.0\ m$.

Sketch the free surface shape along the channel. Name the surface profiles that appear in your sketch and specify the relevant values of the water depth (the depths at the beginning and end of each profile). Calculate also the channel depth upstream the gate, h_U. Assume all stretches to be long, so that asymptotic tendencies are reached.

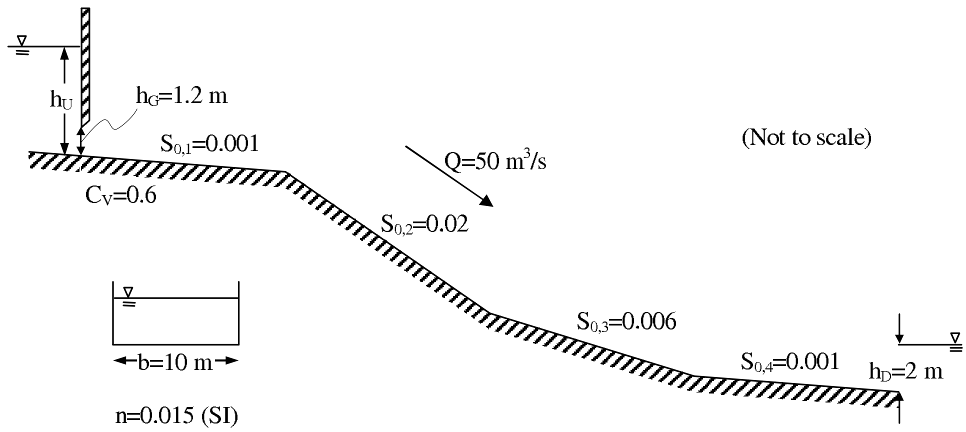


Figure 1: Channel with various slopes in Problem 1.

Problem 2

a) Consider a channel of horizontal slope ($S_0 = 0$). Show in a skecth how each of the possible profiles (Z1 through Z3) looks like.

b) Consider a channel of adverse slope ($S_0 < 0$). Show in a skecth how each of the possible profiles (A1 through A3) looks like.

Problem 3*

A rectangular channel of width $b = 5\ m$ discharges a flowrate of $Q = 10\ m^3/s$. The channel is comprised of two long stretches of slopes $S_{0,1} = 0.005$ and $S_{0,2} = 0.008$, as shown in Figure 2. The stretches are connected by a gradual step up, of total height $H = 0.5\ m$, which takes place over a short horizontal distance. The channel discharges into a large reservoir whose level is located 0.5 m above the channel bottom.

a) In the initial years after the channel is built, its Manning's coefficient is $n = 0.015$. Sketch the free surface shape along the channel. Name the surface profiles that appear in your sketch and specify the relevant values of the water depth. Assume all stretches to be long, so that asymptotic tendencies are reached.

 # - Adapted from: Puertas Agudo, Jerónimo; Sánchez Juny, Martí. *Apuntes de Hidráulica de Canales.* Escuela Técnica Superior de Ingenieros de Caminos, Canales y Puertos. Universidad de La Coruña. February, 2000.

b) Estimate the minimum value of L so that the assumption of long stretches is a good one.

c) Due to lack of maintenance, the roughness of the channel increases. Repeat part **a** with $n = 0.03$. Again, assume all stretches to be long.

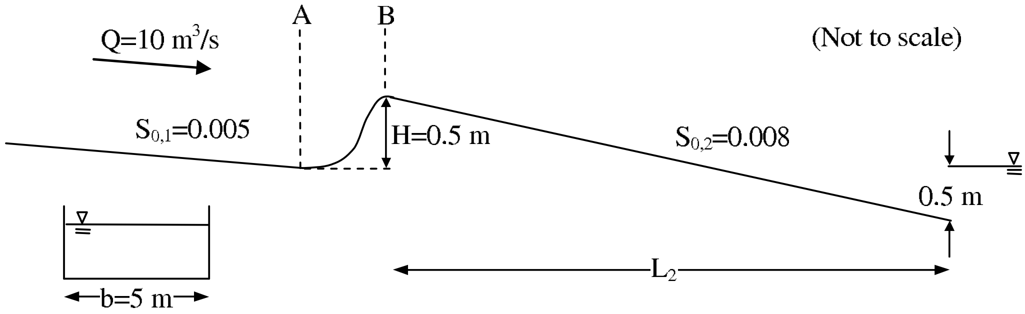


Figure 2: Step on channel bottom in Problem 3.

