Problem sets

- Late policy (5% off per day, but the weekend counts as only one day). E.g.,
  - Friday: -5%
  - Monday: -15%
  - Tuesday: -20%
  - Thursday: -30%

Outline

- Final thoughts on hierarchical Bayesian models and MCMC
- Bayesian classification
- Bayesian concept learning

MCMC methods

- Gibbs sampling
  - Factorize hypotheses $h = <h_1, h_2, \ldots, h_n>$
  - Cycle through variables $h_1, h_2, \ldots, h_n$
  - Draw $h_i^{(t+1)}$ from $P(h_i|h_{-i}, evidence)$
- Metropolis-Hastings
  - Propose changes to hypothesis from some distribution $Q(h^{(t+1)}| h^{(t)})$
  - Accept proposals with probability

$$\mathrm{A}(h^{(t+1)}| h^{(t)}) = \min\{ 1, \frac{P(h^{(t+1)}|evidence)\, Q(h^{(t)}| h^{(t+1)})}{P(h^{(t)}|evidence)\, Q(h^{(t+1)}| h^{(t)})} \}$$

Why MCMC is important

- Simple
- Can be used with just about any kind of probabilistic model, including complex hierarchical structures
- Always works pretty well, if you're willing to wait a long time

(cf. Back-propagation for neural networks.)

A model for cognitive development?

- Some features of cognitive development:
  - Small, random, dumb, local steps
  - Takes a long time
  - Can get stuck in plateaus or stages
  - "Two steps forward, one step back"
  - Over time, intuitive theories get consistently better (more veridical, more powerful, broader scope).
  - Everyone reaches basically the same state (though some take longer than others).

Topic models of semantic structure: e.g., Latent Dirichlet Allocation (Blei, Ng, Jordan)

- Each document in a corpus is associated with a distribution θ over topics.
- Each topic t is associated with a distribution $\phi(t)$ over words.

Image removed due to copyright considerations. Please see:
Blei, David, Andrew Ng, and Michael Jordan. "Latent Dirichlet Allocation." *Journal of Machine Learning Research* 3 (Jan 2003): 993-1022.

Choose mixture weights for each document, generate "bag of words"

$\theta = \{P(z = 1), P(z = 2)\}$

$\{0, 1\}$	MATHEMATICS KNOWLEDGE RESEARCH WORK MATHEMATICS RESEARCH WORK SCIENTIFIC MATHEMATICS WORK
$\{0.25, 0.75\}$	SCIENTIFIC KNOWLEDGE MATHEMATICS SCIENTIFIC HEART LOVE TEARS KNOWLEDGE HEART
$\{0.5, 0.5\}$	MATHEMATICS HEART RESEARCH LOVE MATHEMATICS WORK TEARS SOUL KNOWLEDGE HEART
$\{0.75, 0.25\}$	WORK JOY SOUL TEARS MATHEMATICS TEARS LOVE LOVE LOVE SOUL
$\{1, 0\}$	TEARS LOVE JOY SOUL LOVE TEARS SOUL SOUL TEARS JOY

Gibbs sampling

- Need full conditional distributions for variables
- Since we only sample *z* we need

$$P(z_i = j|\mathbf{z}_{-i}, \mathbf{w}) \propto P(w_i|z_i = j, \mathbf{z}_{-i}, \mathbf{w}_{-i}) P(z_i = j|\mathbf{z}_{-i})$$

$$= \frac{n_{-i,j}^{(w_i)} + \beta}{n_{-i,j}^{(\cdot)} + W\beta} \frac{n_{-i,j}^{(d_i)} + \alpha}{n_{-i,\cdot}^{(d_i)} + T\alpha}$$

$n_j^{(w)}$	number of times word *w* assigned to topic *j*
$n_j^{(d)}$	number of times topic *j* used in document *d*

Gibbs sampling

			iteration 1
i	w_i	d_i	z_i
1	MATHEMATICS	1	2
2	KNOWLEDGE	1	2
3	RESEARCH	1	1
4	WORK	1	2
5	MATHEMATICS	1	1
6	RESEARCH	1	2
7	WORK	1	2
8	SCIENTIFIC	1	1
9	MATHEMATICS	1	2
10	WORK	1	1
11	SCIENTIFIC	2	1
12	KNOWLEDGE	2	1
.	.	.	.
.	.	.	.
.	.	.	.
50	JOY	5	2

Gibbs sampling

			iteration 1	2
i	w_i	d_i	z_i	z_i
1	MATHEMATICS	1	2	?
2	KNOWLEDGE	1	2	
3	RESEARCH	1	1	
4	WORK	1	2	
5	MATHEMATICS	1	1	
6	RESEARCH	1	2	
7	WORK	1	2	
8	SCIENTIFIC	1	1	
9	MATHEMATICS	1	2	
10	WORK	1	1	
11	SCIENTIFIC	2	1	
12	KNOWLEDGE	2	1	
.	.	.	.	
.	.	.	.	
.	.	.	.	
50	JOY	5	2	

Gibbs sampling

			iteration	
			1	2
i	w_i	d_i	z_i	z_i
1	MATHEMATICS	1	2	?
2	KNOWLEDGE	1	2	
3	RESEARCH	1	1	
4	WORK	1	2	
5	MATHEMATICS	1	1	
6	RESEARCH	1	2	
7	WORK	1	2	
8	SCIENTIFIC	1	1	
9	MATHEMATICS	1	2	
10	WORK	1	1	
11	SCIENTIFIC	2	1	
12	KNOWLEDGE	2	1	
.	.	.	.	
.	.	.	.	
.	.	.	.	
50	JOY	5	2	

$$P(z_i = j | \mathbf{z}_{-i}, \mathbf{w}) \propto \frac{n_{-i,j}^{(w_i)} + \beta}{n_{-i,j}^{(\cdot)} + W\beta} \frac{n_{-i,j}^{(d_i)} + \alpha}{n_{-i,\cdot}^{(d_i)} + T\alpha}$$

