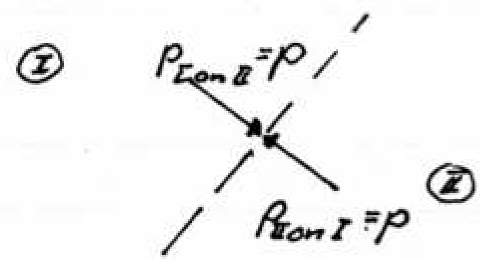
LECTURE #3

1.060 ENGINEERING MECHANICS II

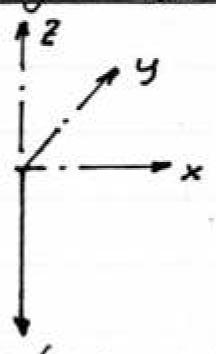
HYDROSTATICS (from Recitation #1)

In a fluid at rest there can be no shear stress only normal stress.

In fluids at rest normal stress is isotropic (same in all directions) and positive if compression. It is referred to a "the pressure".



Hydrostatic Pressure Distribution



$$\text{grad}\, p = \rho \vec{g} \qquad p = 0 \text{ on surfaces} \perp \vec{g}$$

$$\frac{\partial p}{\partial z} = -\rho g \approx \text{constant}$$

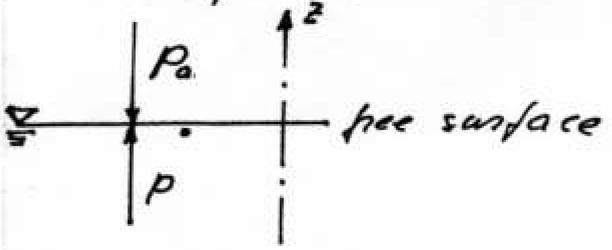
$\vec{g} = (0, 0, -g)$ $\quad \underline{p + \rho g z = p_0 + \rho g z_0 = \text{CONSTANT}}$

within a constant density fluid at rest.

To apply this:

Need pressure p_0 at specified elevation z_0

For liquids with a free surface:



We choose $z = 0 = z_0$ at free surface.

$p_0 = p$ as $z \to 0^-$ = pressure above surface

(except in cases when surface tension counts)

So:

$$\underline{p + \rho g z = p_0 = \text{constant}}$$

If "fluid" above the free surface is air at atmospheric pressure, then

$$p_0 = p_{atm} \approx 101.3 \text{ kPa} \approx 1.013 \text{ bar} = 1013 \text{ mbar (Standard)}$$

and

$$p - p_{atm} + \rho g z = 0$$

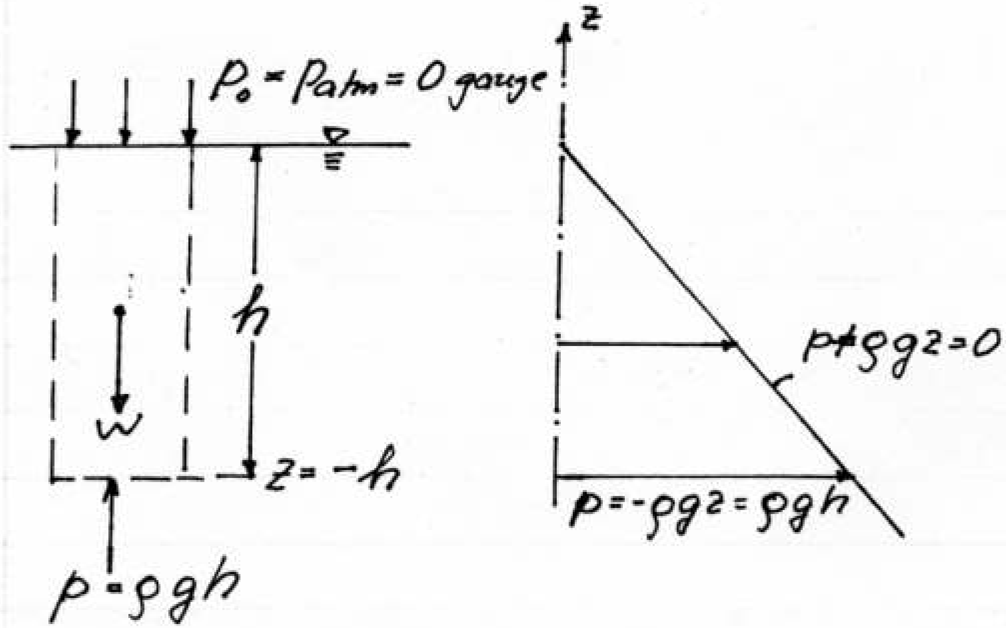
We make a deal: From now on

"p" = $\underline{p - p_{atm}}$ = PRESSURE (gauge pressure)

$p_{atm} = 0$ (gauge) by definition

$p_{atm} = 101.3$ kPa (absolute)

$\underline{p_{absolute} = p + p_{atm}}$



Pressure at depth h simply balances the weight per unit horizontal area of the fluid above. (plus pressure at the free surface, if this is not atmospheric, i.e. 0 gauge).