Problem 4*

A team of engineers is designing an aqueduct to cross a cliff (see Figure 3). The aqueduct channel will consist of two straight segments, AB and BC, each spanning a horizontal length of 50 m. Segment BC is located above a river and it is not possible to place any pilar along it. For this reason, the engineers want to minimize the weight of water on BC. To this end, they determine that flow must be supercritical along BC.

The channel will have a rectangular section of width $b = 1\ m$ and sidewall height $h_{SW} = 1.15\ m$. To avoid spills to the cliff, the minimum clearance along the aqueduct is 15 cm (i.e., $h \leq 1\ m$). The discharge is $Q = 1\ m^3/s$ and Manning's $n = 0.015$. The channel continues upstream and downstream of the aqueduct with a slope $S_{0,3} = 0.004$. Both segments AB and BC are assumed long, so that asymptotic tendencies are reached. The water volume on BC is estimated by approximating the free surface curve to a straight line.

a) Determine the range of slopes $S_{0,1}$ and $S_{0,2}$ which (1) yield supercritical flow everywhere along BC and (2) satisfy the minimum clearance.

b) What are the values of $S_{0,1}$ and $S_{0,2}$ that minimize the weight of water on BC?

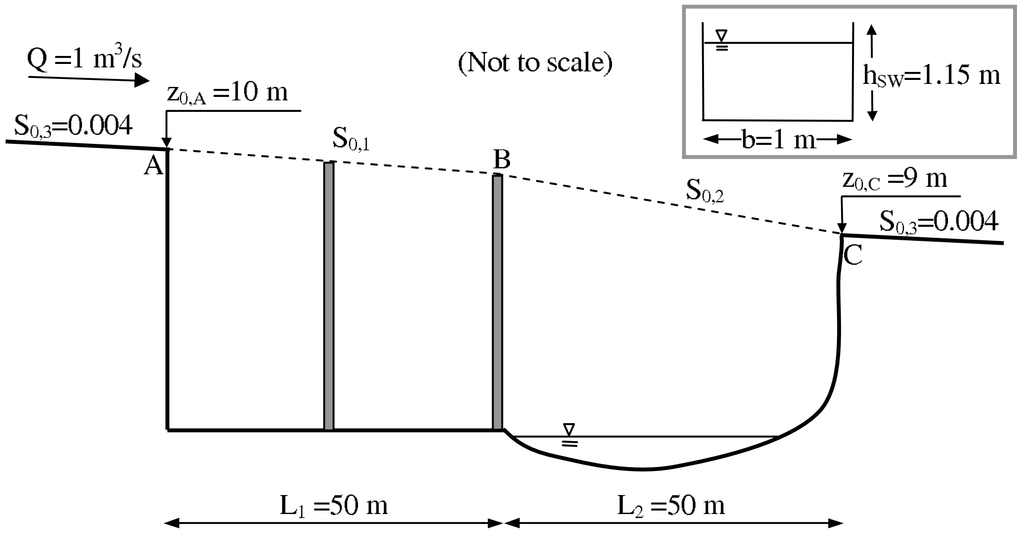


Figure 3: Aqueduct crossing a cliff in Problem 4.

*** - Adapted from:** Bonillo Martínez, Juan J.; Puertas Agudo, Jerónimo; Juncosa Rivera, Ricardo. *Problemas de Hidráulica*. Escuela Técnica Superior de Ingenieros de Caminos, Canales y Puertos. Universidad de La Coruña. July, 2002.

RECITATION 10 – SOLUTIONS

– PROBLEM Nº 1:

Since the critical depth depends only on the geometry of the section and the flowrate, it has the same value for all four stretches:

$$h_C = \left(\frac{Q^2}{b^2 g}\right)^{1/3} = 1'36\ m$$

The normal depths are calculated from Manning's equation by iteration:

$$h_n^{(k+1)} = \left(\frac{Qn}{b\sqrt{S_0}}\right)^{3/5}\left(1+2\,\frac{h_n^{(k)}}{b}\right)^{2/5},\ \text{take } h_n^{(0)}=0.$$