Gibbs sampling

			iteration 1	iteration 2
i	w_i	d_i	z_i	z_i
1	MATHEMATICS	1	2	?
2	KNOWLEDGE	1	**2**	
3	RESEARCH	1	**1**	
4	WORK	1	**2**	
5	MATHEMATICS	1	**1**	
6	RESEARCH	1	**2**	
7	WORK	1	**2**	
8	SCIENTIFIC	1	**1**	
9	MATHEMATICS	1	**2**	
10	WORK	1	**1**	
11	SCIENTIFIC	2	**1**	
12	KNOWLEDGE	2	**1**	
.	.	.	**.**	
.	.	.	**.**	
.	.	.	**.**	
50	JOY	5	**2**	

$$P(z_i = j | \mathbf{z}_{-i}, \mathbf{w}) \propto \frac{n_{-i,j}^{(w_i)} + \beta}{n_{-i,j}^{(\cdot)} + W\beta} \frac{n_{-i,j}^{(d_i)} + \alpha}{n_{-i,\cdot}^{(d_i)} + T\alpha}$$

Gibbs sampling

			iteration	
			1	2
i	w_i	d_i	z_i	z_i
1	MATHEMATICS	1	2	**2**
2	KNOWLEDGE	1	2	?
3	RESEARCH	1	**1**	
4	WORK	1	**2**	
5	MATHEMATICS	1	**1**	
6	RESEARCH	1	**2**	
7	WORK	1	**2**	
8	SCIENTIFIC	1	**1**	
9	MATHEMATICS	1	**2**	
10	WORK	1	**1**	
11	SCIENTIFIC	2	**1**	
12	KNOWLEDGE	2	**1**	
.	.	.	**.**	
.	.	.	**.**	
.	.	.	**.**	
50	JOY	5	**2**	

$$P(z_i = j | \mathbf{z}_{-i}, \mathbf{w}) \propto \frac{n_{-i,j}^{(w_i)} + \beta}{n_{-i,j}^{(\cdot)} + W\beta} \frac{n_{-i,j}^{(d_i)} + \alpha}{n_{-i,\cdot}^{(d_i)} + T\alpha}$$

Gibbs sampling

			iteration 1	2
i	w_i	d_i	z_i	z_i
1	MATHEMATICS	1	2	**2**
2	KNOWLEDGE	1	2	**1**
3	RESEARCH	1	1	?
4	WORK	1	**2**	
5	MATHEMATICS	1	**1**	
6	RESEARCH	1	**2**	
7	WORK	1	**2**	
8	SCIENTIFIC	1	**1**	
9	MATHEMATICS	1	**2**	
10	WORK	1	**1**	
11	SCIENTIFIC	2	**1**	
12	KNOWLEDGE	2	**1**	
.	.	.	**.**	
.	.	.	**.**	
.	.	.	**.**	
50	JOY	5	**2**	

$$P(z_i = j|\mathbf{z}_{-i}, \mathbf{w}) \propto \frac{n_{-i,j}^{(w_i)} + \beta}{n_{-i,j}^{(\cdot)} + W\beta} \frac{n_{-i,j}^{(d_i)} + \alpha}{n_{-i,\cdot}^{(d_i)} + T\alpha}$$

Gibbs sampling

			iteration 1	2
i	w_i	d_i	z_i	z_i
1	MATHEMATICS	1	2	**2**
2	KNOWLEDGE	1	2	**1**
3	RESEARCH	1	1	**1**
4	WORK	1	2	?
5	MATHEMATICS	1	**1**	
6	RESEARCH	1	**2**	
7	WORK	1	**2**	
8	SCIENTIFIC	1	**1**	
9	MATHEMATICS	1	**2**	
10	WORK	1	**1**	
11	SCIENTIFIC	2	**1**	
12	KNOWLEDGE	2	**1**	
.	.	.	.	
.	.	.	.	
.	.	.	.	
50	JOY	5	**2**	

$$P(z_i = j | \mathbf{z}_{-i}, \mathbf{w}) \propto \frac{n_{-i,j}^{(w_i)} + \beta}{n_{-i,j}^{(\cdot)} + W\beta} \frac{n_{-i,j}^{(d_i)} + \alpha}{n_{-i,\cdot}^{(d_i)} + T\alpha}$$

Gibbs sampling

			iteration	
			1	2
i	w_i	d_i	z_i	z_i
1	MATHEMATICS	1	2	**2**
2	KNOWLEDGE	1	2	**1**
3	RESEARCH	1	1	**1**
4	WORK	1	2	**2**
5	MATHEMATICS	1	1	?
6	RESEARCH	1	**2**	
7	WORK	1	**2**	
8	SCIENTIFIC	1	**1**	
9	MATHEMATICS	1	**2**	
10	WORK	1	**1**	
11	SCIENTIFIC	2	**1**	
12	KNOWLEDGE	2	**1**	
.	.	.	**.**	
.	.	.	**.**	
.	.	.	**.**	
50	JOY	5	**2**	

$$P(z_i = j | \mathbf{z}_{-i}, \mathbf{w}) \propto \frac{n_{-i,j}^{(w_i)} + \beta}{n_{-i,j}^{(\cdot)} + W\beta} \frac{n_{-i,j}^{(d_i)} + \alpha}{n_{-i,\cdot}^{(d_i)} + T\alpha}$$

Gibbs sampling

			iteration			
			1	2	…	1000
i	w_i	d_i	z_i	z_i		z_i
1	MATHEMATICS	1	2	2		2
2	KNOWLEDGE	1	2	1		2
3	RESEARCH	1	1	1		2
4	WORK	1	2	2		1
5	MATHEMATICS	1	1	2		2
6	RESEARCH	1	2	2		2
7	WORK	1	2	2		2
8	SCIENTIFIC	1	1	1	…	1
9	MATHEMATICS	1	2	2		2
10	WORK	1	1	2		2
11	SCIENTIFIC	2	1	1		2
12	KNOWLEDGE	2	1	2		2
.	.	.	.	.		.
.	.	.	.	.		.
.	.	.	.	.		.
50	JOY	5	2	1		1

$$P(z_i = j | \mathbf{z}_{-i}, \mathbf{w}) \propto \frac{n_{-i,j}^{(w_i)} + \beta}{n_{-i,j}^{(\cdot)} + W\beta} \frac{n_{-i,j}^{(d_i)} + \alpha}{n_{-i,\cdot}^{(d_i)} + T\alpha}$$

A selection of topics (TASA)

