For this problem set, I will accept submissions with one or two names.

(1) Beginning with linearized material and energy balances for the shower, expressed in physical terms,

$$F = F_h + F_c$$

$$T = \frac{F_{hr}T_{hr} + F_{cr}T_{cr}}{F_{hr} + F_{cr}} + \left[\frac{T_{hr}}{F_{hr} + F_{cr}} - \frac{F_{hr}T_{hr} + F_{cr}T_{cr}}{(F_{hr} + F_{cr})^2}\right](F_h - F_{hr}) + \frac{F_{hr}}{F_{hr} + F_{cr}}(T_h - T_{hr})$$
$$+ \left[\frac{T_{cr}}{F_{hr} + F_{cr}} - \frac{F_{hr}T_{hr} + F_{cr}T_{cr}}{(F_{hr} + F_{cr})^2}\right](F_c - F_{cr}) + \frac{F_{cr}}{F_{hr} + F_{cr}}(T_c - T_{cr})$$

derive an expression for each individual element λ_{ij} of the RGA from the definition

$$\lambda_{ij} \equiv \frac{\text{gain of the } x_{mj} \to y_i \text{ loop with other loops open}}{\text{gain of the } x_{mj} \to y_i \text{ loop with other loops perfect}}$$

$$= \frac{\left.\dfrac{\partial y_i}{\partial x_{mj}}\right|_{x_{mk} \neq x_{mj} \text{ constant}}}{\left.\dfrac{\partial y_i}{\partial x_{mj}}\right|_{y_k \neq y_i \text{ constant}}}$$

- (please use the nomenclature of the Lecture Notes)

(2) Beginning with linearized material and energy balances for the shower, expressed in scaled deviation terms,

$$F^{*'} = \frac{\Delta F_h}{\Delta F} F_h^{*'} + \frac{\Delta F_c}{\Delta F} F_c^{*'}$$

$$T^{*'} = \left[\frac{T_{hr} - T_r}{F_r}\right]\frac{\Delta F_h}{\Delta T} F_h^{*'} + \left[\frac{T_{cr} - T_r}{F_r}\right]\frac{\Delta F_c}{\Delta T} F_c^{*'} + \frac{F_{hr}}{F_r}\frac{\Delta T_h}{\Delta T} T_h^{*'} + \frac{F_{cr}}{F_r}\frac{\Delta T_c}{\Delta T} T_c^{*'}$$

derive the RGA matrix $\underline{\underline{\Lambda}}$ from the matrix of gain coefficients $\underline{\underline{P}}_m^{\ *}$.

$$\underline{\underline{\Lambda}} = \underline{\underline{P}}_m^* \otimes \left(\underline{\underline{P}}_m^{*\ -1}\right)^{\mathrm{T}}$$

- (please use the nomenclature of the Lecture Notes)

(3) For each variable in the shower process, state your reference value and scaling (max-min) limits. Be sure to include units. Then calculate numeric values for the RGA.
- Which pairing of variables do you recommend for control?
- Can you find other reference values that are reasonable, but would lead you to the opposite conclusion?

(4) Calculate DC for various combinations of the disturbance variables. The disturbance variables should be set at their minimum, zero, and maximum deviations. Express the results as a plot of DC versus the angle of the disturbance vector, as shown in the Notes.
- (the example in the Notes presumed that each reference value was centered in its operating range – your reference and scaling values may well be different, so that your angles may not be uniformly spaced at 45°.)