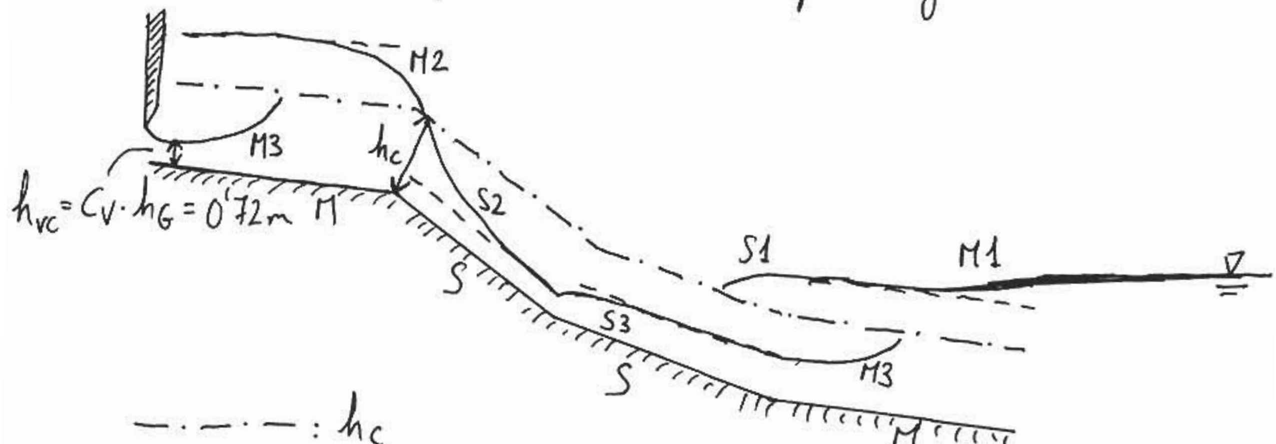
Since the channel is rectangular, the conjugate depth of h_n is given by

$$\frac{h_{n,\,conj}}{h_n} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_n^2}\right]$$

Applying these expressions, we construct the following table:

STRETCH	S_0	h_n	$h_{n,\,conj}$	h_c	M or S?
1 & 4	0'001	1'911 m	0'93 m	1'36 m	M
3	0'006	1'06 m	1'72 m	"	S
2	0'02	0'72 m	2'33 m	"	S

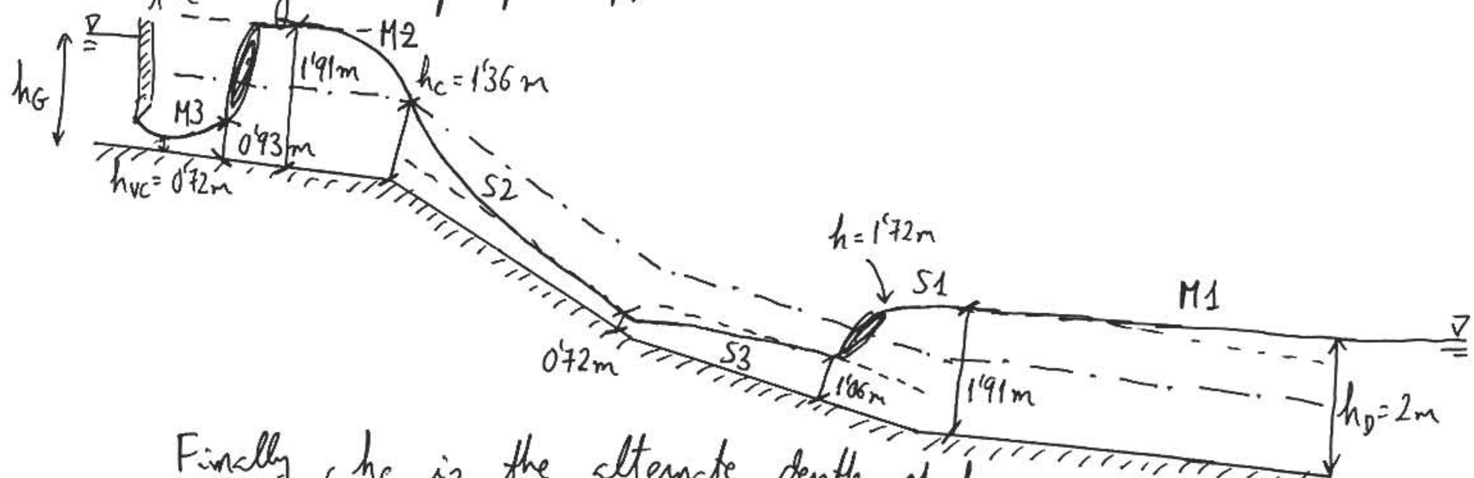
With this information, and using the boundary conditions for each streach, we draw the following sketch:



