5.73

Quiz 8 ANSWERS

1.

$$\int_{x_-(E)}^{x_+(E)} p_E(x')dx' = \frac{h}{2}(n+1/2)$$

$$p_E(x) = \left[2m\left(E - V(x)\right)\right]^{1/2}$$

Even though WKB cannot be valid for a potential of the form

$V = 0$ $\quad |x| \geq L/2$
$V = -V_0$ $\quad |x| < L/2$

A. Evaluate the quantization integral at $E = 0$ and determine the number of bound levels, n_{max},.in the potential.

$$\int_{-L/2}^{L/2}\left[2mV_0\right]^{1/2} dx = \left[2mV_0\right]^{1/2} L = \frac{h}{2}(n+1/2)$$

$$n_{max} = \frac{2}{h}\left[2mV_0\right]^{1/2} L - 1/2$$

levels is $n_{max} + 1$ because $n = 0$ is a level
levels is an integer between $n_{max} - 1/2$ and $n_{max} + 1/2$

B. Calculate $\frac{dn_{max}}{dL}$.

$$\frac{dn_{max}}{dL} = \frac{2}{h}\left[2mV_0\right]^{1/2}$$

C. Calculate $\frac{dn_{max}}{dV_0}$.

$$\frac{dn_{max}}{dV_0} = \frac{2}{h}(2m)^{1/2}\frac{1}{2}V_0^{-1/2}$$

(*over*)

D. Which leads to a larger increase in n_{max}, a 10% increase in L or a 10% increase in V_0?

An increase of L by 10% leads to an increase in n_{max} by 10%.
An increase of V_0 by 10% leads to an increase in n_{max} by 5%.

E. Consider the "bifurcated potential": $V = 0$ $\quad |x| > 20L$
$V = 0$ $\quad |x| < 19.5L$
$V = -V_0$ $\quad 19.5L \leq |x| \leq 20L$

Without doing any new calculations but keeping the result of part A clearly in mind, compare the number of bound levels in the bifurcated potential to those in the original finite square well that is the subject of part A.

As far as WKB-QC is concerned, the two potentials bind exactly the same number of levels. In reality, the bifurcated potential might bind one additional level than the non-bifurcated potential. WKB-QC over-estimates the number of bound levels for both potentials by no more than one level.

5.73 Quantum Mechanics I
Fall 2018