# 5.73

## Quiz 23 ANSWERS

$$\mathbf{J}^2\left|JM\right\rangle = \hbar^2 J(J+1)\left|JM\right\rangle$$
$$\mathbf{J}_z\left|JM\right\rangle = \hbar M\left|JM\right\rangle$$
$$\mathbf{J}_\pm = \mathbf{J}_x \pm i\mathbf{J}_y$$
$$\mathbf{J}_\pm\left|JM\right\rangle = \left[J(J+1) - M(M\pm1)\right]^{1/2}\left|JM\pm1\right\rangle$$

A. What are the ΔJ and ΔM selection rules for the following operators:

*(i)* $\mathbf{J}^4$ $\boxed{\Delta J = 0,\ \Delta M = 0}$

(*ii*) $(\mathbf{J}_+)^2$ $\boxed{\Delta J = 0,\ \Delta M = +2}$

(*iii*) $\mathbf{J}_+\mathbf{J}_-$ $\boxed{\Delta J = 0,\ \Delta M = 0}$

(*iv*) $\mathbf{J}_x$ $\boxed{\Delta J = 0,\ \Delta M = \pm1}$

(*v*) $\vec{\mathbf{J}}$ $\boxed{\Delta J = 0, \Delta M = \left(\hat{i}(\pm1), \hat{j}(\pm1), (\hat{k}0)\right)}$

B. What are the values of the following matrix elements:

*(i)* $\left\langle JM+1\left|\mathbf{J}^2\right|JM\right\rangle$ $\boxed{= 0}$

(*ii*) $\left\langle JM\left|\mathbf{J}^2\mathbf{J}_z\right|JM\right\rangle$ $\boxed{= \hbar^3 \mathrm{J(J+1)M}}$

(*iii*) $\left\langle JM\left|\mathbf{J}_+\mathbf{J}_-\right|JM\right\rangle$ $\boxed{= \hbar^2([\mathrm{J(J+1)} - \mathrm{M(M+1)}][\mathrm{J(J+1)}–\mathrm{M(M–1)}])^{1/2}}$

(*iv*) $\left\langle JM\left|\mathbf{J}_+\mathbf{J}_- - \mathbf{J}_-\mathbf{J}_+\right|JM\right\rangle$ $\boxed{= 0}$

(*v*) $\left\langle JM+1\left|\mathbf{J}_x\right|JM\right\rangle$

$$\boxed{= \left\langle JM+1\left|\frac{1}{2}\left(J_+ + J_-\right)\right|JM\right\rangle = \frac{1}{2}\hbar[J(J+1) - M(M+1)]^{1/2}}$$

C. What is the value of the commutator $[\mathbf{J}_+,\mathbf{J}_-]$ = ?

$$\boxed{\left[J_+, J_-\right] = \left[J_x + iJ_y, J_x - iJ_y\right] = i\hbar\left[0 - iJ_z - iJ_z + 0\right] = 2\hbar J_z}$$

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