PROBLEM SET 7 - SOLUTIONS

Comments on Problem Set 7

PROBLEM 1:

- This time I have a comment on my own solution, because some of you did better than me on this problem. I only calculated a lower limit of the relative roughness for which Manning's approximation of f is accurate. However, as some of you pointed out, there is also an upper limit of the relative roughness: Given that Moody's diagram gives information up to a value of the relative roughness of 0.2 (that is, 4*0.05), using Moody you can only conclude that Manning's approximation is good for a relative roughness from about 0.004 up to 0.2. The approximation will probably work for values of the relative roughness somewhat larger than 0.2, but we don't have data to guarantee this. In any case, there will certainly be an upper limit for the approximation to be valid, since new effects need to be taken into account if the length scale of the roughness elements becomes comparable to the channel size.

PROBLEM 2:

- Do not split the channel section into sub-sections when bottom roughness is uniform. Doing this is less accurate than applying Manning to the whole section. When you split the section, you neglect friction on the dividing fluid-fluid interfaces. Even if this is a necessary assumption to make when the bottom is not uniform (because there is no other easy way for solving the problem), you don't need to make this approximation (and you shouldn't) when the bottom is uniform, as is the case here.

- Remember that we defined $S_0 = \sin \alpha$. If α is small (as is here), then $\sin \alpha \approx \tan \alpha$. This is why it is correct to say $S_0 \approx 1/1000 = 0.001$. I point this out because approximating S_0 by $\tan \alpha$ would not be correct if α was large, in which case you would have to calculate $S_0 = \sin \alpha$.

- A nice ornament to the solution of this problem (worth one bonus point) is to check that flow is indeed rough turbulent, so that to apply Manning is correct (see my solution). Usually, flow in natural channels is rough turbulent, so you are generally safe by using Manning. But it doesn't hurt to check at the end, if you suspect that the assumption of R.T. flow may be invalid or if you have some time to spare.

PROBLEM 3:

- As opposed to problem 2, here you need to subdivide the section, because the bottom is not uniform. You should divide it into three subsections, according to the three different bottom roughnesses, but no more. Don't subdivide the central subsection, because this is inaccurate if you don't account for fluid-fluid friction on the interfaces.

- To compute the flowrate contribution of the central subsection, you cannot just use the result from problem 2, because now the depth in the central subsection is larger (4 meters as opposed to 2 meters before). You cannot add a contribution from the additional upper rectangle to the solution of problem 2, either. Instead, you need to calculate a flowrate for the whole central subsection from scratch (as most of you did), because this is the only way of getting the right value of the hydraulic radius.

PROBLEM 4: Everyone did well on this problem. Most groups had a point or two taken off for an unclear or incomplete explanation in part (c). See solution for details.

PROBLEM 5: No comments here. Good job.

PROBLEM 6: Most groups did well on this problem, also. A few groups got tripped up on part (c) - h1 is best solved for by using Manning's Equation and Continuity.

PROBLEM SET 7 - SOLUTIONS

- PROBLEM N° 1:

a)

For a number of values of the relative roughness, $\frac{\varepsilon}{4R_h}$, we calculate f from Moody diagram (for fully rough turbulent flow) and from the approximation used to derive Manning's equation, $f = 0'113\,(\varepsilon/R_h)^{1/3}$. Then we compute the relative error of the latter with respect to the former.

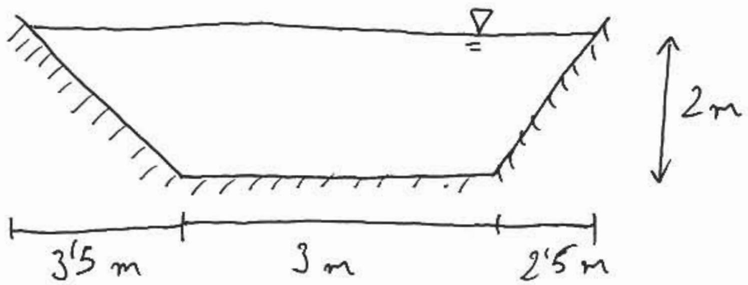
$\varepsilon/4R_h$	f (Moody)	f (Manning)	error $= \left\lvert \frac{f_{MANNING} - f_{MOODY}}{f_{MOODY}} \right\rvert$
0'015	0'044	0'044	0
0'01	0'038	0'039	2'63%
0'004	0'029	0'028	3'45%
0'001	0'02	0'018	10%
0'0001	0'012	0'0083	30'83%

From table, when $\varepsilon/4R_h < 0'001$, an error larger than 10% is observed. Consequently, the approximation of f used in Manning's formula is accurate within 10% if $\underline{\underline{\frac{\varepsilon}{R_h} > 0'004}}$.

b)

Manning's equation assumes rough turbulent flow, which doesn't happen on smooth surfaces like glass. Besides, as shown in part a, the approximation of f used in Manning becomes inaccurate when $\frac{\varepsilon}{R_h} \to 0$. Therefore, for smooth channels, Darcy-Weisbach (Moody) must be used. Reporting a value of "n" for glass is unjustified. However, people use Manning for smooth channels, because the results are usually not too bad (see Recit. 8, problem 2).

