LECTURE # 16

1.060 ENGINEERING MECHANICS II

FRICTIONAL HEAD LOSS IN CIRCULAR PIPES

$$H_1 - H_2 = \Delta H_f = \frac{\tau_s P}{\rho g A} l_{12}$$

For circular pipe of diameter D

$$P = \pi D \; ; \; A = \frac{\pi}{4} D^2$$

and with

$$\tau_s = \frac{1}{8} \rho f V^2 \quad ; \; V = Q/A = Q/(\tfrac{\pi}{4}D^2)$$

we have

$$H_1 - H_2 = f \cdot \frac{V^2}{2g} \cdot \frac{l_{12}}{D} = \Delta H_f$$

[Note: The artificial introduction of a factor of (1/8) in the definition of the Darcy-Weisbach friction factor relationship has produced this pleasing result for ΔH_f: proportional to "f", "the velocity head" ($V^2/2g$), and the non-dimensional length of the pipe section, l_{12}/D].

From dimensional analysis we obtained that

f = Darcy-Weisbach Friction Factor =

$$f\left(Re = \frac{DV}{\nu},\ \frac{\varepsilon}{D}\right)$$

Experimental results for ΔH_f with given fluid (ρ & ν known) and discharge Q, knowledge of diameter, D, gives $V = Q/(\frac{\pi}{4}D^2)$ and a value of "f" can be obtained along with the corresponding value of the Reynolds Number, $Re = DV/\nu$. But what is the value of the relative roughness, ε/D, or rather how do we "define" the pipe wall roughness, ε?

Nikuradse did this for us in 1933 by conducting a large number of experiments in which uniform sand grains were glued onto the inside of smooth pipes. In this manner he could assign an unambiguous value for the pipe roughness as the diameter of sand grains. Thus, with $\varepsilon = d_{sand}$ both,

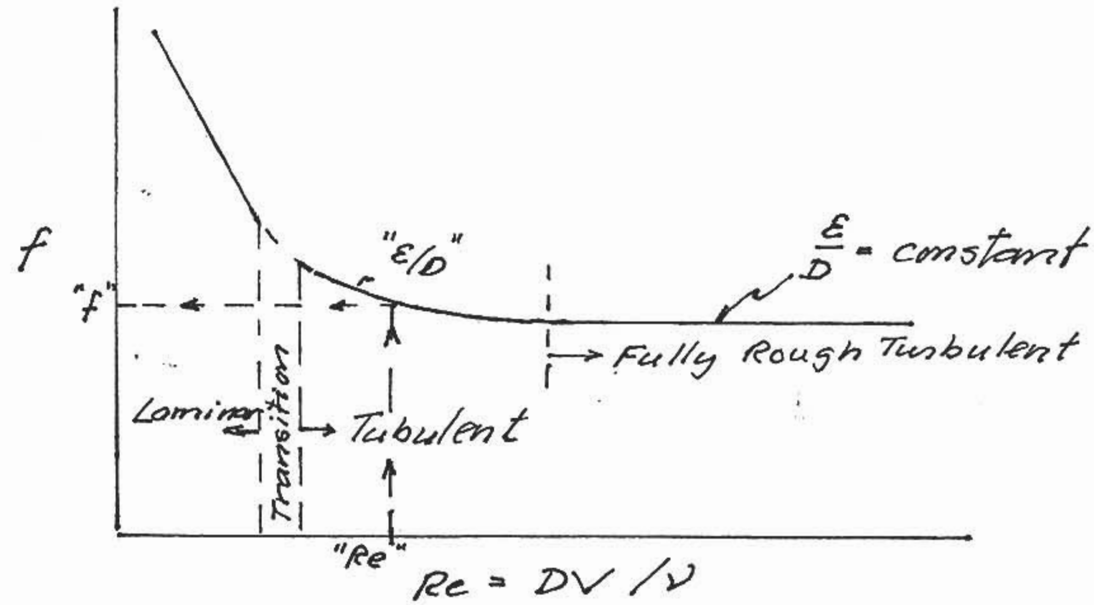
$$Re = DV/\nu \text{ and } \varepsilon/D$$

were known for a given experiment, and an empirical determination of

$$f = f(DV/\nu,\ \varepsilon/D)$$

was obtained.