DISEASE	WATER	MIND	STORY	FIELD	SCIENCE	BALL	JOB
BACTERIA	FISH	WORLD	STORIES	MAGNETIC	STUDY	GAME	WORK
DISEASES	SEA	DREAM	TELL	MAGNET	SCIENTISTS	TEAM	JOBS
GERMS	SWIM	DREAMS	CHARACTER	WIRE	SCIENTIFIC	FOOTBALL	CAREER
FEVER	SWIMMING	THOUGHT	CHARACTERS	NEEDLE	KNOWLEDGE	BASEBALL	EXPERIENCE
CAUSE	POOL	IMAGINATION	AUTHOR	CURRENT	WORK	PLAYERS	EMPLOYMENT
CAUSED	LIKE	MOMENT	READ	COIL	RESEARCH	PLAY	OPPORTUNITIES
SPREAD	SHELL	THOUGHTS	TOLD	POLES	CHEMISTRY	FIELD	WORKING
VIRUSES	SHARK	OWN	SETTING	IRON	TECHNOLOGY	PLAYER	TRAINING
INFECTION	TANK	REAL	TALES	COMPASS	MANY	BASKETBALL	SKILLS
VIRUS	SHELLS	LIFE	PLOT	LINES	MATHEMATICS	COACH	CAREERS
MICROORGANISMS	SHARKS	IMAGINE	TELLING	CORE	BIOLOGY	PLAYED	POSITIONS
PERSON	DIVING	SENSE	SHORT	ELECTRIC	FIELD	PLAYING	FIND
INFECTIOUS	DOLPHINS	CONSCIOUSNESS	FICTION	DIRECTION	PHYSICS	HIT	POSITION
COMMON	SWAM	STRANGE	ACTION	FORCE	LABORATORY	TENNIS	FIELD
CAUSING	LONG	FEELING	TRUE	MAGNETS	STUDIES	TEAMS	OCCUPATIONS
SMALLPOX	SEAL	WHOLE	EVENTS	BE	WORLD	GAMES	REQUIRE
BODY	DIVE	BEING	TELLS	MAGNETISM	SCIENTIST	SPORTS	OPPORTUNITY
INFECTIONS	DOLPHIN	MIGHT	TALE	POLE	STUDYING	BAT	EARN
CERTAIN	UNDERWATER	HOPE	NOVEL	INDUCED	SCIENCES	TERRY	ABLE

A selection of topics (TASA)

DISEASE	WATER	MIND	STORY	**FIELD**	SCIENCE	BALL	JOB
BACTERIA	FISH	WORLD	STORIES	MAGNETIC	STUDY	GAME	WORK
DISEASES	SEA	DREAM	TELL	MAGNET	SCIENTISTS	TEAM	JOBS
GERMS	SWIM	DREAMS	CHARACTER	WIRE	SCIENTIFIC	FOOTBALL	CAREER
FEVER	SWIMMING	THOUGHT	CHARACTERS	NEEDLE	KNOWLEDGE	BASEBALL	EXPERIENCE
CAUSE	POOL	IMAGINATION	AUTHOR	CURRENT	WORK	PLAYERS	EMPLOYMENT
CAUSED	LIKE	MOMENT	READ	COIL	RESEARCH	PLAY	OPPORTUNITIES
SPREAD	SHELL	THOUGHTS	TOLD	POLES	CHEMISTRY	**FIELD**	WORKING
VIRUSES	SHARK	OWN	SETTING	IRON	TECHNOLOGY	PLAYER	TRAINING
INFECTION	TANK	REAL	TALES	COMPASS	MANY	BASKETBALL	SKILLS
VIRUS	SHELLS	LIFE	PLOT	LINES	MATHEMATICS	COACH	CAREERS
MICROORGANISMS	SHARKS	IMAGINE	TELLING	CORE	BIOLOGY	PLAYED	POSITIONS
PERSON	DIVING	SENSE	SHORT	ELECTRIC	**FIELD**	PLAYING	FIND
INFECTIOUS	DOLPHINS	CONSCIOUSNESS	FICTION	DIRECTION	PHYSICS	HIT	POSITION
COMMON	SWAM	STRANGE	ACTION	FORCE	LABORATORY	TENNIS	**FIELD**
CAUSING	LONG	FEELING	TRUE	MAGNETS	STUDIES	TEAMS	OCCUPATIONS
SMALLPOX	SEAL	WHOLE	EVENTS	BE	WORLD	GAMES	REQUIRE
BODY	DIVE	BEING	TELLS	MAGNETISM	SCIENTIST	SPORTS	OPPORTUNITY
INFECTIONS	DOLPHIN	MIGHT	TALE	POLE	STUDYING	BAT	EARN
CERTAIN	UNDERWATER	HOPE	NOVEL	INDUCED	SCIENCES	TERRY	ABLE

The[14] “shape[7]” of[4] a[23] female[115] mating[115] preference[125] is[32] the[14] relationship[7] between[4] a[23] male[115] trait[15] and[37] the[14] probability[7] of[4] acceptance[21] as[43] a[23] mating[115] partner[20], The[14] shape[7] of[4] preferences[115] is[32] important[49] in[5] many[39] models[6] of[4] sexual[115] selection[46], mate[115] recognition[125], communication[9], and[37] speciation[46], yet[50] it[41] has[18] rarely[19] been[33] measured[17] precisely[19], Here[12] I[9] examine[34] preference[7] shape[7] for[5] male[115] calling[115] song[125] in[22] a[23] bushcricket[*13] (katydid[*48]). Preferences[115] change[46] dramatically[19] between[22] races[46] of[4] a[23] species[15], from[22] strongly[19] directional[11] to[31] broadly[19] stabilizing[45] (but[50] with[21] a[23] net[49] directional[46] effect[46]), Preference[115] shape[46] generally[19] matches[10] the[14] distribution[16] of[4] the[14] male[115] trait[15], This[41] is[32] compatible[29] with[21] a[23] coevolutionary[46] model[20] of[4] signal[9]-preference[115] evolution[46], although[50] it[41] does[33] not[37] rule[20] out[17] an[23] alternative[11] model[20], sensory[125] exploitation[150]. Preference[46] shapes[40] are[8] shown[35] to[31] be[44] genetic[11] in[5] origin[7].