- PROBLEM N° 2:



Clean, straight channel:
$n = 0'030$
(Young et al., Table 10.1)

a)
Assume R.T. flow and use Manning's equation:

$$Q = A \frac{1}{n} R_h^{2/3} S_0^{1/2}$$

$$A = \frac{1}{2} \cdot 2 \cdot 3'5 + 3 \cdot 2 + \frac{1}{2} \cdot 2 \cdot 2'5 = 12 \; m^2$$

$$P = \sqrt{3'5^2 + 2^2} + 3 + \sqrt{2'5^2 + 2^2} = 10'2 \; m$$

$$R_h = \frac{A}{P} = 1'17 \; m$$

$$S_0 = \frac{1}{1000} = 10^{-3}$$

$$\underline{\underline{Q}} = 12 \cdot \frac{1}{0'03} \cdot 1'17^{2/3} \cdot (10^{-3})^{1/2} = \underline{\underline{14'0 \; m^3/s}}$$

b)

$$\underline{\underline{V}} = \frac{Q}{A} = \frac{14'0}{12} = \underline{\underline{1'17 \; m/s}}$$

Check R.T. assumption:

$$\left. \begin{aligned} Re &= \frac{V(4R_h)}{\nu} = 5'5 \cdot 10^6 \\ \varepsilon = \left(\frac{n}{0'038}\right)^6 = 0'242 \; m \Rightarrow \frac{\varepsilon}{4R_h} &= 0'052 \end{aligned} \right\} \xrightarrow{\text{MOODY}} \text{R.T. flow} \checkmark$$

In general, flow in channels is R.T. However, to be rigorous, you should check that flow is R.T. when using Manning.

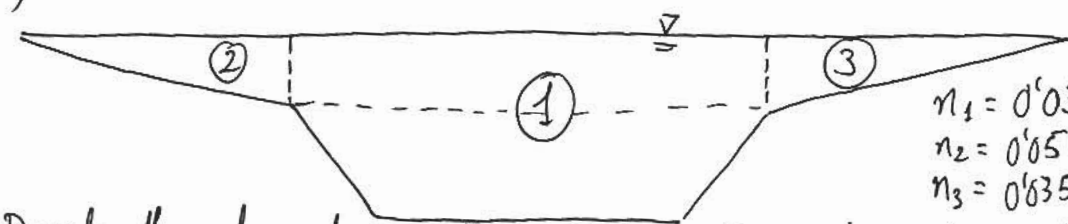
c)

$$h_m = \frac{A}{b_s} = \frac{12}{3'5 + 3 + 2'5} = 1'33 \; m \; ; \; \underline{\underline{Fr}} = \frac{V}{\sqrt{g h_m}} = \frac{1'17}{\sqrt{9'8 \cdot 1'33}} = \underline{\underline{0'324}}$$

d) $Fr = 0'324 < 1 \Rightarrow$ SUBCRITICAL FLOW

- PROBLEM N°3 :

a)



$n_1 = 0'030$
$n_2 = 0'050$
$n_3 = 0'035$
From Table 10.1

Divide the channel in three sections (neglecting shear stresses on the interfaces) and apply Manning's formula to each section (assuming R.T. flow).

$$\begin{cases} A_1 = \text{area of creek} + \text{area above creek} = 12 + 9 \cdot 2 = 30\,m^2 \\ A_2 = \frac{1}{2} \cdot 25 \cdot 2 = 25\,m^2 \\ A_3 = \frac{1}{2} \cdot 40 \cdot 2 = 40\,m^2 \end{cases}$$

$$A_{TOTAL} = 30 + 25 + 40 = 95\,m^2$$

$$\begin{cases} P_1 = 10'2\,m \text{ (from Problem 2)} \\ P_2 = \sqrt{25^2 + 2^2} = 25'1\,m \\ P_3 = \sqrt{40^2 + 2^2} = 40'0\,m \end{cases} \qquad \begin{cases} Rh_1 = \frac{A_1}{P_1} = 2'94\,m \\ Rh_2 = \frac{A_2}{P_2} = 1'00\,m \\ Rh_3 = \frac{A_3}{P_3} = 1'00\,m \end{cases}$$

