5.73

Quiz 14

1.

Quartic Oscillator

$$\mathbf{H} = \frac{\mathbf{p}^2}{2m} + \frac{1}{2}k\mathbf{x}^2 + b\mathbf{x}^4$$

$$\mathbf{H}^\circ = \frac{1}{2}(\mathbf{a}^\dagger\mathbf{a} + \mathbf{a}\mathbf{a}^\dagger)\mathbf{h}\omega$$

$$\mathbf{x} = \left(\frac{\mathbf{h}}{m\omega}\right)^{1/2}\underset{\sim}{\mathbf{x}} = \left(\frac{\mathbf{h}}{2m\omega}\right)^{1/2}(\mathbf{a} + \mathbf{a}^\dagger)$$

$$[\mathbf{a}, \mathbf{a}^\dagger] = \mathbb{1}$$

$$\mathbf{x}^4 = \left(\frac{\mathbf{h}}{2m\omega}\right)^2\left\{\mathbf{a}^4 + 2\mathbf{a}^2(2\mathbf{a}^\dagger\mathbf{a} - 1) + \left[6\mathbf{a}^\dagger\mathbf{a}(\mathbf{a}^\dagger\mathbf{a} + 1) + 3\right] + 2\mathbf{a}^{\dagger 2}(2\mathbf{a}^\dagger\mathbf{a} + 3) + \mathbf{a}^{\dagger 4}\right\}$$

A. Give the general formula for $E_n^{(0)} = H_{nn}^{(0)}$

$$E_n^{(0)} = [\text{some function of } n]\hbar\omega\ .$$

B. $\mathbf{H}^{(1)}=b\mathbf{x}^4$. Give the general formula for $E_n^{(1)} = H_{nn}^{(1)}$

$$E_n^{(1)} = [\text{some function of } n]b\left(\frac{\hbar}{2m\omega}\right)^2.$$

C. The expression for the second-order correction to the energy is

$$E_n^{(2)} = \sum_k \frac{\left|H_{nk}^{(1)}\right|^2}{E_n^{(0)} - E_k^{(0)}}$$

(i) For which 5 values of k will $H_{nk}^{(1)}$ be nonzero?
$k = n + 4, k = ?$, etc.

(ii) For each of those values of k, what is $E_n^{(0)} - E_k^{(0)}$?
$E_n^{(0)} - E_{k=n+4}^{(0)} = \hbar\omega[\text{some integer}]$, etc.

(iii) For k = n + 4, what is $H_{nk}^{(1)}$?

$$H_{nn+4}^{(1)} = b\left[\frac{\hbar}{2m\omega}\right]^2 [\text{some function of } n]$$

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