(graylevel = membership in topic 115)

Ritchie, Michael G. "The Shape of Female Mating Preferences." *PNAS* 93 (1996): 14628-14631.

The[14] "shape[7]" of[4] a[23] female[115] mating[115] preference[125] is[32] the[14] relationship[7] between[4] a[23] male[115] trait[15] and[37] the[14] probability[7] of[4] acceptance[21] as[43] a[23] mating[115] partner[20], The[14] shape[7] of[4] preferences[115] is[32] important[49] in[5] many[39] models[6] of[4] sexual[115] selection[46], mate[115] recognition[125], communication[9], and[37] speciation[46], yet[50] it[41] has[18] rarely[19] been[33] measured[17] precisely[19], Here[12] I[9] examine[34] preference[7] shape[7] for[5] male[115] calling[115] song[125] in[22] a[23] bushcricket[*13] (katydid[*48]). Preferences[115] change[46] dramatically[19] between[22] races[46] of[4] a[23] species[15], from[22] strongly[19] directional[11] to[31] broadly[19] stabilizing[45] (but[50] with[21] a[23] net[49] directional[46] effect[46]), Preference[115] shape[46] generally[19] matches[10] the[14] distribution[16] of[4] the[14] male[115] trait[15], This[41] is[32] compatible[29] with[21] a[23] coevolutionary[46] model[20] of[4] signal[9]-preference[115] evolution[46], although[50] it[41] does[33] not[37] rule[20] out[17] an[23] alternative[11] model[20], sensory[125] exploitation[150]. Preference[46] shapes[40] are[8] shown[35] to[31] be[44] genetic[11] in[5] origin[7].

(graylevel = membership in topic 115, 46)

The[14] “shape[7]” of[4] a[23] female[115] mating[115] preference[125] is[32] the[14] relationship[7] between[4] a[23] male[115] trait[15] and[37] the[14] probability[7] of[4] acceptance[21] as[43] a[23] mating[115] partner[20], The[14] shape[7] of[4] preferences[115] is[32] important[49] in[5] many[39] models[6] of[4] sexual[115] selection[46], mate[115] recognition[125], communication[9], and[37] speciation[46], yet[50] it[41] has[18] rarely[19] been[33] measured[17] precisely[19], Here[12] I[9] examine[34] preference[7] shape[7] for[5] male[115] calling[115] song[125] in[22] a[23] bushcricket[*13] (katydid[*48]). Preferences[115] change[46] dramatically[19] between[22] races[46] of[4] a[23] species[15], from[22] strongly[19] directional[11] to[31] broadly[19] stabilizing[45] (but[50] with[21] a[23] net[49] directional[46] effect[46]), Preference[115] shape[46] generally[19] matches[10] the[14] distribution[16] of[4] the[14] male[115] trait[15], This[41] is[32] compatible[29] with[21] a[23] coevolutionary[46] model[20] of[4] signal[9]-preference[115] evolution[46], although[50] it[41] does[33] not[37] rule[20] out[17] an[23] alternative[11] model[20], sensory[125] exploitation[150]. Preference[46] shapes[40] are[8] shown[35] to[31] be[44] genetic[11] in[5] origin[7].

(graylevel = membership in topic 115, 46, 125)

Ritchie, Michael G. "The Shape of Female Mating Preferences." *PNAS* 93 (1996): 14628-14631.

Joint models of syntax and semantics

(Griffiths, Steyvers, Blei & Tenenbaum, NIPS 2004)

- Embed topics model inside an *n*th order Hidden Markov Model:

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Griffiths, T. L., M. Steyvers, D. M. Blei, and J. B. Tenenbaum. "Integrating Topics and Syntax." *Advances in Neural Information Processing Systems* 17 (2005).

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Griffiths, T. L., M. Steyvers, D. M. Blei, and J. B. Tenenbaum. "Integrating Topics and Syntax." *Advances in Neural Information Processing Systems* 17 (2005).

Semantic classes

FOOD	MAP	DOCTOR	BOOK	GOLD	BEHAVIOR	CELLS	PLANTS
FOODS	NORTH	PATIENT	BOOKS	IRON	SELF	CELL	PLANT
BODY	EARTH	HEALTH	READING	SILVER	INDIVIDUAL	ORGANISMS	LEAVES
NUTRIENTS	SOUTH	HOSPITAL	INFORMATION	COPPER	PERSONALITY	ALGAE	SEEDS
DIET	POLE	MEDICAL	LIBRARY	METAL	RESPONSE	BACTERIA	SOIL
FAT	MAPS	CARE	REPORT	METALS	SOCIAL	MICROSCOPE	ROOTS
SUGAR	EQUATOR	PATIENTS	PAGE	STEEL	EMOTIONAL	MEMBRANE	FLOWERS
ENERGY	WEST	NURSE	TITLE	CLAY	LEARNING	ORGANISM	WATER
MILK	LINES	DOCTORS	SUBJECT	LEAD	FEELINGS	FOOD	FOOD
EATING	EAST	MEDICINE	PAGES	ADAM	PSYCHOLOGISTS	LIVING	GREEN
FRUITS	AUSTRALIA	NURSING	GUIDE	ORE	INDIVIDUALS	FUNGI	SEED
VEGETABLES	GLOBE	TREATMENT	WORDS	ALUMINUM	PSYCHOLOGICAL	MOLD	STEMS
WEIGHT	POLES	NURSES	MATERIAL	MINERAL	EXPERIENCES	MATERIALS	FLOWER
FATS	HEMISPHERE	PHYSICIAN	ARTICLE	MINE	ENVIRONMENT	NUCLEUS	STEM
NEEDS	LATITUDE	HOSPITALS	ARTICLES	STONE	HUMAN	CELLED	LEAF
CARBOHYDRATES	PLACES	DR	WORD	MINERALS	RESPONSES	STRUCTURES	ANIMALS
VITAMINS	LAND	SICK	FACTS	POT	BEHAVIORS	MATERIAL	ROOT
CALORIES	WORLD	ASSISTANT	AUTHOR	MINING	ATTITUDES	STRUCTURE	POLLEN
PROTEIN	COMPASS	EMERGENCY	REFERENCE	MINERS	PSYCHOLOGY	GREEN	GROWING
MINERALS	CONTINENTS	PRACTICE	NOTE	TIN	PERSON	MOLDS	GROW

Image removed due to copyright considerations. Please see:
Griffiths, T. L., M. Steyvers, D. M. Blei, and J. B. Tenenbaum. "Integrating Topics and Syntax." *Advances in Neural Information Processing Systems* 17 (2005).