$$P_{TOTAL} = 10'2 + 25'1 + 40'0 = 75'3$$

$$\text{Manning: } Q = V \cdot A = A \frac{1}{n} (Rh)^{2/3} (S_0)^{1/2}$$

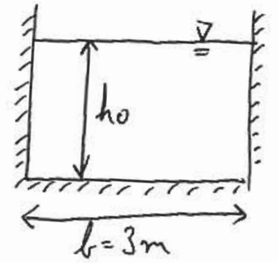
$$\begin{cases} Q_1 = 30 \cdot \frac{1}{0'03} (2'94)^{2/3} \sqrt{10^{-3}} = 64'9\,m^3/s \\ Q_2 = 25 \cdot \frac{1}{0'05} (1'00)^{2/3} \sqrt{10^{-3}} = 15'8\,m^3/s \\ Q_3 = 40 \cdot \frac{1}{0'035} (1'00)^{2/3} \sqrt{10^{-3}} = 36'1\,m^3/s \end{cases} \quad \underline{\underline{Q_{TOTAL}}} = 64'9 + 15'8 + 36'1 = \underline{\underline{116'8\,m^3/s}}$$

b)

$$\begin{cases} h_m = \frac{A}{b_s} = \frac{95}{25+9+40} = 1'28\,m \\ V = \frac{Q}{A} = \frac{116'8}{95} = 1'23\,m/s \end{cases} \quad \underline{\underline{Fr}} = \frac{V}{\sqrt{g h_m}} = \frac{1'23}{\sqrt{9'8 \cdot 1'28}} = \underline{\underline{0'35}}$$

(Subcritical flow)

- PROBLEM N° 4 :



$\beta = 0'03° \Rightarrow S_0 = \sin\beta = 5\cdot10^{-4}$
$Q = 12\ m^3/s$
$\varepsilon = 1\,mm$

a)
Darcy – Weisbach: $Q = A\cdot V = A\sqrt{\frac{8g}{f}R_h S_0} \Rightarrow 12 = 3h_0\sqrt{\frac{8\cdot 9'8}{f}}\sqrt{\frac{3h_0}{3+2h_0}5\cdot10^{-4}} \Rightarrow$
$\Rightarrow h_0^3 = 136\,f\,(3+2h_0)$. Solve by iteration.

1ST ITERATION : $f = 0'02 \Rightarrow h_0^{(k+1)} = \left[2'72\left(3+2h_0^{(k)}\right)\right]^{1/3}$

Solve for h_0 by iteration, with $h_0^{(0)} = 0$

k	0	1	2	3	4	5	6
$h_0^{(k)}$	0	2'01	2'67	2'83	2'87	2'88	2'88

$\Rightarrow h_0 = 2'88\,m$

With $h_0 = 2'88$ m we obtain:

$V = \frac{Q}{3h_0} = 1'39\ m/s \Rightarrow Re = \frac{V\cdot 4R_h}{\nu} = \frac{4V3h_0}{\nu(3+2h_0)} = 5'48\cdot10^6$

$\frac{\varepsilon}{4R_h} = \frac{0'001}{4\cdot\frac{3h_0}{3+2h_0}} = 0'000253$

MOODY

$f = 0'0145$

2ND ITERATION : $f = 0'0145 \Rightarrow h_0^{(k+1)} = \left[1'972\left(3+2h_0^{(k)}\right)\right]^{1/3}$
Solve by iteration, with $h_0^{(0)} = 2'88\,m$ $\Big\} \Rightarrow h_0 = 2'55\,m \Rightarrow$

$\Rightarrow \begin{cases} V = 1'57\ m/s \Rightarrow Re = 5'93\cdot10^6 \\ \varepsilon/4R_h = 0'000265 \end{cases} \Rightarrow f = 0'0145$ CONVERGED ✓

Thus, $\underline{\underline{h_0 = 2'55\,m}}$

b)
Manning: $n = 0'038\,\varepsilon^{1/6} = 0'038\cdot(10^{-3})^{1/6} = 0'012\ s\,m^{-1/3}$

$Q = V\cdot A = A\,\frac{1}{n}R_h^{2/3}S_0^{1/2} \Rightarrow 12 = 3h_0\frac{1}{0'012}\left(\frac{3h_0}{3+2h_0}\right)^{2/3}\sqrt{5\cdot10^{-4}} \Rightarrow$

$\Rightarrow h_0^{(k+1)} = 1'58\left(1+\frac{2}{3}h_0^{(k)}\right)^{2/5}$; solve by iteration with $h_0^{(0)} = 0 \Rightarrow$