THE MOODY DIAGRAM

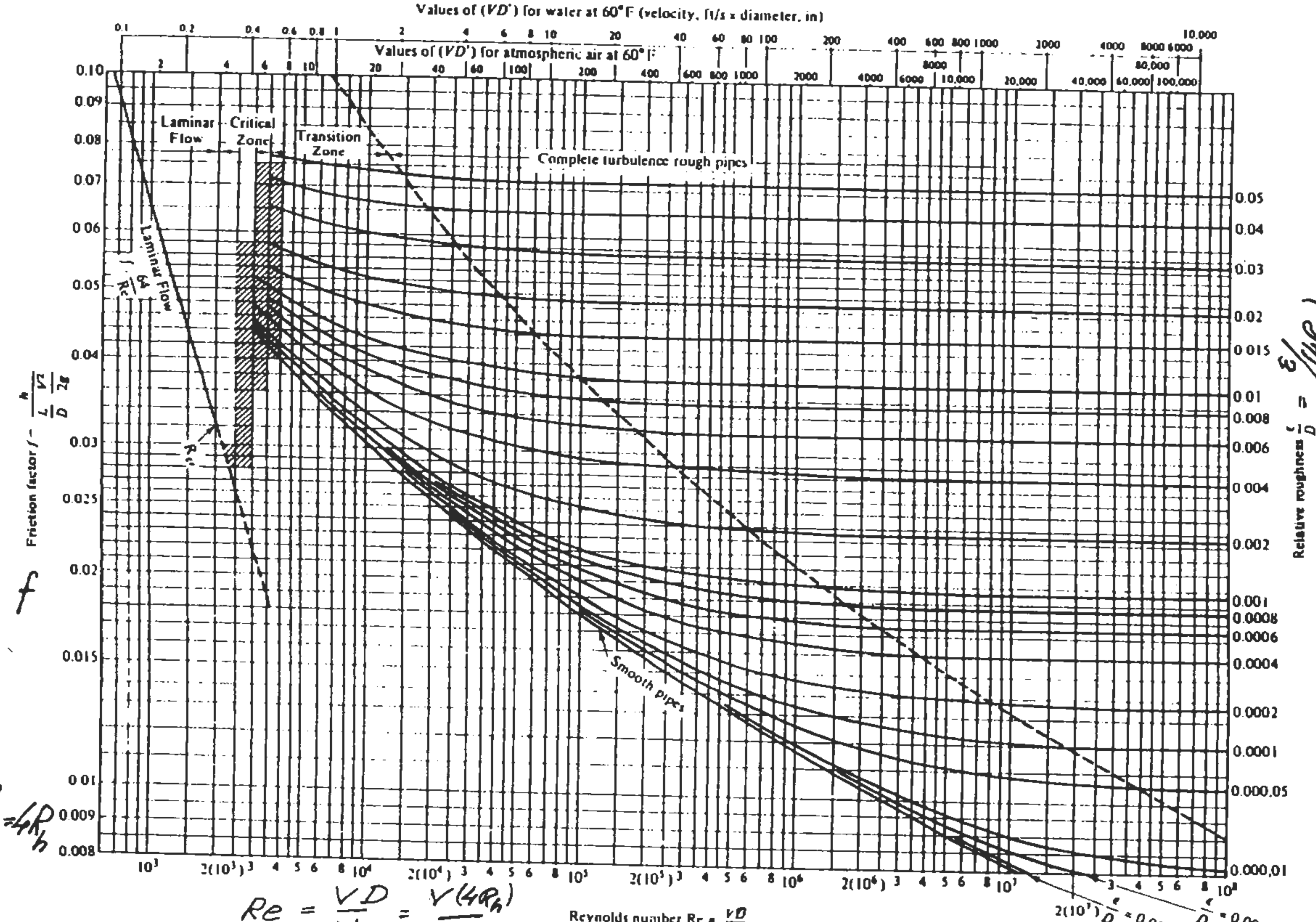


For given discharge, Q, of a known fluid, ρ & ν can be obtained if temperature is known, and a pipe of diameter, D, made from a standard material (consult Table 8.1 in Text for corresponding value of ε = the equivalent Nikuradse sand grain roughness) one obtains

$$\text{"}Re\text{"} = DV/\nu \quad \text{and} \quad \text{"}\varepsilon/D\text{"}$$

and "f" = Darcy-Weisbach's friction factor is obtained from the Moody Diagram as shown.

