10.302
Fall 2004
Exam 1
Wednesday, October 13, 2004

Problem 1 (40 points)

A very long composite cylinder consists of two concentric cylinders. See attached sketch. The inner cylinder of radius r_1 has no heat generation. The outer cylindrical shell, with inner radius r_1 and outer radius r_2, is capable of being heated at a uniform volumetric heat generation rate of $\dot{q}$. The two cylinders are in intimate thermal contact. The outside of the cylindrical shell is immersed in a flowing fluid with temperature T_∞ far from the cylinder and heat transfer coefficient h. The thermal conductivity, heat capacity, and density of the inner cylinder are k_1, c_1, and ρ_1, respectively. The values of these three thermal properties for the cylindrical shell are $k_2, c_2,$ and ρ_2. Initially, the temperature of the entire composite cylinder, T_i, is equal to the ambient temperature, T_∞, and there is no heat generation $\dot{q} = 0$ in the outer cylindrical shell. At $t = 0$ the heater is turned on and the outer cylindrical shell is heated at rate $\dot{q}$.

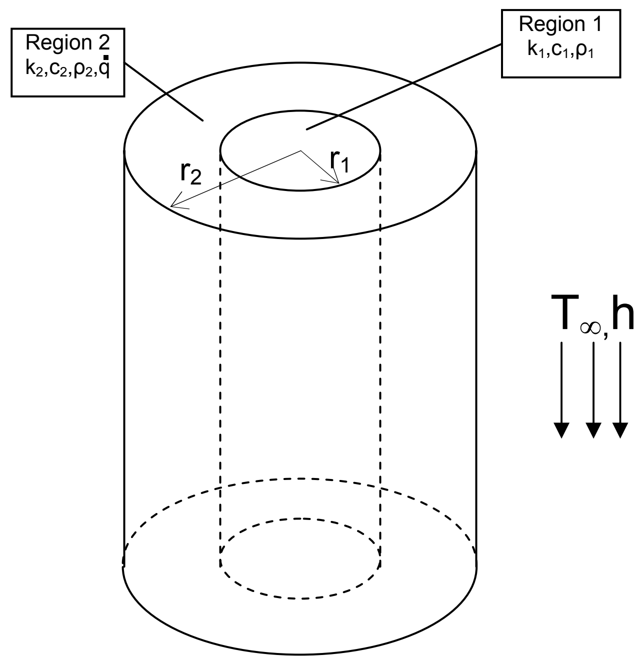
a) (5) Write the heat diffusion equation that describes the unsteady heat transfer within the inner cylinder. Specify appropriate boundary and initial conditions.

b) (5) Write the heat diffusion equation that describes the unsteady heat transfer rate within the outer cylindrical shell. Specify appropriate boundary and initial conditions.

c) (15) Sketch a series of temperature profiles versus radial position at different times. Include the initial profile at t = 0, the steady state profile and two intermediate profiles. Be sure to identify the shapes of the curves (e.g. linear, hyperbolic, parabolic, etc.). Specify slopes and relative magnitudes at r_1, and r_2. Use the attached template for the sketch.

d) (10) At steady state, find an equation for the temperature profile in the inner cylinder and another equation for the temperature profile in the cylindrical shell.