Syntactic classes

SAID	THE	MORE	ON	GOOD	ONE	HE	BE
ASKED	HIS	SUCH	AT	SMALL	SOME	YOU	MAKE
THOUGHT	THEIR	LESS	INTO	NEW	MANY	THEY	GET
TOLD	YOUR	MUCH	FROM	IMPORTANT	TWO	I	HAVE
SAYS	HER	KNOWN	WITH	GREAT	EACH	SHE	GO
MEANS	ITS	JUST	THROUGH	LITTLE	ALL	WE	TAKE
CALLED	MY	BETTER	OVER	LARGE	MOST	IT	DO
CRIED	OUR	RATHER	AROUND	*	ANY	PEOPLE	FIND
SHOWS	THIS	GREATER	AGAINST	BIG	THREE	EVERYONE	USE
ANSWERED	THESE	HIGHER	ACROSS	LONG	THIS	OTHERS	SEE
TELLS	A	LARGER	UPON	HIGH	EVERY	SCIENTISTS	HELP
REPLIED	AN	LONGER	TOWARD	DIFFERENT	SEVERAL	SOMEONE	KEEP
SHOUTED	THAT	FASTER	UNDER	SPECIAL	FOUR	WHO	GIVE
EXPLAINED	NEW	EXACTLY	ALONG	OLD	FIVE	NOBODY	LOOK
LAUGHED	THOSE	SMALLER	NEAR	STRONG	BOTH	ONE	COME
MEANT	EACH	SOMETHING	BEHIND	YOUNG	TEN	SOMETHING	WORK
WROTE	MR	BIGGER	OFF	COMMON	SIX	ANYONE	MOVE
SHOWED	ANY	FEWER	ABOVE	WHITE	MUCH	EVERYBODY	LIVE
BELIEVED	MRS	LOWER	DOWN	SINGLE	TWENTY	SOME	EAT
WHISPERED	ALL	ALMOST	BEFORE	CERTAIN	EIGHT	THEN	BECOME

Corpus-specific factorization (NIPS)

Image removed due to copyright considerations. Please see:
Griffiths, T. L., M. Steyvers, D. M. Blei, and J. B. Tenenbaum. "Integrating Topics and Syntax." *Advances in Neural Information Processing Systems* 17 (2005).

Syntactic classes in PNAS

5	8	14	25	26	30	33
IN	ARE	THE	SUGGEST	LEVELS	RESULTS	BEEN
FOR	WERE	THIS	INDICATE	NUMBER	ANALYSIS	MAY
ON	WAS	ITS	SUGGESTING	LEVEL	DATA	CAN
BETWEEN	IS	THEIR	SUGGESTS	RATE	STUDIES	COULD
DURING	WHEN	AN	SHOWED	TIME	STUDY	WELL
AMONG	REMAIN	EACH	REVEALED	CONCENTRATIONS	FINDINGS	DID
FROM	REMAINS	ONE	SHOW	VARIETY	EXPERIMENTS	DOES
UNDER	REMAINED	ANY	DEMONSTRATE	RANGE	OBSERVATIONS	DO
WITHIN	PREVIOUSLY	INCREASED	INDICATING	CONCENTRATION	HYPOTHESIS	MIGHT
THROUGHOUT	BECOME	EXOGENOUS	PROVIDE	DOSE	ANALYSES	SHOULD
THROUGH	BECAME	OUR	SUPPORT	FAMILY	ASSAYS	WILL
TOWARD	BEING	RECOMBINANT	INDICATES	SET	POSSIBILITY	WOULD
INTO	BUT	ENDOGENOUS	PROVIDES	FREQUENCY	MICROSCOPY	MUST
AT	GIVE	TOTAL	INDICATED	SERIES	PAPER	CANNOT
INVOLVING	MERE	PURIFIED	DEMONSTRATED	AMOUNTS	WORK	REMAINED
AFTER	APPEARED	TILE	SHOWS	RATES	EVIDENCE	ALSO
ACROSS	APPEAR	FULL	SO	CLASS	FINDING	THEY
AGAINST	ALLOWED	CHRONIC	REVEAL	VALUES	MUTAGENESIS	BECOME
WHEN	NORMALLY	ANOTHER	DEMONSTRATES	AMOUNT	OBSERVATION	MAG
ALONG	EACH	EXCESS	SUGGESTED	SITES	MEASUREMENTS	LIKELY

Semantic highlighting

Darker words are more likely to have been generated from the topic-based "semantics" module:

In contrast to this approach, we study here how the overall network activity can control single cell parameters such as input resistance, as well as time and space constants, parameters that are crucial for excitability and spariotemporal (sic) integration.

The integrated architecture in this paper combines feed forward control and error feedback adaptive control using neural networks.

In other words, for our proof of convergence, we require the softassign algorithm to return a doubly stochastic matrix as *sinkhorn theorem guarantees that it will instead of a matrix which is merely close to being doubly stochastic based on some reasonable metric.

The aim is to construct a portfolio with a maximal expected return for a given risk level and time horizon while simultaneously obeying *institutional or *legally required constraints.

The left graph is the standard experiment the right from a training with # samples.

The graph G is called the *guest graph, and H is called the host graph.

