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Quiz 22 $\boxed{\text{ANSWERS}}$

$$\left[\mathbf{L}_i,\mathbf{p}_j\right]=i\hbar\sum_k \varepsilon_{ijk}\mathbf{p}_k$$

$$\left[\mathbf{L}_i,\mathbf{p}_j\right]=i\hbar\sum \varepsilon_{ijk}\mathbf{p}_k$$

ε_{ijk} = +1 if ijk are in cyclic order (i.e. xyz, yzx, or zxy)
–1 if ijk are in anti-cyclic order
0 if any index is repeated.

$$\mathbf{L}=(\mathbf{q}\times\mathbf{p})=\begin{pmatrix}\hat{i} & \hat{j} & \hat{k}\\ \mathbf{x} & \mathbf{y} & \mathbf{z}\\ \mathbf{p}_x & \mathbf{p}_y & \mathbf{p}_z\end{pmatrix}$$

A. What are $\mathbf{L}_y$ and $\mathbf{L}_z$ in terms of $(\mathbf{x},\mathbf{y},\mathbf{z})$ and $(\mathbf{p}_x, \mathbf{p}_y, \mathbf{p}_z)$?

$\mathbf{L}_x = \mathbf{y}\mathbf{p}_z - \mathbf{z}\mathbf{p}_y$
$\mathbf{L}_y = -\mathbf{x}\mathbf{p}_z + \mathbf{z}\mathbf{p}_x$
$\mathbf{L}_z = \mathbf{x}\mathbf{p}_y - \mathbf{y}\mathbf{p}_x$

B. Use ε_{ijk} notation to evaluate $[\mathbf{L}_x, \mathbf{x}]$, $[\mathbf{L}_x, \mathbf{z}]$, $[\mathbf{L}_x, \mathbf{p}_x]$, and $[\mathbf{L}_x, \mathbf{p}_z]$.

$[\mathbf{L}_x, \mathbf{x}] = \boxed{0}$

$[\mathbf{L}_x, \mathbf{z}] = \boxed{-i\hbar y}$

$[\mathbf{L}_x, \mathbf{p}_x] = \boxed{0}$

$[\mathbf{L}_z, \mathbf{p}_z] = \boxed{-i\hbar p_y}$

C. Use the results of part B to show that $[\mathbf{L}_x, \mathbf{L}_y] = i\hbar\mathbf{L}_z$. Recall that $[\mathbf{A},\mathbf{BC}] = \mathbf{B}[\mathbf{A},\mathbf{C}] + [\mathbf{A},\mathbf{B}]\mathbf{C}$.

$[\mathbf{L}_x, -xp_z + zp_x] = -[\mathbf{L}_x, xp_z] + [\mathbf{L}_x, zp_x]$
$[\mathbf{L}_x, xp_z] = x[\mathbf{L}_x, p_z] + [\mathbf{L}_x, x]p_z = -i\hbar xp_y + 0$
$[\mathbf{L}_x, zp_x] = z[\mathbf{L}_x, p_x] + [\mathbf{L}_x, z]p_x = -i\hbar yp_x$
$[\mathbf{L}_x, \mathbf{L}_y] = i\hbar[xp_y - yp_x] = i\hbar\mathbf{L}_z$