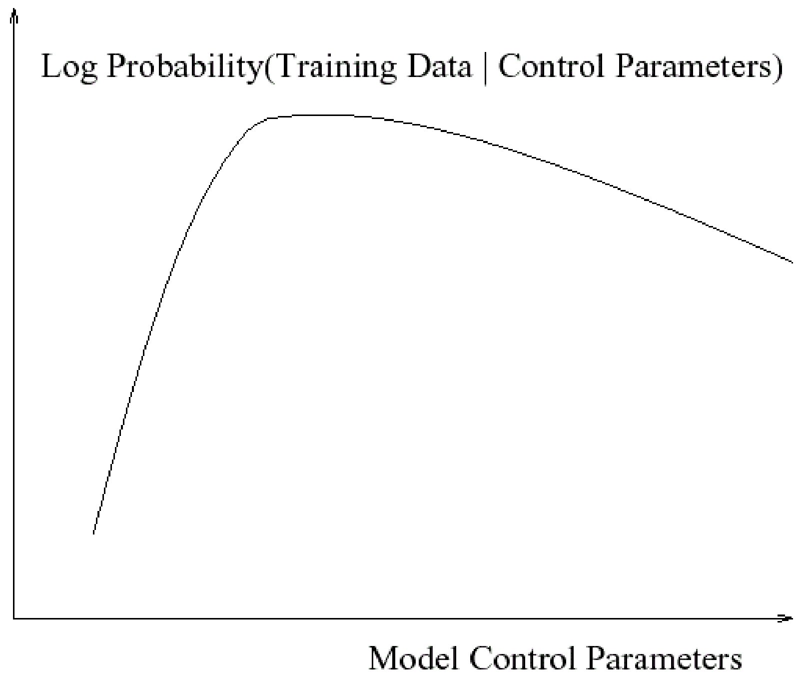
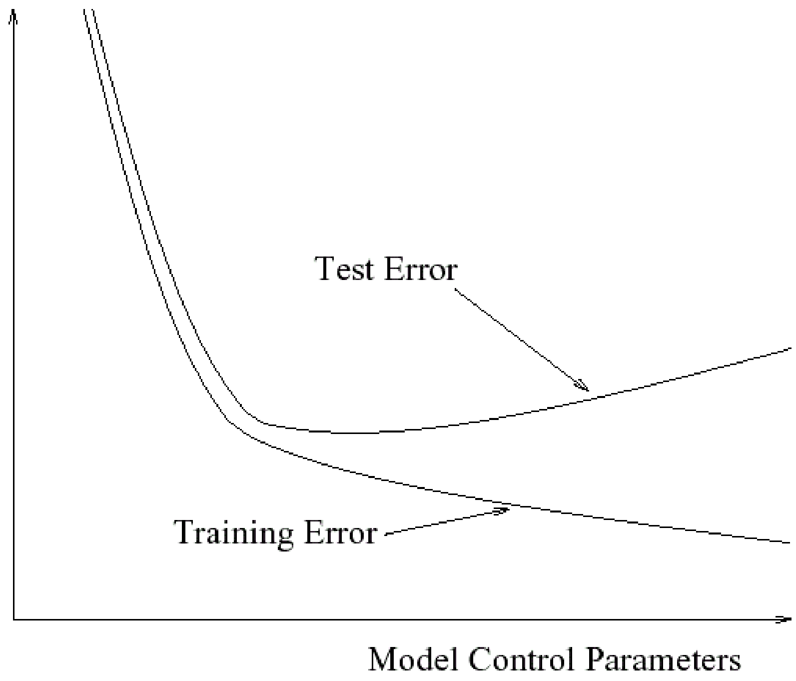
Outline

- Controlling complexity in Bayesian neural networks
- Controlling complexity in infinite mixture models
- Discussion
  - Computational strengths and weaknesses
  - Cognitive relevance

How to choose control parameters?

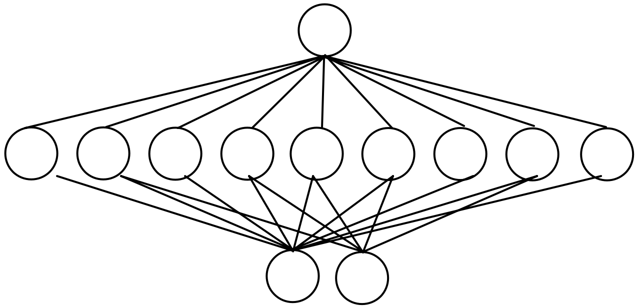
- Bayesian Occam's razor





Demo

- Smaller weights (higher α) yield simpler models
  - neural_net.m
  - architecture:
    - 2 inputs
    - 1 output
    - 100 hidden units

