1.010 Uncertainty in Engineering
Fall 2008

1. Two continuous variables X and Y have joint probability density function:

$$f_{X,Y}(x,y) = \begin{cases} \dfrac{1}{8}(x+y) & 0 \le x \le 2 \, and \, 0 \le y \le 2 \\ 0 & elsewhere \end{cases}$$

a) Find and plot the marginal probability density function (PDF) of X.
b) Find and plot the marginal cumulative distribution function (CDF) of X.
c) Find and plot the conditional PDF of $(Y|X=1)$.
d) Are X and Y independent? Comment.

2. According to schedule, Train A arrives at station S at 10:55 am and Train B departs from the same station at 11:05 am. Due to delays, the arrival time of Train A is uniformly distributed between 10:55 and 11:10 and the departure time of Train B is uniformly distributed between 11:05 and 11:15. If the arrival and departure times of the two trains are statistically independent, what is the probability that a passenger on Train A misses the connection with Train B?[Hint: Let T_A and T_B be the times when train A arrives and train B departs, respectively. Plot the joint range of (T_A, T_B) on the (T_A, T_B)-plane and find the region that corresponds to missing the connection.]

3. Show that the function below is the PDF of R, the distance between the epicenter of an earthquake and the site of a dam, when the epicenter is equally likely to be at any location along a neighboring fault (see figure below). You may restrict your attention to a length of fault l that is within a distance r_o of the site because earthquakes at greater distances will have negligible effect at the site.

$$f_R(r) = \frac{2r}{l}\,(r^2 - d^2)^{-1/2} \; , \quad d \le r \le r_o$$

Sketch the function.