LECTURE #2

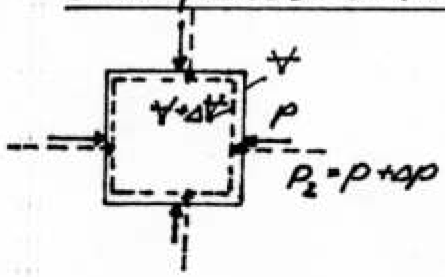
1.060 ENGINEERING MECHANICS II

Continuum Hypothesis

$$\frac{\partial u}{\partial x} = \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x} = \lim_{\Delta x \to \text{"}0\text{"}} \frac{\Delta u}{\Delta x}$$

where "0" is a scale much smaller than any in which we are interested.

Compressibility of Fluids



$$E \frac{\Delta \forall}{\forall} = -\nabla p$$

E_v = bulk modulus

$$E \frac{\Delta \rho}{\rho} = \nabla p$$

For water:

$E_v = 2.15 \cdot 10^9 \frac{N}{m^2}$; $\nabla p = 10^8 \frac{N}{m^2}$ (~ 10 km depth)

$\rho \cong 10^3 kg/m^3$

$$\left|\frac{\Delta \forall}{\forall}\right| = \left|\frac{\Delta \rho}{\rho}\right| \cong 5\% \quad \underline{\text{NOT MUCH}}$$

For air

$E_v = 1.4 \cdot 10^5 \frac{N}{m^2}$; $\nabla p = 10^4 \frac{N}{m^2}$ (~ 800 m height)

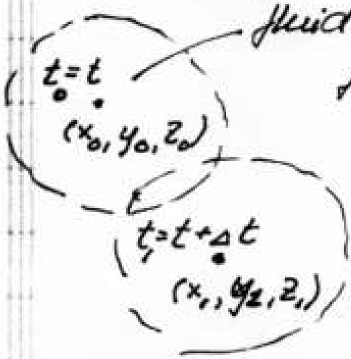
$\rho = 1.25 kg/m^3$

$$\left|\frac{\Delta \forall}{\forall}\right| = \left|\frac{\Delta \rho}{\rho}\right| \cong 7\% \quad \underline{\text{NOT MUCH}}$$

$$C = \text{speed of sound} = \sqrt{\frac{\partial p}{\partial \rho}} = \sqrt{\frac{\Delta p}{\Delta \rho}} = \sqrt{\frac{E}{\rho}} \cong \begin{cases} 1{,}500 \text{ m/s (water)} \\ 335 \text{ m/s (air)} \end{cases}$$

If V = fluid velocity $\ll$ c = speed of sound in fluid, the fluid can be considered ~incompressible when pressure variations are not excessive.

Fluid Velocity



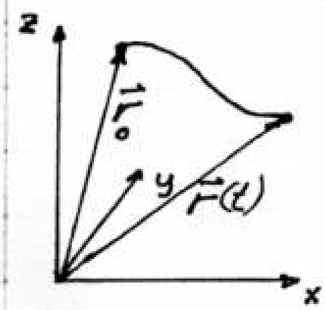
"fluid particle" = a small volume δV = "0" that consists of "the same" molecules.

$$\vec{q} = \vec{q}(x_0, y_0, z_0, t) = \lim_{\Delta t \to \text{"}0\text{"}} \frac{(x_1 - x_0,\ y_1 - y_0,\ z_1 - z_0)}{t_1 - t_0} =$$

$$\lim_{\Delta t \to 0} \frac{\Delta x, \Delta y, \Delta z}{\Delta t} = (u_0, v_0, w_0) =$$

velocity (vector) at point (x_0, y_0, z_0) at time t_0

Choice of Coordinate System

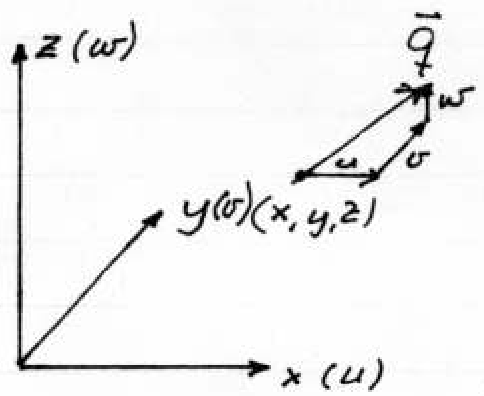


Lagrangean Coordinates

Identify a fluid particle by its position $\vec{r}_0$ at $t=0$ and determine its position $\vec{r}(t)$ at any subsequent time

$$\underset{\text{position}}{\vec{r}(\vec{r}_0, t)} \Rightarrow \underset{\text{velocity}}{\vec{q} = \frac{d\vec{r}}{dt}} \Rightarrow \underset{\text{acceleration}}{\frac{d\vec{q}}{dt} = \frac{d^2\vec{r}}{dt^2}}$$