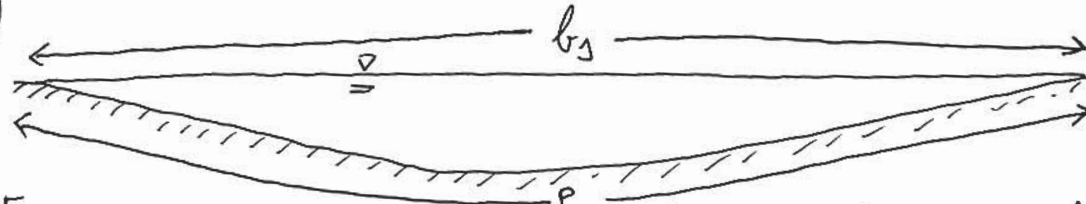
$\Rightarrow \underline{\underline{h_0 = 2'29\,m}}$ (10% error with respect to Darcy – Weisbach).

c)

For the values of $Re = 5'93 \cdot 10^6$ and $\frac{\varepsilon}{4R_h} = 0'00265$ obtained in (a), flow is on the limit of R.T. (see Moody diagram), and R.T. flow is assumed by Manning. Besides,
$\frac{\varepsilon}{4R_h} = 0'000265 < 0'001$, and the error in Manning's approximation of "f" is larger than 10% (see Problem 1), so inaccuracy in Manning's solution was expected.

-PROBLEM N° 5:

a)



For a very wide and shallow channel, $P \approx b_s \Rightarrow R_h = \frac{A}{P} \approx \frac{A}{b_s} = h_m$.
Thus, Darcy-Weisbach's formulation yields

$$V = \left(\frac{8g}{f}\right)^{1/2} (h_m S_0)^{1/2} \Rightarrow \underline{\underline{Fr}} = \frac{V}{\sqrt{g h_m}} = \underline{\underline{\sqrt{\frac{8 S_0}{f}}}}$$

b)

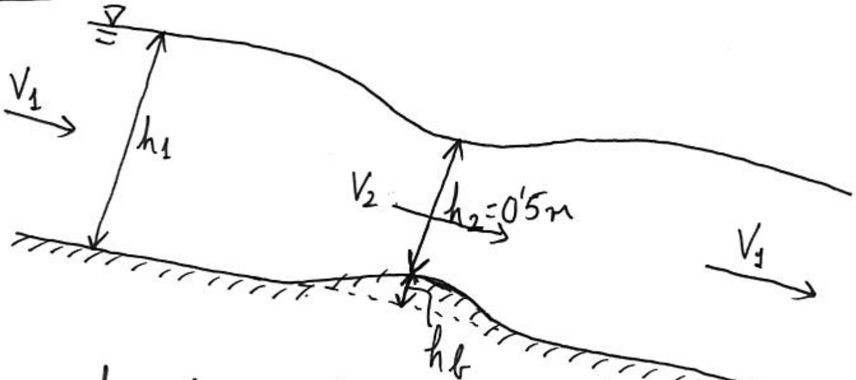
$$Fr = 1 = \sqrt{\frac{8 S_{0,cr}}{f}} \Rightarrow \underline{\underline{S_{0,cr} = \frac{f}{8}}}$$

c)

$$S_0 > S_{0,cr} \Rightarrow \sqrt{\frac{8 S_0}{f}} > \sqrt{\frac{8 S_{0,cr}}{f}} \Rightarrow Fr > 1 \Rightarrow \text{SUPERCRITICAL FLOW}$$

Conversely, $S_0 < S_{0,cr} \Rightarrow$ SUBCRITICAL FLOW.

- PROBLEM N° 6 :



Wide rectangular channel:
$R_h \approx h_m \approx h$

Brick channel: $n = 0'015$ (Table 10.1)

$\beta = 0'020 \Rightarrow S_0 = \sin\beta = 3'5 \cdot 10^{-4}$

a) Critical velocity over bump $\Rightarrow Fr = 1$

$$Fr = \frac{V_2}{\sqrt{g h_2}} = 1 \Rightarrow \underline{\underline{V_2}} = \sqrt{9'8 \cdot 0'5} = \underline{\underline{2'2 \ m/s}}$$

b) $\underline{\underline{Q}} = V_2 \, h_2 = 2'2 \cdot 0'5 = \underline{\underline{1'1 \ m^3/ms}}$

c) Continuity $\Rightarrow V_1 = \dfrac{Q}{h_1} = \dfrac{1'1}{h_1}$

Manning $\Rightarrow V_1 = \dfrac{1}{n} h_1^{2/3} S_0^{1/2}$

$$\frac{1'1}{h_1} = \frac{1}{0'015} h_1^{2/3} (3'5 \cdot 10^{-4})^{1/2}$$

$$h_1^{5/3} = 0'882 \Rightarrow \underline{\underline{h_1 = 0'93 \ m}}$$

$$\underline{\underline{V_1}} = \frac{1'1}{h_1} = \underline{\underline{1'2 \ m/s}}$$