Note: For a given ε/D the value of "f" is independent of Re = DV/ν for Re sufficiently large. This region is "fully rough turbulent flow"



$$D = \frac{4A}{P} = 4R_h$$

$$Re = \frac{VD}{\nu} = \frac{V(4R_h)}{\nu}$$

Fig. 6.13 The Moody chart for pipe friction with smooth and rough walls. This chart is identical to Eq. (6.64) for turbulent flow. (*From Ref. 8, by permission of the ASME.*)

FRICTIONAL LOSS IN CONDUITS

Although developed for circular pipes the Moody Diagram may be used to estimate the friction factor also for non-circular conduits, by making the following substitution:

$$R_h = \text{Hydraulic Radius} = \frac{A}{P}$$

For a circular pipe

$$R_h = \frac{(\pi/4)\,D^2}{\pi D} = \frac{D}{4} \Rightarrow D = 4R_h$$

Thus, we may use the Moody Diagram to obtain f for any conduit by taking

$$\text{"}D\text{"} = 4R_h = 4\frac{A}{P}$$

i.e.

$$Re = \frac{DV}{\nu} = \frac{(4R_h)V}{\nu}\ ;\quad \varepsilon/D = \varepsilon/(4R_h)$$

in Moody to get "f".

Then the frictional head loss is obtained from

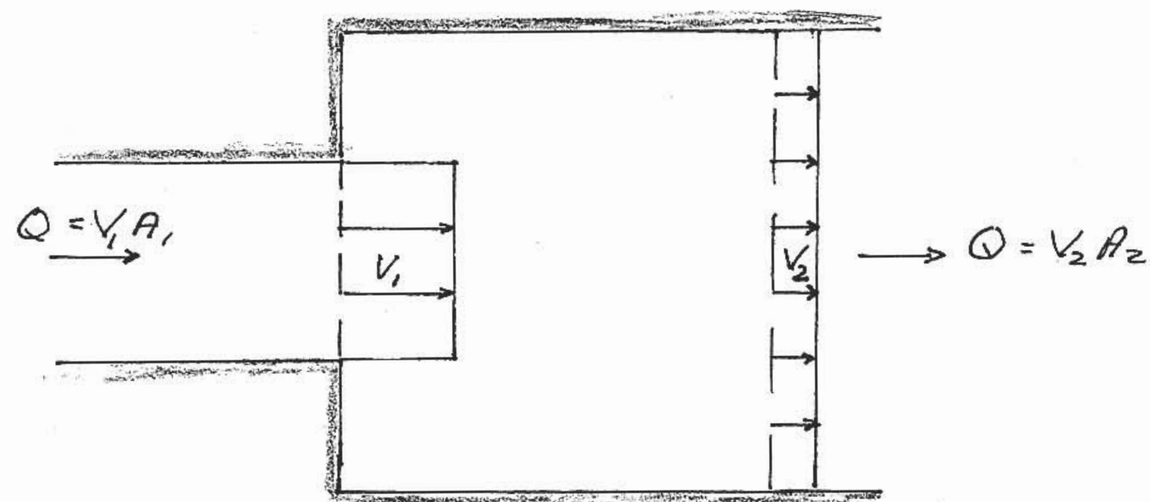
$$\Delta H_f = f\,\frac{V^2}{2g}\,\frac{\ell_{12}}{4R_h}.$$

MINOR LOSSES

In addition to frictional head losses along straight sections of pipes, there are also so-called minor losses.

Minor losses are without exceptions associated with some sort of flow expansion in the direction of flow.

Recall the general expression for a head loss associated with an expansion (Lecture #13)



$$\Delta H_{exp} = \frac{(V_1 - V_2)^2}{2g} = \left(1 - V_2/V_1\right)^2 \frac{V_1^2}{2g} = \left(\frac{V_1}{V_2} - 1\right)^2 \frac{V_2^2}{2g}$$

or

$$\Delta H_{exp} = K_{L1} \frac{V_1^2}{2g} \quad \text{or} \quad \Delta H_{exp} = K_{L2} \frac{V_2^2}{2g}$$