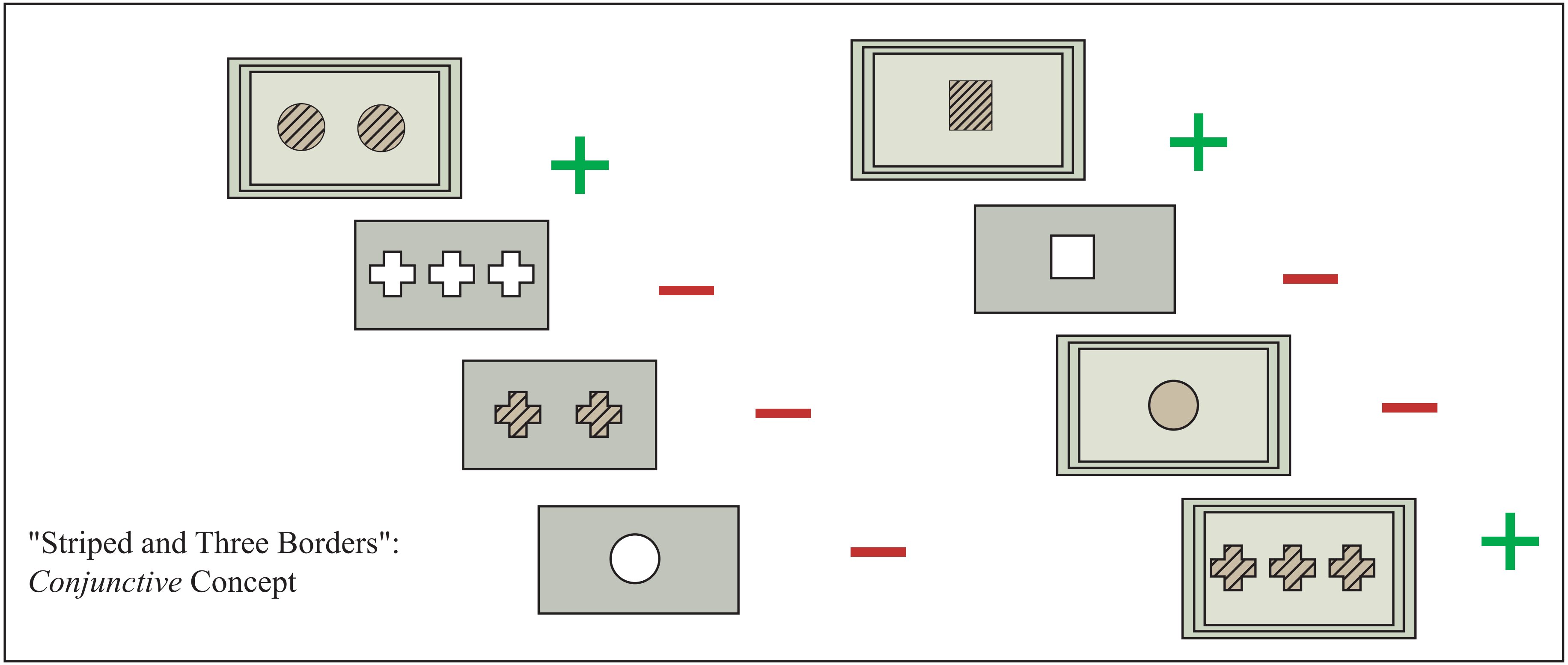
Outline

- Final thoughts on hierarchical Bayesian models and MCMC
- Bayesian classification
- Bayesian concept learning

Concepts and categories

- A category is a set of objects that are treated equivalently for some purpose.
- A concept is a mental representation of the category.
- Functions for concepts:
  - Categorization/classification
  - Prediction
  - Inductive generalization
  - Explanation
  - Reference in communication and thought

- Classical view of concepts (1950's-1960's): Concepts are rules or symbolic representations for classifying.
- Examples
  - Psychology: Bruner et al.



"Striped and Three Borders": *Conjunctive* Concept

Figure by MIT OCW.

- Classical view of concepts (1950's-1960's): Concepts are rules or symbolic representations
- Examples
  - AI: Winston's arch learner

Image removed due to copyright considerations. Please see:

Winston, P. H., ed. *The Psychology of Computer Vision*. New York, NY: McGaw-Hill, 1975. ISBN: 0070710481.

http://www.rci.rutgers.edu/~cfs/472_html/Learn/LearnGifs/ArchExSeq.gif

- Statistical view of concepts (1960’s-1970’s)
- Examples
  – Machine learning/statistics: Iris classification

Images removed due to copyright considerations.

- Standard version (1960’s-1970’s): Concepts are statistical representations for classifying.
- Examples
  - Psychology: Posner and Keele

Image removed due to copyright considerations. Please see:
Posner, M. I., and S. W. Keele. "On the Genesis of Abstract Ideas." *Journal of Experimental Psychology* 77 (1968): 353-363.

Different levels of random distortion:

Images removed due to copyright considerations.

Statistical pattern recognition

Two-class classification problem:

Images removed due to copyright considerations.

The task: Given an object generated from class 1 or class 2, infer the generating class.

Formalizing two-class classification:

Images removed due to copyright considerations.

The task: Observe x generated from c_1 or c_2, compute:

$$p(c_1 \mid x) = \frac{p(x \mid c_1)\, p(c_1)}{p(x \mid c_1)\, p(c_1) + p(x \mid c_2)\, p(c_2)}$$

Different approaches vary in how they represent $p(x|c_j)$.

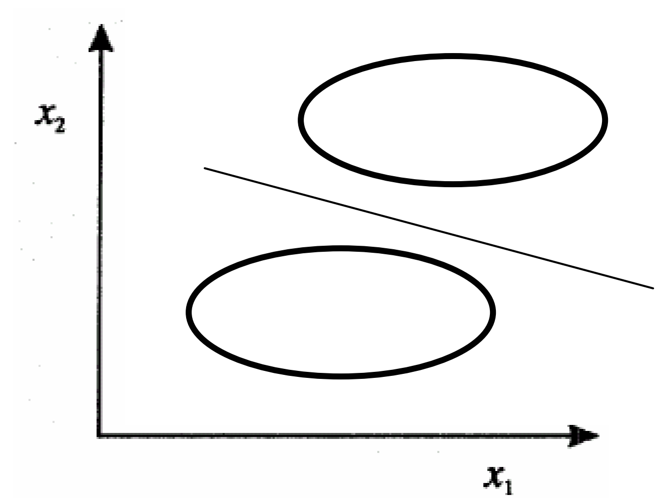
Parametric approach

- Assume a simple canonical form for $p(x|c_j)$.
- E.g., Gaussian distributions:

Images removed due to copyright considerations.

Parametric approach

- Assume a simple canonical form for $p(x|c_j)$.
- The simplest Gaussians have all dimensions independent, variances equal for all classes:
  - Classification based on distance to means.
  - Covariance ellipse determines the distance metric.