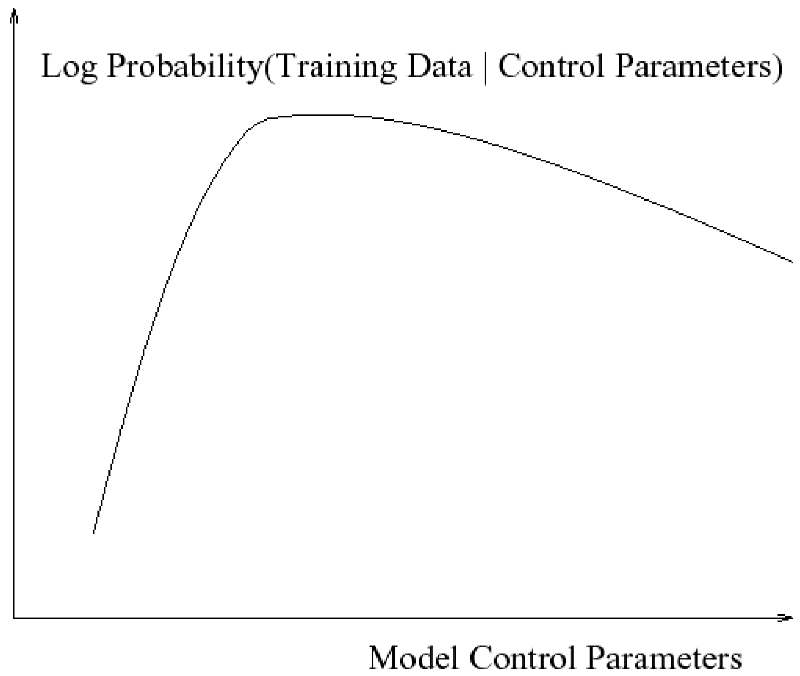


Two approaches to choosing control parameters

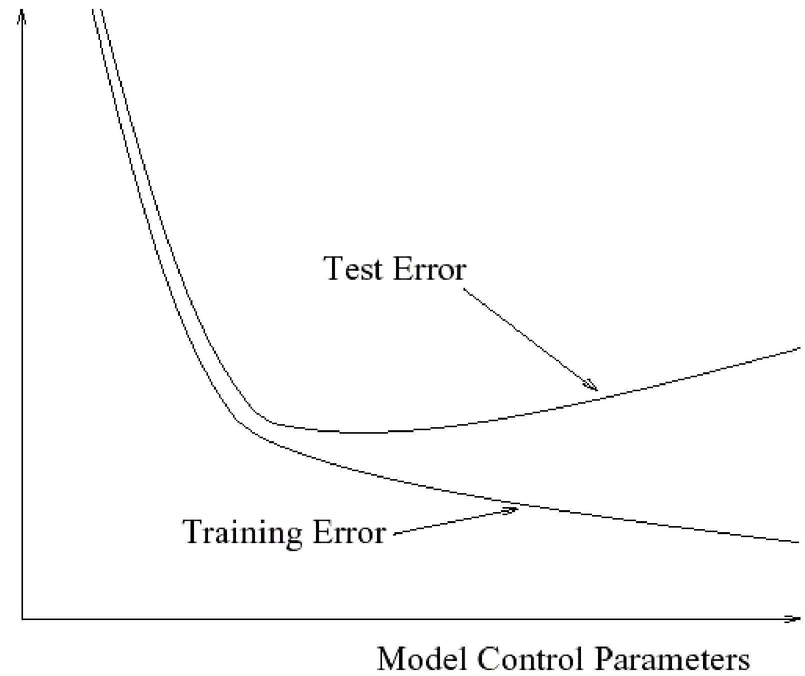
- Evidence maximization (traditional Bayesian Occam’s razor).
- Automatic relevance determination (ARD).

How to choose control parameter?

- Bayesian Occam's razor



$$\mathcal{H} = \alpha$$



Evidence maximization

evidence $\quad p(\mathbf{y}|X,\boldsymbol{\alpha}) = \int p(\mathbf{y}|X,\boldsymbol{\theta})p(\boldsymbol{\theta}|\boldsymbol{\alpha})\,d\boldsymbol{\theta}$

θ:
Weight
space

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Bayesian Occam's Razor

$$P(D\,|\mathcal{H}_i) = \int P(D|\mathbf{w}, \mathcal{H}_i) P(\mathbf{w}|\mathcal{H}_i)\, \mathrm{d}\mathbf{w}$$

$\mathcal{H} = \alpha$

D $P(D|\mathbf{w}, \mathcal{H}_i)$ $P(\mathbf{w}|\mathcal{H}_i)$ $P(D|\mathbf{w}, \mathcal{H}_i) P(\mathbf{w}|\mathcal{H}_i)$

$P(D\,|\mathcal{H}_i) \simeq$ peak height x width

Bayesian Occam's Razor

$$P(D\,|\mathcal{H}_i) = \int P(D|\mathbf{w},\mathcal{H}_i)P(\mathbf{w}|\mathcal{H}_i)\,\mathrm{d}\mathbf{w}$$

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$P(D\,|\mathcal{H}_i) \simeq$ peak height x width

$$P(D\,|\mathcal{H}_i) \simeq P(D\,|\mathbf{w}_{\mathrm{MP}},\mathcal{H}_i) \times P(\mathbf{w}_{\mathrm{MP}}|\mathcal{H}_i)\,\sigma_{w|D}$$

