1.010 Uncertainty in Engineering
Fall 2008

1.010 - Brief Notes # 3
Random Variables: Continuous Distributions

- **Continuous Distributions**

  - Cumulative distribution function (CDF)

    $F_X(x) = P[X \leq x]$

    $P[x_1 < X \leq x_2] = F_X(x_2) - F_X(x_1)$

  - Average probability density in an interval $[x_1, x_2]$

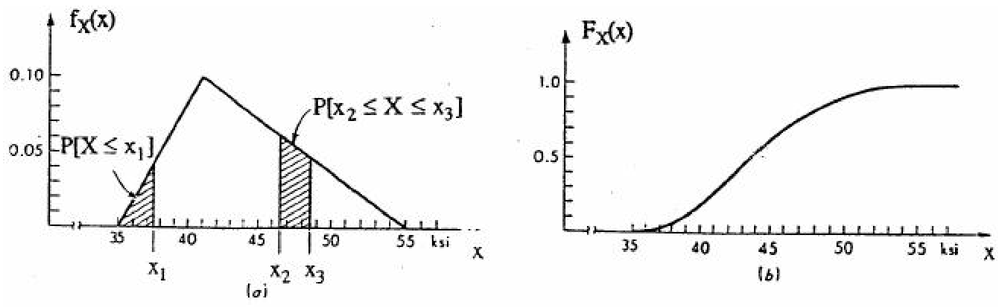
    $$\frac{P[x_1 < X \leq x_2]}{x_2 - x_1} = \frac{F_X(x_2) - F_X(x_1)}{x_2 - x_1}$$

  - Probability density function (PDF)

    $$f_X(x_1) = \lim_{x_2 \to x_1} \frac{P[x_1 < X \leq x_2]}{x_2 - x_1} = \left.\frac{dF_X}{dX}\right|_{x_1}$$

    $\int_{x_1}^{x_2} f_X(x)dx = P[x_1 < X \leq x_2] = F_X(x_2) - F_X(x_1)$

  - Properties of the PDF
    1. $f_X(x) \geq 0$
    2. $\int_{-\infty}^{\infty} f_X(x)dx = 1$
    3. $\int_{-\infty}^{u} f_X(x)dx = F_X(u)$



Example of PDF and corresponding CDF of a continuous random variable: steel-yield-stress.
(a) Probability density function; (b) cumulative distribution function.

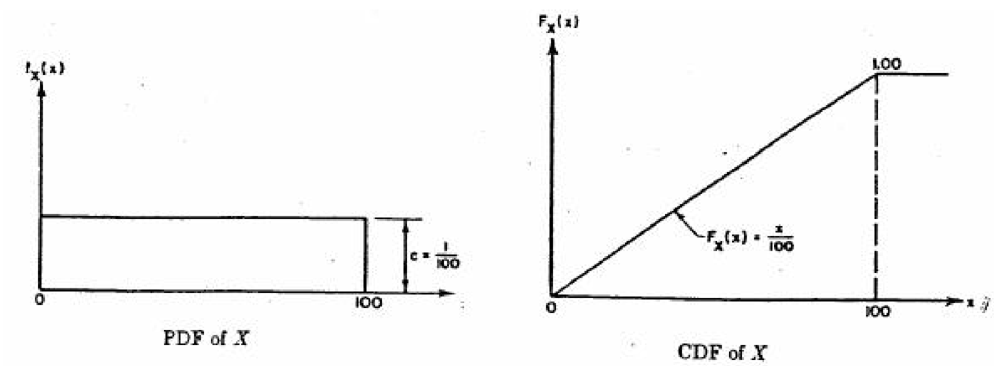
- **Examples of continuous probability distributions**

  - Uniform distribution

$$f_X(x) = \begin{cases} c, & \text{where c is constant, } a < x < b \\ 0, & \text{otherwise.} \end{cases}$$

Then $c = \frac{1}{b-a}$

$$F_X(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \le x \le b \\ 1, & x > b \end{cases}$$



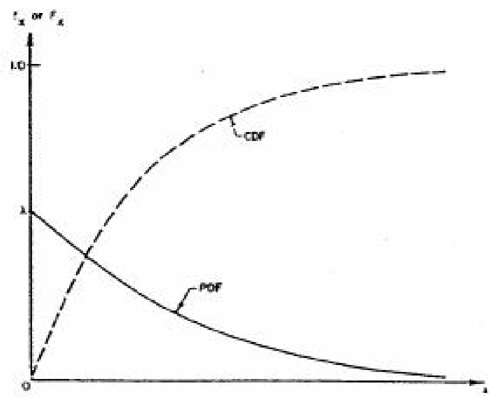
Example of uniform distribution.

- Exponential Distribution

Let T = time to first arrival in a Poisson point process

$$F_T(t) = P[T \le t] = 1 - P[T > t]$$
$$= 1 - P[\text{no occurrence in } [0, t]]$$
$$= 1 - e^{-\lambda t}$$

$$f_T(t) = \lambda e^{-\lambda t}, \quad t \ge 0$$



PDF and CDF of the exponential distribution.