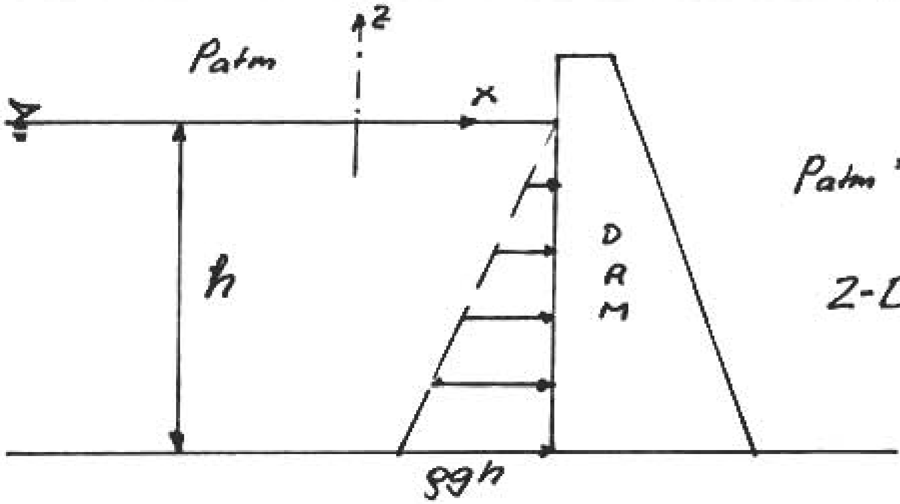
In a fluid at rest the pressure varies linearly in the vertical direction so long as the fluid density is constant

$$p_2 - p_1 = -\rho g (z_2 - z_1) = -\rho g \Delta z$$

If $\Delta z > 0$, i.e. going <u>up</u> from ① to ②, then $\Delta p = p_2 - p_1 < 0$ (less fluid on top at ②!) and if going <u>down</u> from ① to ② then $\Delta z < 0$ and $\Delta p > 0$ (more fluid on top at ②!).

In many cases involving a gas, e.g. air, ρ is so small that $\rho g \Delta z \simeq 0$ and $p_2 = p_1 =$ constant is OK.

HYDROSTATIC FORCE ON A PLANE AREA



2-D problem

1) Pressure varies linearly along upstream face of dam from 0 to $\rho g h$: $p = -\rho g z$

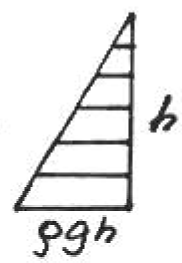
2) Pressure is always $\perp$ surface upon which it acts : $\vec{P}$ is in $+x$-direction

Per <u>unit length</u> (into paper) the pressure force is

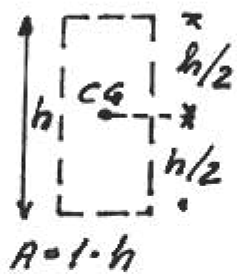
$$\vec{P} = F_x = \int_{-h}^{0} p\, dA = \int_{-h}^{0} -\rho g z \cdot 1\, dz = \tfrac{1}{2}\rho g h^2$$

Two interpretations of this result:

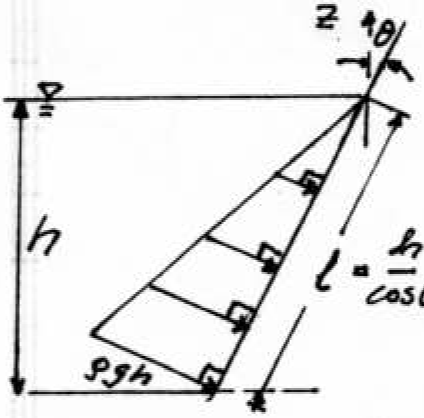
F_x = area of the pressure prism = $\frac{1}{2}(\rho g h)(h) = \frac{1}{2}\rho g h^2$



F_x = pressure at the center of gravity of Area upon which pressure acts multiplied by the Area = $p_{CG} \cdot A$ = $(\rho g \frac{1}{2} h)(1 \cdot h) = \frac{1}{2}\rho g h^2$



Same "rules" apply if the plane surface is inclined to vertical, so long as it is recalled that the pressure and hence the pressure force is $\perp$ to the surface upon which it acts.



Total force on inclined surface (θ = angle of surface from vertical) of length $l = h/\cos\theta$ =

$$F_{\perp} = \tfrac{1}{2}(\rho g h)\, l = \tfrac{1}{2}\rho g h^2 \frac{1}{\cos\theta} =$$

Area of pressure prism $= (\tfrac{1}{2}\rho g h)\cdot l = p_{CG}\cdot A$ = pressure at center of gravity of Area × Area.