Bayesian Occam's Razor

$$P(D\,|\mathcal{H}_i) = \int P(D|\mathbf{w}, \mathcal{H}_i) P(\mathbf{w}|\mathcal{H}_i)\,\mathrm{d}\mathbf{w}$$

D $P(D|\mathbf{w}, \mathcal{H}_i)$ $P(\mathbf{w}|\mathcal{H}_i)$ $P(D|\mathbf{w}, \mathcal{H}_i) P(\mathbf{w}|\mathcal{H}_i)$

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$$\underbrace{P(D\,|\mathcal{H}_i)}_{\text{Evidence}} \simeq \underbrace{P(D\,|\mathbf{w}_{\mathrm{MP}}, \mathcal{H}_i)}_{\text{Best fit likelihood}} \times \underbrace{P(\mathbf{w}_{\mathrm{MP}}|\mathcal{H}_i)\,\sigma_{w|D}}_{\text{Occam factor}}$$

Bayesian Occam's Razor

$$P(D\,|\mathcal{H}_i) = \int P(D|\mathbf{w}, \mathcal{H}_i) P(\mathbf{w}|\mathcal{H}_i)\,\mathrm{d}\mathbf{w}$$

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$$\underbrace{P(D\,|\mathcal{H}_i)}_{\text{Evidence}} \simeq \underbrace{P(D\,|\mathbf{w}_{\mathrm{MP}}, H_i)}_{\text{Best fit likelihood}} \times \underbrace{P(\mathbf{w}_{\mathrm{MP}}|\mathcal{H}_i)\,\det^{-\frac{1}{2}}(\mathbf{A}/2\pi)}_{\text{Occam factor}}$$

$$\mathbf{A} = -\nabla\nabla \log P(\mathbf{w}|D, \mathcal{H}_i)$$

Multiple levels of inference

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Different architectures:
number of hidden layers, kinds of hidden units, etc.

Automatic Relevance Determination

- Relation to Kruschke's "Backprop with attentional weights on inputs".
- Could specify different classes of features, and learn which class is most relevant for a given classification.
  - Shape and material properties in word learning
  - Internal anatomy versus surface markings in biological classification.
- Applied to weights from hidden units to output units, can effectively infer "size" of bottleneck hidden layer.
- Can apply the same idea to other probabilistic models, e.g., sparseness priors in generative models.

Comparison with cross-validation

- Advantages:
  - Clear theoretical justification.
  - Uses all of the data.
  - Works with many control parameters.
  - Optimize over control parameters in parallel to (or instead of) optimizing over model parameters.
  - Works well in practice (Neal’s ARD triumph)
- Disadvantages
  - Not as intuitive

Comparison with SVMs

- A deep similarity
  - Classification using a model with as many free parameters as possible.
  - Control complexity via sparseness
- Some differences
  - SVM (max margin hyperplane) uses data vectors sparsely, while ARD uses features sparsely.
  - SVM is rotationally invariant; ARD is not.
  - ARD solution may be more interpretable.
  - ARD idea more extendable.

Comparison with SVMs

- What makes a good model?
  - SVM (PAC learning approach): high probability of good generalization
  - Bayesian Occam’s razor: most likely to be the model that generated the data.
- In a non-parametric setting, generalization guarantees seem desirable.
  - PAC-Bayesian theorems (MacAllester, 1998 ff)

- PAC-Bayes error bounds for stochastic model selection (McAllester 1998):
  - Given model class *T*, classify by choosing consistent hypotheses in *T* in proportion to their probability.
  - For any model class *T* and any d > 0, with probability 1- d over the choice of an I.I.D. sample of *m* labeled instances Y_{obs}, the expected error rate of classifying based on is bounded by:

$$\frac{\ln \frac{1}{p(Y_{obs} \mid T)} + \frac{1}{\delta} + 2\ln m + 1}{m}$$

Label evidence:

The better the model class fits the observed labels, the tighter the bound on generalization.

Comparison with SVMs

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- In a non-parametric setting, generalization guarantees seem desirable.
  - PAC-Bayesian theorems (MacAllester, 1998 ff)
  - PAC-Bayes-MDL (Langford and Blum)

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Advantages of the infinite mixture relative to finite model w/ Bayesian Occam's razor

- Allows number of classes to grow as indicated by the data.
- Doesn't require that we commit to a fixed -- or even finite -- number of classes.
- Computationally much simpler than applying Bayesian Occam's razor to finite mixture models of varying sizes, or thorough cross-validation procedures. *Experience this yourself*….
- Use of MCMC avoids problem of local minima in EM approach to learning finite mixture models.
- BUT: Do we lose the "objective" nature of our complexity control?

Unsupervised learning of topic hierarchies

(Blei, Griffiths, Jordan & Tenenbaum, NIPS 2003)

Image removed due to copyright considerations. Please see:
Blei, D., T. L. Griffiths, M. I. Jordan, and J. B. Tenenbaum. "Hierarchical Topic Models and the Nested Chinese Restaurant Process." *Advances in Neural Information Processing Systems* 16 (2004).

A generative model for hierarchies

Nested Chinese Restaurant Process:

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J. ACM abstracts

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