Eulerian Coordinates



Determine the velocity vector, $\vec{q}$, at a fixed point, (x, y, z), as a function of time:

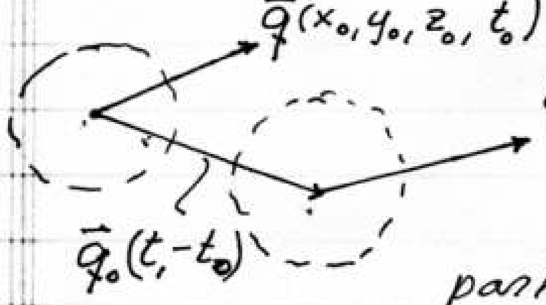
$$\vec{q}\,(x, y, z, t) = (u, v, w)$$

This coordinate system - Eulerian - is favored over Lagrange's in Fluid Mechanics.

If $\vec{q}\,(x, y, z, t)$ is not a function of time, i.e.

$$\frac{\partial \vec{q}}{\partial t} = \left(\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}, \frac{\partial w}{\partial t} \right) = (0,0,0) = \underline{0}$$

the flow is referred to as <u>STEADY FLOW</u>, but $\partial \vec{q} / \partial t = \underline{0}$ does not imply that the fluid is not accelerating.



Following the same fluid particle as we must according to Newton's Law, we have

$$\vec{a} = \frac{d\vec{q}}{dt} = \frac{D\vec{q}}{Dt} = \lim_{\Delta t = t_1 - t_0 \to 0} \frac{\vec{q}\,(x_1, y_1, z_1, t) - \vec{q}\,(x_0, y_0, z_0, t_0)}{t_1 - t_0}$$

or, with $x_1 - x_0 = u_0(t_1 - t_0)$ and analogous we have

$$\frac{d\vec{q}}{dt} = \frac{D\vec{q}}{Dt} = \lim_{\Delta t \to 0} \frac{\frac{\partial \vec{q}}{\partial x} u_0 \Delta t + \frac{\partial \vec{q}}{\partial y} v_0 \Delta t + \frac{\partial \vec{q}}{\partial z} w_0 \Delta t + \frac{\partial \vec{q}}{\partial t} \Delta t}{\Delta t} =$$

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \mathrm{grad})\vec{q} = \frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q}$$

where

∇ = "del" operator = $\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

In words:

$\frac{D\vec{q}}{Dt}$ Total derivative or Material Derivative = rate of change (in this case of velocity $\vec{q}$) following a fluid particle

=

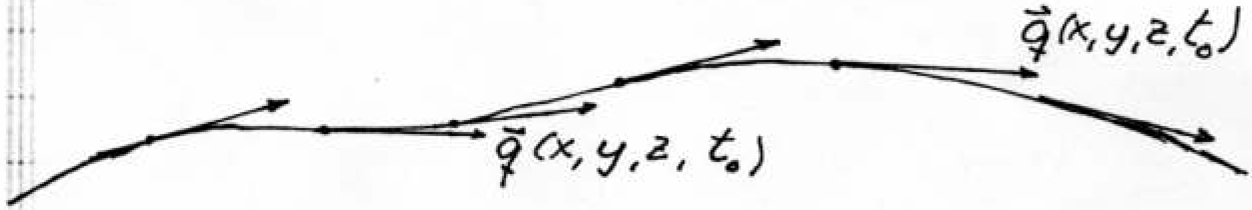
$\frac{\partial \vec{q}}{\partial t}$ Local rate of change, i.e. the rate of change taking place at the fixed location (x, y, z). Note this would be zero if flow is steady

+

$(\vec{q} \cdot \nabla)\vec{q}$ The convective rate of change, i.e. the rate of change associated with the particle moving to a new location where conditions (here the velocity) has changed relative to the original location. Note this term would be zero if $\vec{q}$ independent of location: UNIFORM FLOW

Streamline

Definition: A streamline is a line that at a given instant of time has the local velocity vector as its tangent at any point along the line.



By definition, it therefore follows that

$$d\vec{s} = \text{infinitesimal element along streamline} = (dx_s, dy_s, dz_s) \propto \vec{q}(s, t_0) = (u_{s0}, v_{s0}, w_{s0})$$

or

$$\frac{dx_s}{u_{s0}} = \frac{dy_s}{v_{s0}} = \frac{dz_s}{w_{s0}}$$

If the flow is STEADY the velocity vector at any point does not vary with time, $\partial\vec{q}/\partial t = 0$. Hence the streamline is independent of time, and since a particle on a streamline always moves tangential to the line, it will follow a path (the PATHLINE) equal to the streamline.