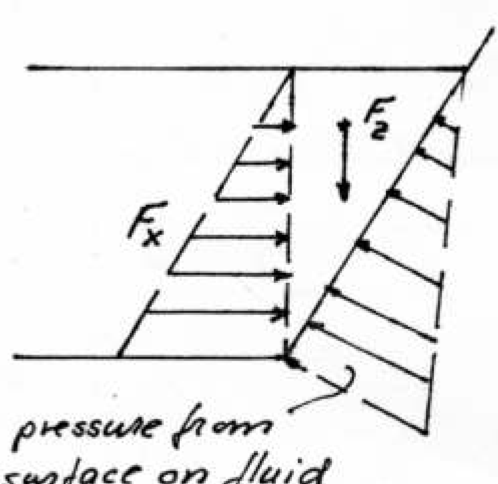
We also have the x- and z-components:

$$F_x = F_{\perp}\cdot\cos\theta = \tfrac{1}{2}\rho g h^2 = \text{Pressure force}$$

on area's projection on a vertical plane

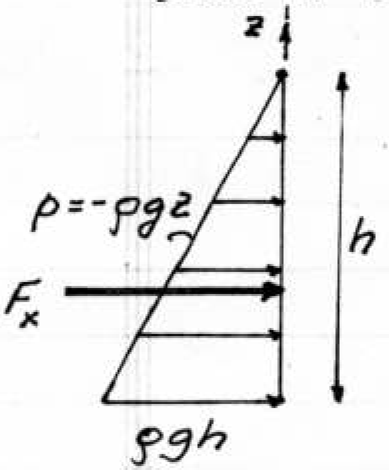
$$F_z = F_{\perp}(-\sin\theta) = -\tfrac{1}{2}\rho g h^2 \tan\theta = -(\text{weight}$$

of fluid above the area $= \tfrac{1}{2}h(h\tan\theta)\rho g = \tfrac{1}{2}\rho g h^2 \tan\theta)$



This makes sense, since the triangle of fluid above the surface must be in equilibrium, i.e.
Net force in x = 0
Net force in z must balance gravity (weight).

We have obtained the total (integrated) pressure force on a plane area, its magnitude ($p_{cg} A$) and direction ($\perp A$), but were does it act, i.e. we need its <u>line of action</u>.



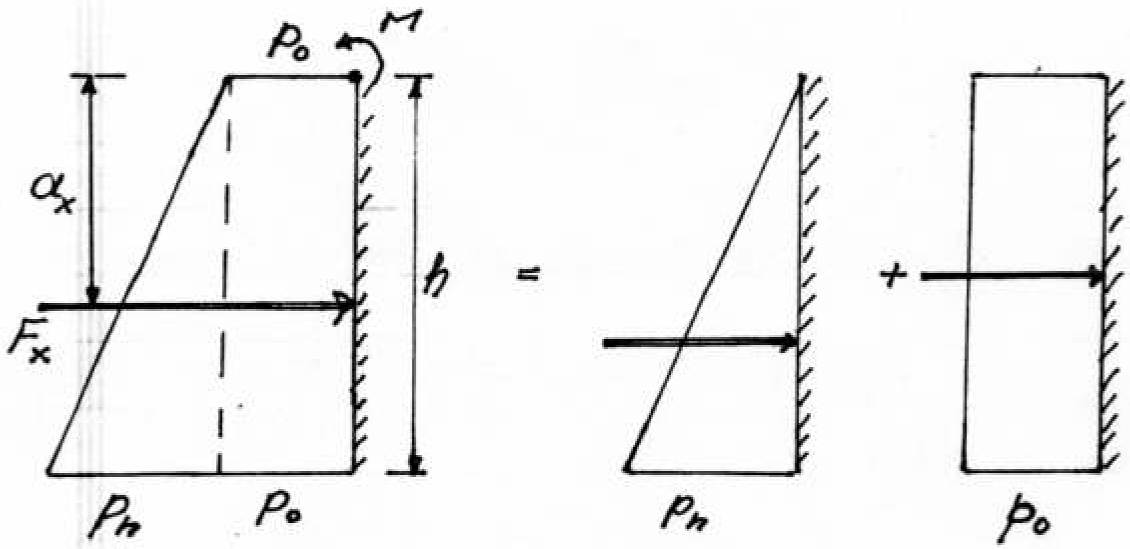
To obtain this, we take the moment of forces around the top of the triangle

$$M = \int_{-h}^{0} p(-z)\,dz = \int_{-h}^{0} \rho g z^2 dz = \frac{1}{3}\rho g h^3 = \left(\frac{2}{3}h\right)\left(\frac{1}{2}\rho g h^2\right) = \left(\frac{2}{3}h\right)F_x$$

Thus, the line of action of the total pressure force for a triangular pressure prism is a distance of (2/3) down from the top, or a distance of (1/3) up from the bottom, i.e.

The line of action passes through the <u>CENTER OF PRESSURE</u> which is the center of gravity of the pressure distribution prism for 2-D problems.

In many cases of 2-D problems the pressure distribution can be represented as the sum of a constant pressure and a linearly varying pressure. To treat this problem we "divide and conquer"



$$F_x = \frac{1}{2} p_h h + p_o h$$

$$M = F_x \cdot a_x = \frac{2}{3} h \left(\frac{1}{2} p_h h\right) + \frac{1}{2} h \left(p_o h\right)$$