$- \cdot - \cdot -$: h_c

$- - - - - -$: h_n

- In the outflow from the gate, since $h_{n1,conj} = 0'93\,m > 0'72\,m = C_V \cdot h_G$, there is a hydraulic jump from M3 to M2 (otherwise, we would have a drowned outflow).
- In the transition between stretches 3 and 4, since $h_{n4,conj} = 0'93\,m < h_{n3} = 1'06\,m$ (or equivalently, since $h_{n3,conj} = 1'72\,m < h_{n4} = 1'91\,m$), h_{n4} has more "MP" than h_{n3} and it "gets into" the steep slope. So the hydraulic jump happens in the steep slope, from S3 to S1.



Finally, h_G is the alternate depth of h_{vc}, i.e.,

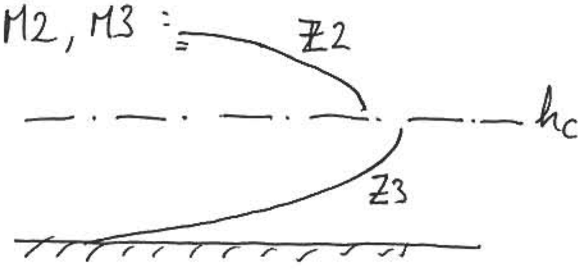
$$h_G + \frac{V_G^2}{2g} = h_{vc} + \frac{V_{vc}^2}{2g} \Rightarrow h_G = 3'18 - \frac{1'276}{h_G^2} \Rightarrow \underline{\underline{h_G = 3'04\,m}}$$

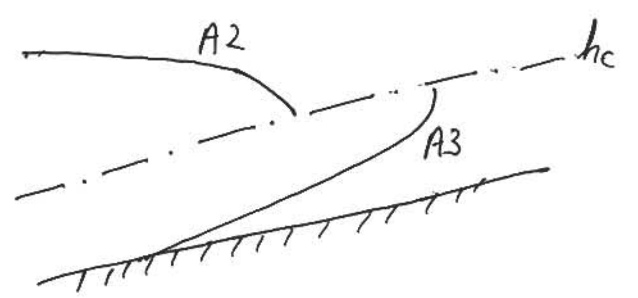
– PROBLEM Nº 2:

Manning : $S_0 = \dfrac{n^2 V_n^2}{Rh_n^{4/3}}$, $S_0 = 0 \Rightarrow \dfrac{n^2 V_n^2}{Rh_n^{4/3}} = 0 \Rightarrow h_n \to \infty$

$S_0 < 0 \Rightarrow \nexists\, h_n$, since $\dfrac{n^2 V^2}{Rh^{4/3}} > 0$ for all h $\Big\}$ No Z1 or A1

Other than that, the curves Z2, Z3 and A2, A3 look similar to M2, M3 :





– PROBLEM Nº 3:

		$n = 0'015$		$n = 0'03$		
STRETCH	S_o	h_n	M or S?	h_n	M or S?	h_c
1	0'005	0'65 m	S	1'04 m	M	0'74 m
2	0'008	0'56 m	S	0'89 m	M	0'74 m

a) $n = 0'015$

Both stretches are S. Hypothesis:

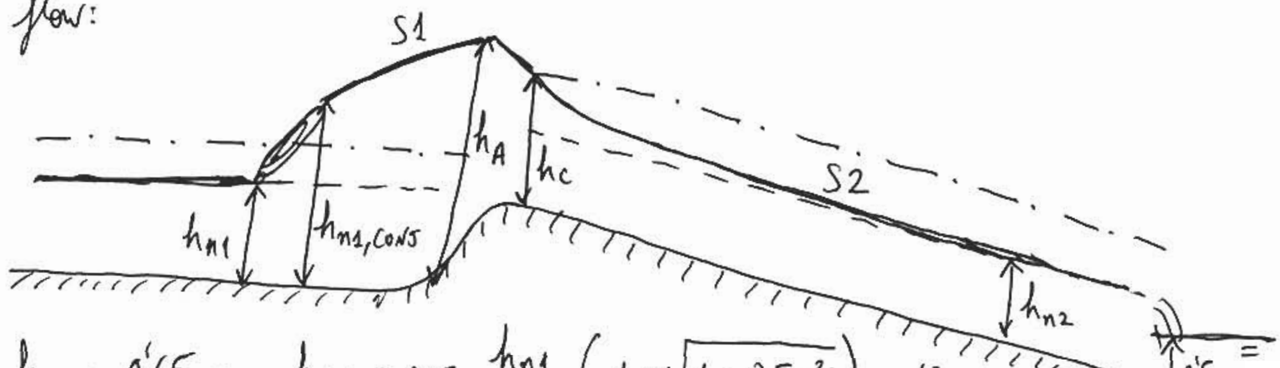
Flow in stretch 1 will proceed with $h = h_{n1}$ if it has enough energy to pass the step. Check:

$$E_{n1} = h_{n1} + \frac{V_{n1}^2}{2g} = 1'13\,m$$

Short transition of converging flow ⇒ Energy is conserved.

$$E_{C2} = \frac{3}{2} h_{C2} = 1'11\,m$$

Since $E_{n1} < E_{C2} + H = 1'61\,m$, water has not enough energy to pass. Water will accumulate before the step, generating subcritical flow:



$$h_{n1} = 0'65\,m, \quad h_{n1,CONJ} = \frac{h_{n1}}{2}\left(-1 + \sqrt{1 + 8\,Fr_{n1}^2}\right) = 0'84\,m$$

$$\left.\begin{aligned} E_A &= h_A + \frac{V_A^2}{2g} = h_A + \frac{Q^2}{2g\,b^2}\frac{1}{h_A^2} \\ E_A &= E_{C2} + H = 1'61\,m \end{aligned}\right\} \Rightarrow h_A = 1'61 - \frac{0'204}{h_A^2} \Rightarrow h_A = 1'52\,m$$

$$h_{n2} = 0'56\,m$$

b) $\dfrac{dh}{dx} = \dfrac{S_0 - S_f}{1 - Fr^2} \Rightarrow L \geq -\dfrac{(h_c - h_{n2}) \cdot (1 - \overline{Fr^2})}{S_0 - \overline{S_f}}$

$$\left.\begin{array}{l} \text{at } h = h_c \rightarrow Fr^2 = 1, \quad S_f = \dfrac{n^2 V^2}{Rh^{4/3}} = 3'5 \cdot 10^{-3} \\ \text{at } h = h_{n2} \rightarrow Fr^2 = \dfrac{V_{n2}^2}{g h_{n2}} = 2'32, \quad S_f = S_0 = 0'008 \end{array}\right\} \overline{Fr^2} = 1'66, \ \overline{S_f} = 5'8 \cdot 10^{-3}$$

$$\underline{\underline{L \geq}} -\frac{(0'74 - 0'56)(1 - 1'66)}{8 \cdot 10^{-3} - 5'8 \cdot 10^{-3}} = \underline{\underline{54 \text{ m}}}$$