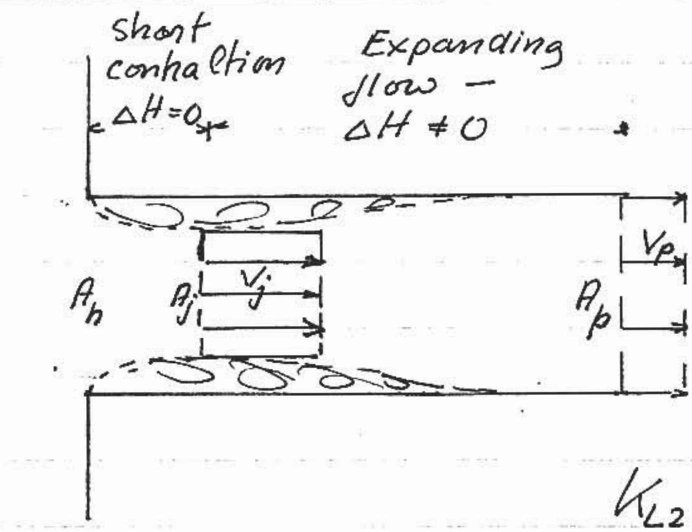
where $K_{L1} = \left(1 - V_2/V_1\right)^2 = \left(1 - A_1/A_2\right)^2$

and $K_{L2} = \left(V_1/V_2 - 1\right)^2 = \left(A_2/A_1 - 1\right)^2$

The choice of K_{L1} or K_{L2} depends on convenience in subsequent calculations

Example of Minor Loss.

Sharp-Edged inlet



$$\Delta H_{inlet} = \Delta H_{exp} = \frac{(V_j - V_p)^2}{2g} = K_{Lp}\frac{V_p^2}{2g}$$

$$K_{L2} = \frac{A_p}{A_j} = \frac{A_h}{A_j} = \frac{1}{C_V}$$

$A_j = C_V A_h$
$C_V \cong 0.6$ as for free outflow

$$K_{Lp} = \left(\frac{1}{C_V} - 1\right)^2 = \left(\frac{1}{0.6} - 1\right)^2 = 0.44$$

Why choose K_{Lp} - referenced to $V_p^2/2g$ - rather than K_{Lj} - referenced to $V_j^2/2g$?

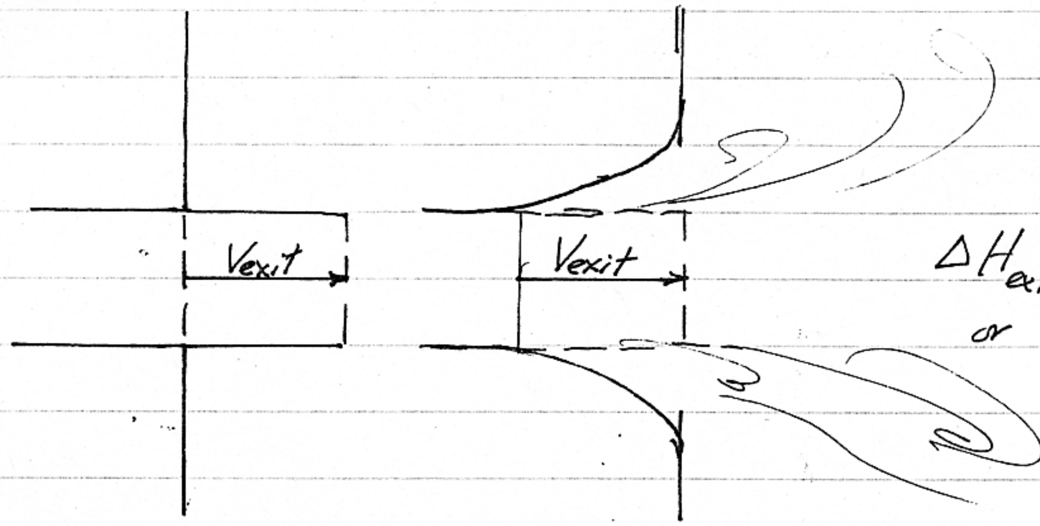
Because pipe flow that follows inlet is governed by head loss due to friction in pipe, i.e. referenced to $V_p^2/2g$, e.g.

$$\Delta H = \Delta H_{inlet} + \Delta H_f = K_{Lp}\frac{V_p^2}{2g} + \left(f\frac{\ell_p}{4R_h}\right)\frac{V_p^2}{2g} =$$

$$\left(K_{Lp} + f\frac{\ell}{4R_h}\right)V_p^2/2g.$$

Rounding corners reduces inlet loss DRAMATICALLY

Outflow (Exit) Loss



$$\Delta H_{exit} = \frac{V_{exit}^2}{2g}$$

or

$$K_{L_{exit}} = 1$$

No effect of rounding corners ⇒ The jet still shoots straight into "pond"