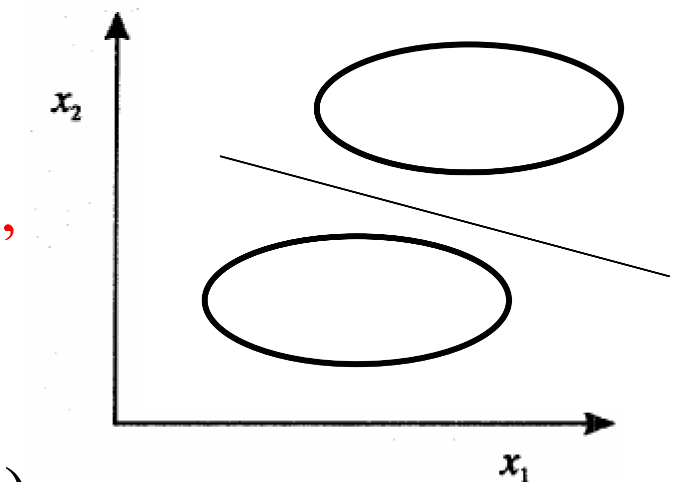


Parametric approach

- Assume a simple canonical form for $p(x|c_j)$.
- The simplest Gaussians have all dimensions independent, variances equal for all classes:
  - Bayes net representation:

$$\begin{tikzcd} & C \arrow[dl] \arrow[dr] & \\ x_1 & & x_2 \end{tikzcd}$$

"naïve Bayes"

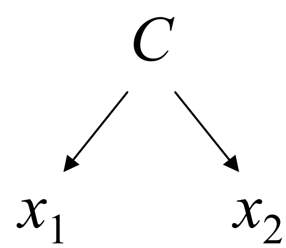


$$p(x|c_j) = p(x_1|c_j) \times p(x_2|c_j)$$

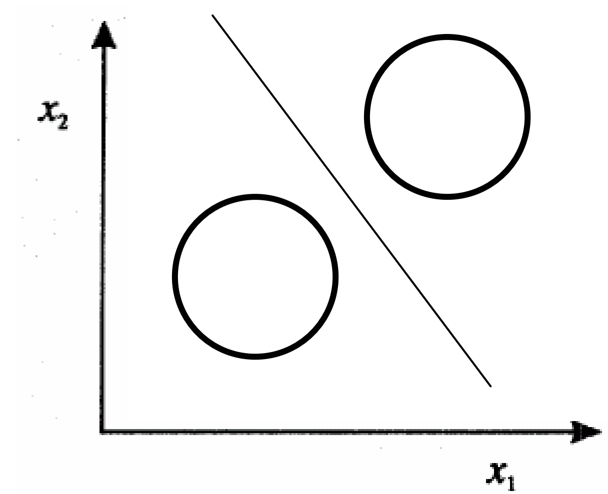
$$p(x_i|c_j) \propto e^{-(x_i - \mu_{ij})^2/(2\sigma_i^2)}$$

Parametric approach

- Other possible forms:
  - All dimensions independent with variances equal across dimensions and classes:



“naïve Bayes”

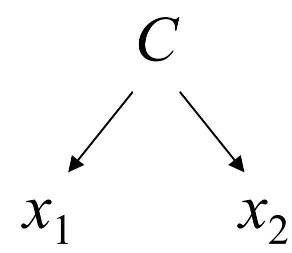


$$p(x \mid c_j) = p(x_1 \mid c_j) \times p(x_2 \mid c_j)$$

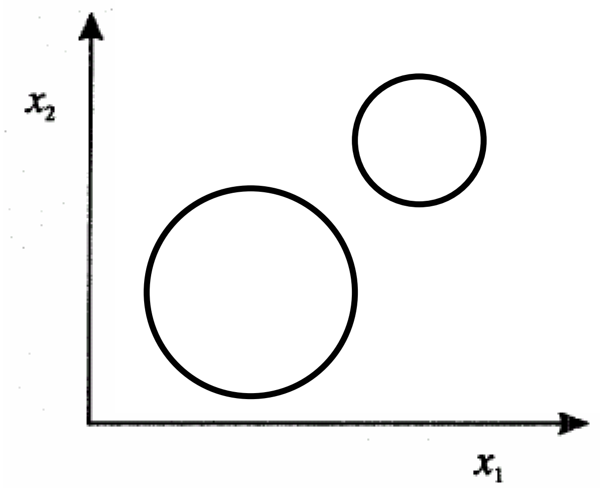
$$p(x_i \mid c_j) \propto e^{-(x_i - \mu_{ij})^2 / (2\sigma^2)}$$

Parametric approach

- Other possible forms:
  - All dimensions independent with equal variances, but variances differ across classes:



"naïve Bayes"



$$p(x \mid c_j) = p(x_1 \mid c_j) \times p(x_2 \mid c_j)$$

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Parametric approach

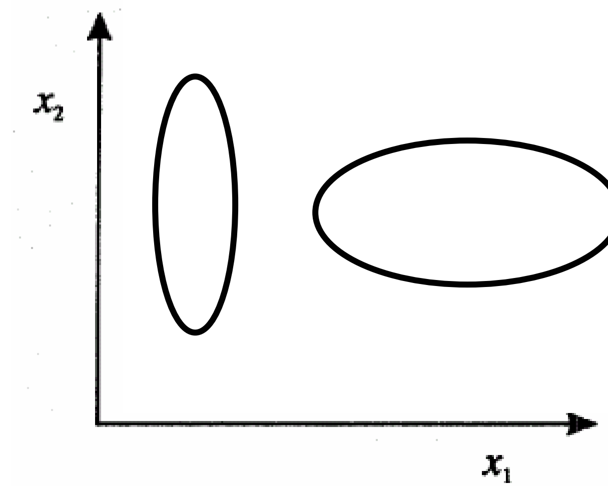
- Other possible forms:
  - All dimensions independent, variances differ across dimensions and across classes:

$$\begin{tikzcd}[column sep=small] & C \arrow[dl] \arrow[dr] & \\ x_1 & & x_2 \end{tikzcd}$$