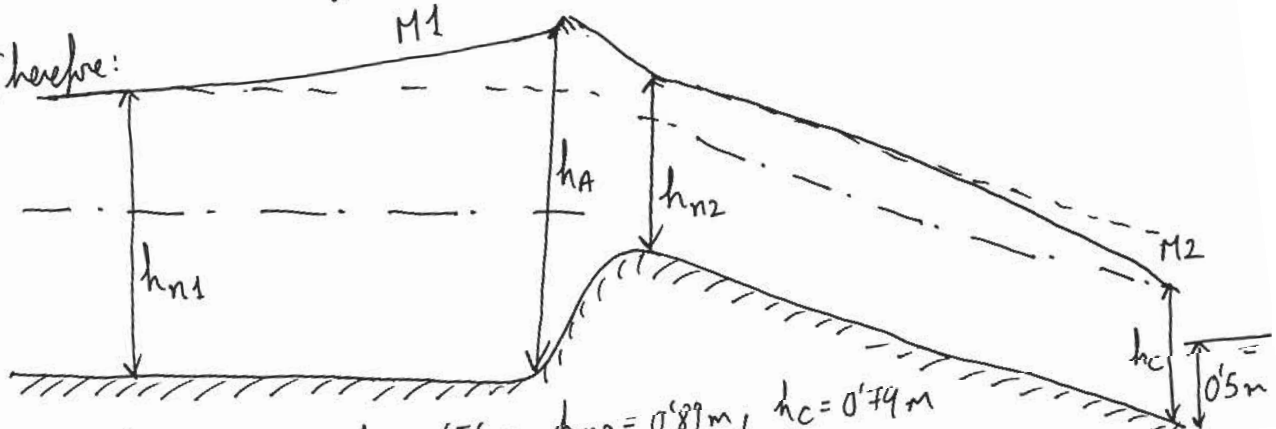
c) $\underline{n = 0'03}$

Conservation of energy between A and B:

$$\left.\begin{array}{l} A \rightarrow E_A = h_A + \dfrac{V_A^2}{2g} \\ B \rightarrow E_B = h_{n2} + \dfrac{V_{n2}^2}{2g} \end{array}\right\} E_A = E_B + H \Rightarrow h_A = 1'56 \text{ m}$$

(is the solution corresponding to subcritical flow)

Therefore:



$h_{n1} = 1'04$ m, $h_A = 1'56$ m, $h_{n2} = 0'89$ m, $h_c = 0'74$ m

- PROBLEM N° 4:

a) Conditions:

- $h_{max} = 1$ m
- $Fr < 1$ along BC
- $50 S_{0,1} + 50 S_{0,2} = 1 \Rightarrow S_{0,1} + S_{0,2} = 0'02$

For any $S_{0,1}$ & $S_{0,2}$, the critical depth is

$$h_C = \sqrt[3]{\frac{Q^2}{b^2 g}} = 0'47 \text{ m}$$

If $S_{0,1} = S_{0,2} = 0'01 \Rightarrow h_{n,1} = h_{n,2} = 0'40$ m $< h_C \Rightarrow$ both are S.

Outside the aqueduct, $S_{0,3} = 0'004 \Rightarrow h_{n,3} = 0'57$ m $> h_C \Rightarrow$ M. The conjugate depth of $h_{n,3}$ is

$$h_{n,3,\,CONJ} = \frac{h_{n3}}{2}\left(1 - \sqrt{1 + 8Fr_{n3}^2}\right) = 0'38 \text{ m}$$

Since $h_{n2} > h_{n3,CONJ}$, the hydraulic jump would happen in BC, which is unacceptable (no subcritical flow in BC). To avoid this, we need to increase $S_{0,2}$ to get a smaller h_{n2} and push the jump outside BC.

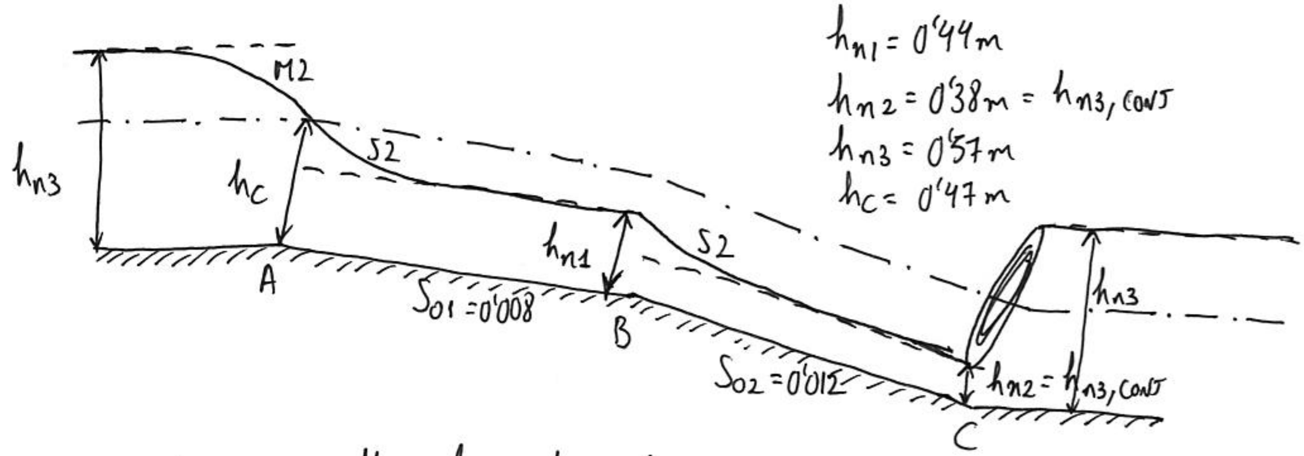
Determination of minimum $S_{0,2}$ to satisfy the conditions:

