5.73

Quiz 1 ANSWERS

A is a complex number $\quad A \equiv a + ib$ (a and b are real)
$A^* \equiv a - ib$
$|A|^2 = AA^*$
Re A means real part of A: Re A = a
Im A means imaginary part of A: Im A = b
$e^{ix} = \cos x + i\sin x$

A. $\quad A = 4 + i3$. Evaluate $|A|^2$.

$|A|^2 = A^*A = (4 - i3)(4 + i3) = 16 + 9 = 25$

B. $\quad$ What is $\mathrm{Im}[(4 + i3)\, e^{i2x}]$?.

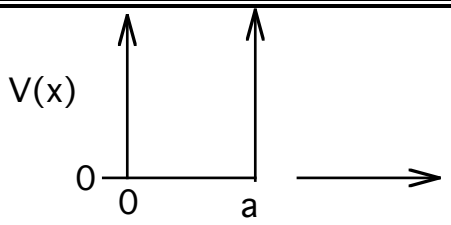
$$\begin{aligned}(4+i3)e^{i2x} &= (4+i3)(\cos 2x + i\sin 2x)\\ &= 4\cos 2x - 3\sin 2x + i(4\sin 2x + 3\cos 2x)\\ \mathrm{Im}\left[(4+i3)e^{i2x}\right] &= 4\sin 2x + 3\cos 2x\end{aligned}$$

C. $\quad |(4 + i3)e^{i2x}|^2$.

$$\begin{aligned}\left|(4+i3)e^{i2x}\right|^2 &= (4+i3)e^{i2x}(4-i3)e^{-i2x}\\ &= 16+9 = 25\end{aligned}$$

2.

The energies and eigenfunctions for



are $E_n = n^2\left[\frac{h^2}{8ma^2}\right]$ $n = 1,2,\ldots\infty$

$$\psi_n = \left(\frac{2}{a}\right)^{1/2}\sin(n\pi x)$$

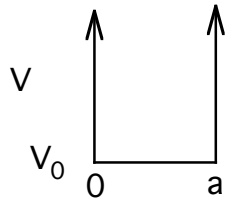
A. Which eigenstates (even n or odd n) have a node at $x = a/2$?

The even n ψ_n have a node at $x = a/2$.
The odd n ψ_n have a maximum at $x = a/2$.

B. There is one internal node in ψ_2. How many internal nodes are there in $\psi_{13}(x)$?

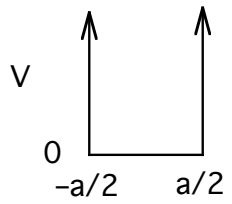
There are 13–1 internal nodes in ψ_{13}.

C. Do the eigenfunctions, $\{\psi_n\}$, change if the potential is shifted up by V_0? Why?



If V_0 is shifted to higher energy, the boundary conditions do not change. All of the energies shift up by the value of V_0. The wavefunctions do not depend on V_0.

D. Is there any change in the energy levels, $\{E_n\}$, if the potential is shifted to the left by a/2?



No change.

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