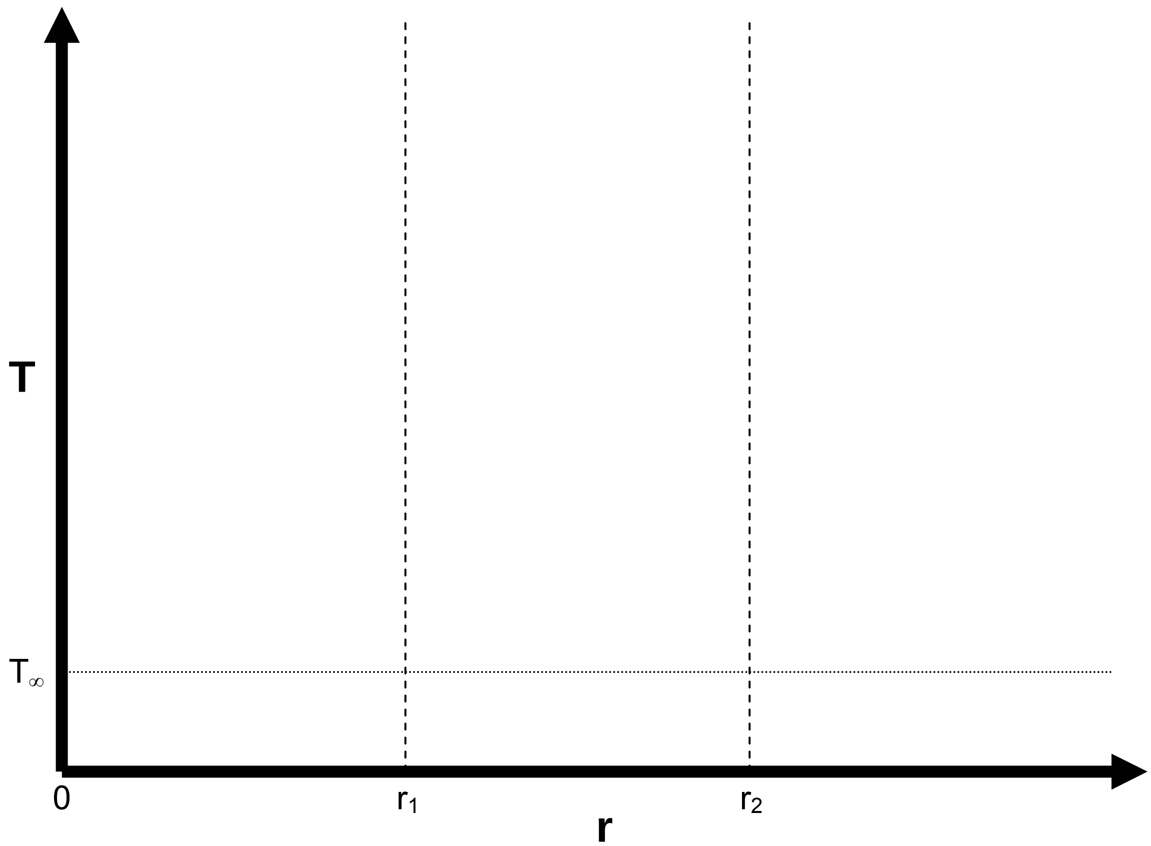
e) (5) Find an expression for the centerline temperature at steady state.


Region 2
$k_2, c_2, \rho_2, \dot{q}$
Region 1
k_1, c_1, ρ_1
r_2
r_1
T_∞, h

TEMPLATE

Problem

Name:___________________



Problem 2 (60 points)

The solid, cylindrical brass rod used in the 10.302 Experiment One, Extended Surface Heat Conduction, is 1 cm in diameter and 35 cm long, with eight thermocouples placed along its length at 5 cm intervals. The thermal conductivity, heat capacity, and density of the rod are $121\frac{\text{W}}{\text{mK}}$, $380\frac{\text{J}}{\text{kgK}}$, and $8500\frac{\text{kg}}{\text{m}^3}$, respectively. The heat transfer coefficient, including contributions from radiation and convection, is $17\frac{\text{W}}{\text{m}^2\text{K}}$.

At steady state, the temperature at the heated end of the rod, T_1, is 60ºC. Ambient temperature is 20ºC.

a) (5) Find the temperature at the tip of the rod, T_8.
b) (5) How much heat q_f is supplied to the rod at T_1, at steady state?
c) (10) It is known that it took 100 minutes for a steady state temperature profile to be established in the fin. How long would it take if the rod were twice as long, i.e., L = 0.70m? An answer which is accurate to within 20% will suffice.

The rod is now covered with a concentric cylinder of high quality foam insulation. The inner diameter of this insulation is 1 cm and the outer diameter is 2 cm. The insulation has negligible thermal mass (low density, low heat capacity) and a thermal conductivity of $0.030\frac{\text{W}}{\text{mK}}$. The same amount of heat q_f is supplied to the rod at T_1 as found in part b. (If you do not have an answer for part b, assume q_f = 2.5W.)

d) (20) Calculate the appropriate new value of the Biot number. Is an analysis of the fin type still appropriate?
e) (5) Calculate the value of the steady state temperature at T_1.
f) (5) Find the steady state temperature at the tip of the rod, T_8.
g) (10) Estimate the time it would now take to attain steady state.

The equation which describes the response of a finite fin to a step change in heat flux is

$$\theta(x,t) = \theta_{ss} - \theta_b \frac{\tanh mL}{mL} e^{-\alpha m^2 t} - \theta_b \sum_{n=1} \frac{2mL \tanh mL}{(mL)^2 + (n\pi)^2} e^{-((n\pi)^2 + (mL)^2)\frac{\alpha t}{L^2}} \cos\frac{n\pi x}{L}$$

$$\theta_{ss} = \theta_b \frac{\cosh m(L - x)}{\cosh mL}$$

Description of Parameters:

Symbol	Definition	Unit
θ_b	$\theta_b = \theta_{ss}$ @ x=0	^{o}C or K
m	m= $(hP/kA)^{1/2}$	m^{-1}
L	Length of the rod	m
h	Heat transfer coefficient	W/m^2K
P	Perimeter	m
k	Thermal conductivity	W/mK
A	Cross sectional area	m^2
α	Thermal diffusivity	m^2/s