The minimum value of $S_{0,2}$ will cause the hydraulic jump to happen exactly at C, i.e., $h_{n,2} = h_{n3,CONJ} = 0'38$ m $\Rightarrow$

$$\Rightarrow S_{0,2} = \frac{n^2 V_{n2}^2}{Rh_{n2}^{4/3}} = 0'012.$$ Therefore, for

$S_{0,2} \geq 0'012$ (or, equivalently, $S_{01} \leq 0'008$), the jump will happen to the right of C.

For the limiting case, $S_{01} = 0'008 \Rightarrow h_{n1} = 0'44$ m $< h_C$ and the situation is:



For this case, the volume of water over BC is

$$V_{BC} \approx \frac{A_B + A_C}{2} \cdot 50 = \frac{h_{n1} \cdot 1 + h_{n2} \cdot 1}{2} \cdot 50 = 20'50 \, m^3 \qquad (1)$$

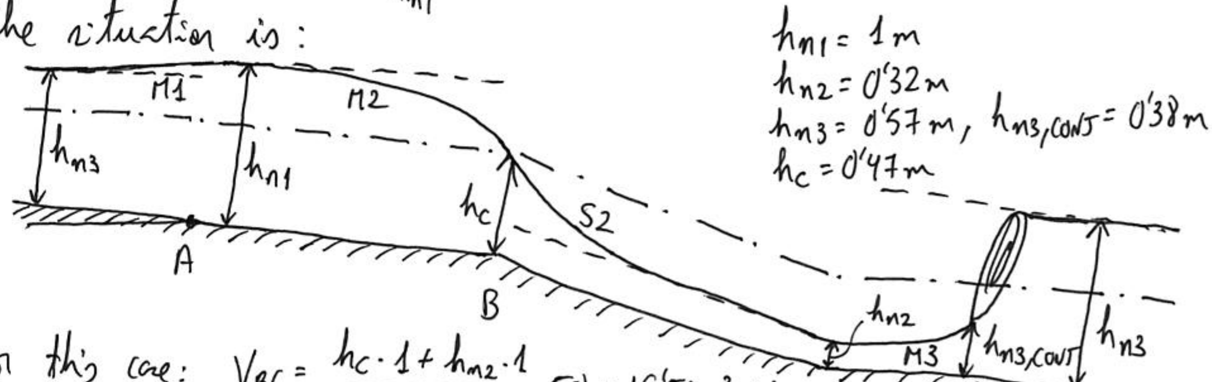
<u>Determination of minimum $S_{0,1}$ to satisfy the conditions:</u>

If we keep increasing S_{02} beyond 0'012, we will reduce h_{n2} and V_{BC}, and thus the weight on BC. But when we increase S_{02}, we have to decrease S_{01}, and thus h_{n1} increases. To avoid water spills, we have to impose $h_{n1} \leq 1 m$.

For the limiting case, $h_{n1} = 1 m > h_c \Rightarrow$ M in stretch 1.

$$h_{n1} = 1 m \Rightarrow S_{01} = \frac{n^2 \cdot V_{n1}^2}{R h_{n1}^{4/3}} = 9'7 \cdot 10^{-4} \Rightarrow S_{02} = 0'019 \Rightarrow h_{n2} = 0'32 m.$$

The situation is:



For this case: $V_{BC} = \frac{h_c \cdot 1 + h_{n2} \cdot 1}{2} \cdot 50 = 19'50 m^3$ (2)

In conclusion: $\boxed{9'7 \cdot 10^{-4} \leq S_{01} \leq 8 \cdot 10^{-3} \; ; \; 0'012 \leq S_{0,2} \leq 0'019 \; ; \; S_{01} + S_{02} = 0'02}$

b) Comparing $V_{BC} = 20'50 \, m^3$ from (1) and $V_{BC} = 19'50 \, m^3$ from (2), we conclude that V_{BC} is minimum (and thus the weight is minimum) for $\boxed{S_{01} = 9'7 \cdot 10^{-4} \simeq 0'001 \; ; \; S_{02} = 0'019}$.