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1.010 Uncertainty in Engineering  
Fall 2008

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1.010 Fall 2008  
Homework Set #7  
Due November 6, 2008 (in class)

1.  $X$  has uniform distribution between 0 and 1. Let  $Y = X^2$ .
  - a) Find the exact mean value and variance of  $X$
  - b) Find the exact mean value and variance of  $Y$ .
  - c) Approximate the mean value and variance of  $Y$  using first-order second-moment (FOSM) analysis. Compare with the results in (b).

2. The natural frequency  $\omega$  at which a one-story building vibrates is given by

$$\omega = \sqrt{\frac{K}{M}}$$

where  $K$  is the lateral stiffness of the building and  $M$  is its mass. If  $K$  is random with mean value  $m_K$  and standard deviation  $\sigma_K$  and  $M$  is known, use FOSM analysis to find in approximation the mean value and variance of  $\omega$ .

3. In planning a building, the number of elevators is chosen on the basis of balancing initial costs versus the expected delay times of the users. These delays are closely related to the number of stops the elevator makes on a trip. If an elevator runs full ( $n$  people) and there are  $k$  floors, we want to find the expected number of stops  $R$  the elevator makes on any trip. Assuming that the passengers act independently and that any passenger chooses a floor with equal probability  $1/k$ . Show that:

$$E[R] = k \left[ 1 - \left( 1 - \frac{1}{k} \right)^n \right]$$

**Hint:** It is often useful to define “indicator random variables” as follows:

Let  $X_i = 1$  if the elevator stops at floor  $i$ , and  $X_i = 0$  if it does not.

Then observe that  $R = \sum_{i=1}^k X_i$ .

Find the expected value of  $X_i$  after finding the probability that  $X_i = 0$ .

4. In the very preliminary planning of some harbor island developments, there was discussion regarding the cost estimates of building five bridges. There had as yet not been a preliminary soil survey in the harbor area, and it was recognized that the bridge costs are highly dependent on soil conditions. Therefore, there was great uncertainty in the preliminary cost estimates. A spokesman, however, made this statement; “I recognize the

uncertainty on the cost estimate of any bridge. But I am much more confident of our estimate for the total cost of all five bridges because the likelihood that a high estimate on one will be balanced by a low estimate on another.”

Discuss this statement. Use your knowledge, simply, of the means and variances of sums of random variables to support your comments. If your initial intuition lies with the spokesman, be sure you resolve in your own mind why it is inconsistent. Compare both the variance and the coefficients of variation of the total cost versus those of an individual cost.

If the five bridge sites are relatively close to one another, soil conditions, although unknown, are probably similar. What implications does this observation have for your analysis? Would your answer change in the case of 10 bridges?