"naïve Bayes"

$$p(x \mid c_j) = p(x_1 \mid c_j) \times p(x_2 \mid c_j)$$

$$p(x_i \mid c_j) \propto e^{-(x_i - \mu_{ij})^2 / (2\sigma_{ij}^2)}$$

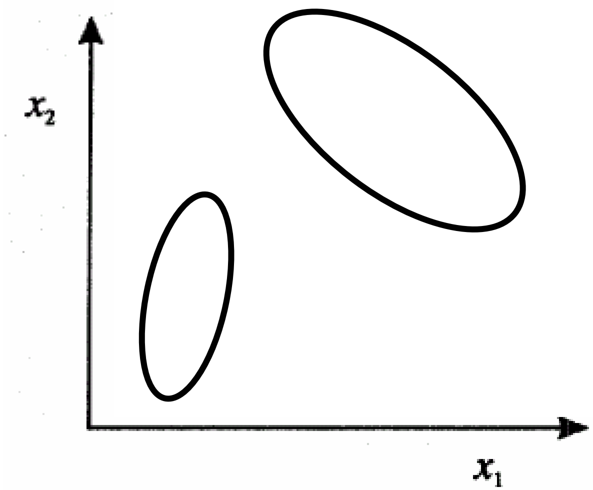


Parametric approach

- Other possible forms:
  - Arbitrary covariance matrices for each class.

$$C \downarrow \mathbf{x} = \{x_1, x_2\}$$

Board formula



Parametric approach

- Assume a simple canonical form for $p(x|c_j)$.
- The simplest Gaussians have all dimensions independent, variances equal for all classes:
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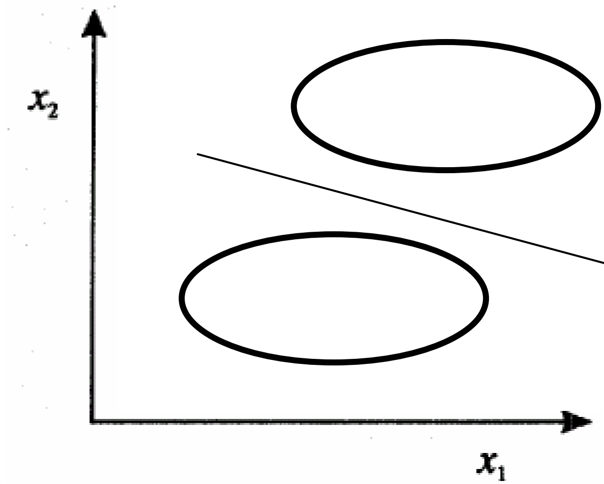


$$p(x \mid c_j) = p(x_1 \mid c_j) \times p(x_2 \mid c_j)$$

$$p(x_i \mid c_j) \propto e^{-(x_i - \mu_{ij})^2 / (2\sigma_i^2)}$$

Learning

- Hypothesis space of possible Gaussians:

Images removed due to copyright considerations.

- Find parameters that maximize likelihood of examples.
  - $\vec{\mu}_j$ = mean of examples of class j.
  - σ_i = standard deviation along dimension i, for examples in each class.

Relevance to human concept learning

- Natural categories often have Gaussian (or other simple parametric forms) in perceptual feature spaces.
- Prototype effects in categorization (Rosch)
- Posner & Keele studies of prototype abstraction in concept learning.

Posner and Keele: design

Image removed due to copyright considerations. Please see:

Posner, M. I., and S. W. Keele. "On the Genesis of Abstract Ideas." *Journal of Experimental Psychology* 77 (1968): 353-363.

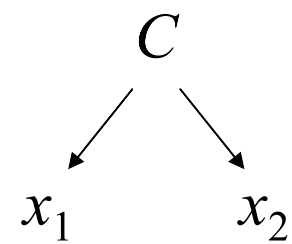
Posner and Keele: results

Image removed due to copyright considerations. Please see:
Posner, M. I., and S. W. Keele. "On the Genesis of Abstract Ideas." *Journal of Experimental Psychology* 77 (1968): 353-363.

Unseen prototype ("Schema") classified as well as memorized variants, and much better than new random variants ("5").

Parametric approach

- Other possible forms:
  - All dimensions independent with variances equal across dimensions and classes:



"naïve Bayes"

$$p(x \mid c_j) = p(x_1 \mid c_j) \times p(x_2 \mid c_j)$$

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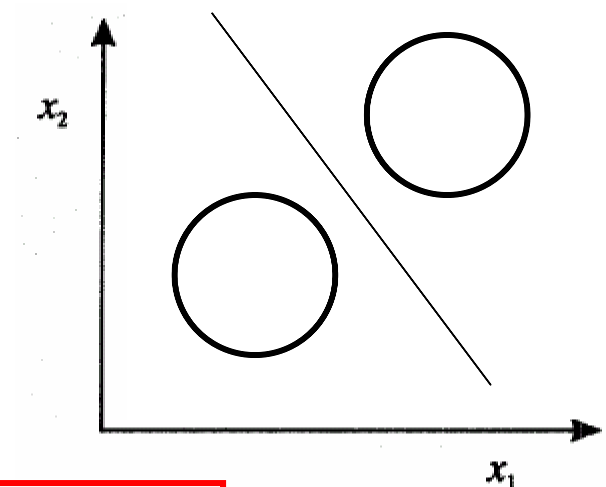

Equivalent to prototype model:
Prototype of class j: $\vec{\mu}_j = \{\mu_{1j}, \mu_{2j}\}$
Variability of categories: σ

Limitations

- Of this empirical paradigm?
- Of this computational approach?

Limitations

- Is categorization just discrimination among mutually exclusive classes?
  - Overlapping concepts? Hierarchies? "None of the above"? Can we learn a single new concept?
- How do we learn concepts from just a few positive examples?
  - Learning with high certainty from little data.
  - Schema abstraction from one imperfect example.
- Are most categories Gaussian, or any simple parametric shape?
  - What about superordinate categories?
  - What about learning rule-based categories?

Limitations

- Is prototypicality = degree of membership?
  - Armstrong et al.: No, for classical rule-based categories
  - Not for complex real-world categories either: “Christmas eve”, “Hollywood actress”, “Californian”, “Professor”
  - For natural kinds, huge variability in prototypicality independent of membership.
- Richer concepts?
  - Meaningful stimuli, background knowledge, theories?
  - Role of causal reasoning? “Essentialism”?
- Difference between “perceptual” and “cognitive” concepts?