5.73

Quiz 17 ANSWERS

1.

$$\psi^{(0)}_{v_1 v_2} = \phi_{v_1}(x_1)\phi_{v_2}(x_2)$$
$$E^{(0)}_{v_1 v_2} = \hbar\left[\omega_1(v_1+1/2)+\omega_2(v_2+1/2)\right]$$

$$\mathbf{H}^{(1)} = k_{122}x_1x_2^2 \qquad \mathbf{H}^{(1)}_{n_1k_1n_2k_2} = k_{122}\left\langle n_1\middle|x_1\middle|k_1\right\rangle\left\langle n_2\middle|x_2^2\middle|k_2\right\rangle$$
$$\mathbf{x}_1 = \left(\frac{m_1\omega_1}{2\hbar}\right)^{1/2}\left(\mathbf{a}_1+\mathbf{a}_1^\dagger\right)$$
$$\mathbf{x}_2 = \left(\frac{m_2\omega_2}{2\hbar}\right)^{1/2}\left(\mathbf{a}_2+\mathbf{a}_2^\dagger\right)$$

A. Matrix elements of $\mathbf{H}^{(1)}$ have four indices, n_1 and k_1 specify the final and initial state quantum numbers for oscillator #1, n_2 and k_2 the final and initial state quantum numbers for oscillator #2. What are the selection rules for nonzero matrix elements of $\mathbf{H}^{(1)}$?

$$n_1 - k_1 = \boxed{\pm 1}$$

$$n_2 - k_2 = \boxed{0, \pm 2}$$

B. What are the zero-order energy differences that correspond to each of the nonzero matrix elements of $\mathbf{H}^{(1)}_{n_1k_1n_2k_2}$?

$$E^{(0)}_{n_1n_2} - E^{(0)}_{k_1k_2} = \hbar\left[\omega_1\left(\boxed{n_1-k_1}\right)+\omega_2\left(\boxed{n_2-k_2}\right)\right]$$

C. Evaluate at least two of the six nonzero values of the off-diagonal elements of $\mathbf{H}^{(1)}$.

(i) e.g. $\mathbf{H}_{n_1 n_1+1 n_2 n_2+2} = \gamma\left[(n_1+1)(n_2+2)(n_2+1)\right]^{1/2}$

(ii) $\mathbf{H}_{n_1 n_1+1 n_2 n_2} = \gamma\left[\boxed{(n_1+1)^{1/2}(2n_2+1)}\right]$

DO THIS ONE! (iii) $\mathbf{H}_{n_1 n_1+1 n_2 n_2-2} = \gamma\left[\boxed{(n_1+1)(n_2)(n_2-1)}\right]^{1/2}$

(iv) $\mathbf{H}_{n_1 n_1-1 n_2 n_2+2} = \gamma\left[\boxed{n_1(n_2+2)(n_2+1)}\right]^{1/2}$

(v) $\mathbf{H}_{n_1 n_1-1 n_2 n_2} = \gamma\left[\boxed{n_1^{1/2}(2n_2+1)}\right]$

(vi) $\mathbf{H}_{n_1 n_1-1 n_2 n_2-2} = \gamma\left[\boxed{n_1 n_2(n_2-1)}\right]^{1/2}$

where $\gamma \equiv \left(\frac{m_1\omega_1}{2\hbar}\right)^{1/2}\left(\frac{m_2\omega_2}{2\hbar}\right)^{1}$.

D. The term $\dfrac{\left|\mathbf{H}^{(1)}_{n_1 n_1+1 n_2 n_2-2}\right|^2}{E^{(0)}_{n_1 n_2} - E^{(0)}_{n_1+1 n_2-2}}$

appears in the second-order perturbation summation for $E^{(2)}_{n_1 n_2}$.

Evaluate this term (based on your answers to parts B and C(iii)).

$$\boxed{\frac{\gamma^2(n_1+1)(n_2)(n_2-1)}{\hbar(2\omega_2-\omega_1)}}$$

E. What happens to the term in part D if $\omega_1 = 2\omega_2$?

Blows up!

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