Book cover removed due to copyright restrictions.
Source: Grodzinsky, Alan. Field, Forces, and Flows in Biological Systems. Garland Science, 2011.

Equations 1.29 - 1.31 removed due to copyright restrictions.
Source: Grodzinsky, Alan. Field, Forces and Flows in Biological Systems. Garland Science, 2011. [Preview with Google Books]

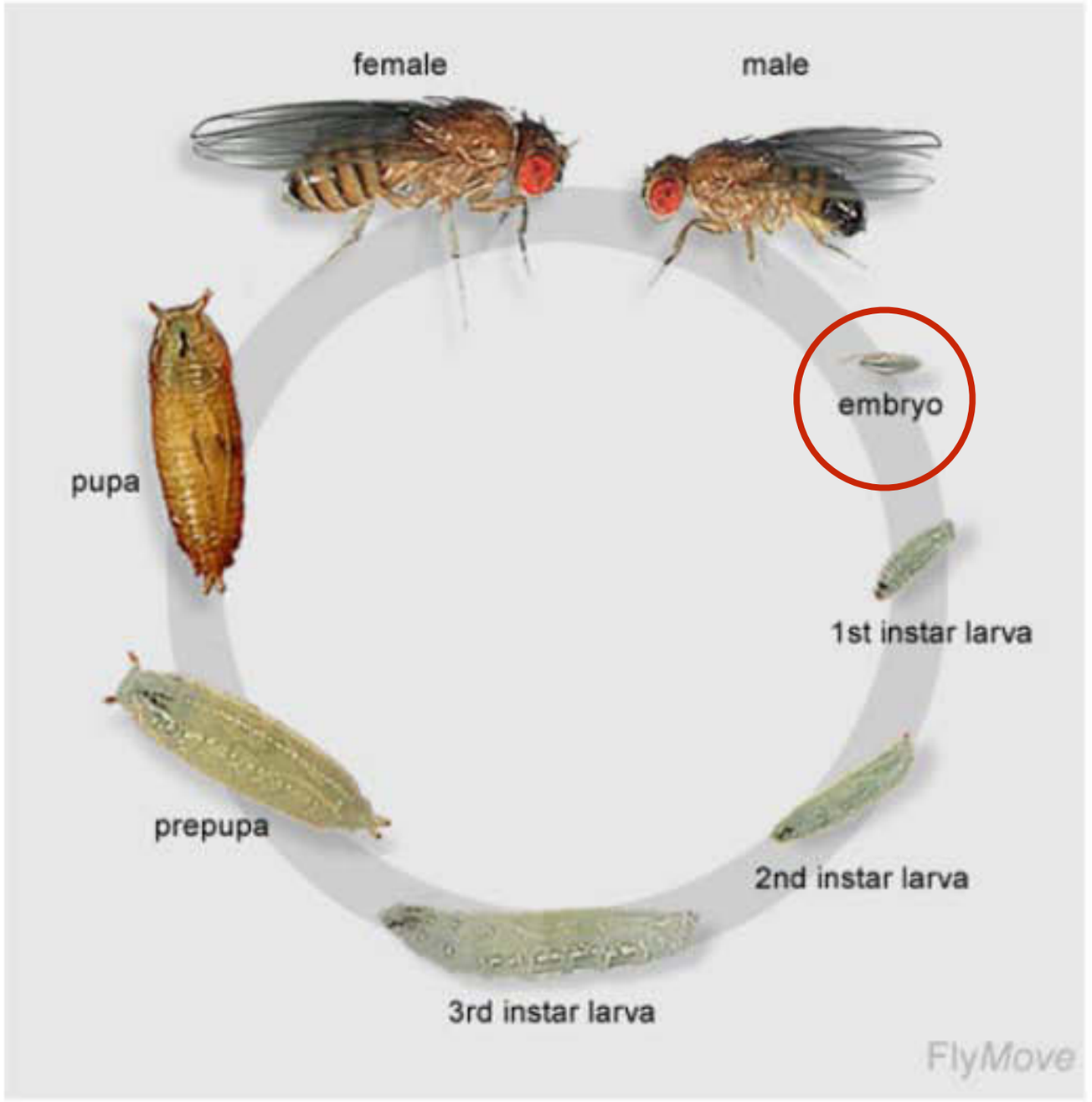
Book cover removed due to copyright restrictions.
Source: Grodzinsky, Alan. Field, Forces, and Flows in Biological Systems. Garland Science, 2011.

Equations 1.35 - 1.39 removed due to copyright restrictions.
Source: Grodzinsky, Alan. Field, Forces and Flows in Biological Systems. Garland Science, 2011. [Preview with Google Books]

Diffusion-reaction in developing early embryos to measure length

Drosophila melanogaster (fruit fly) developmental cycle

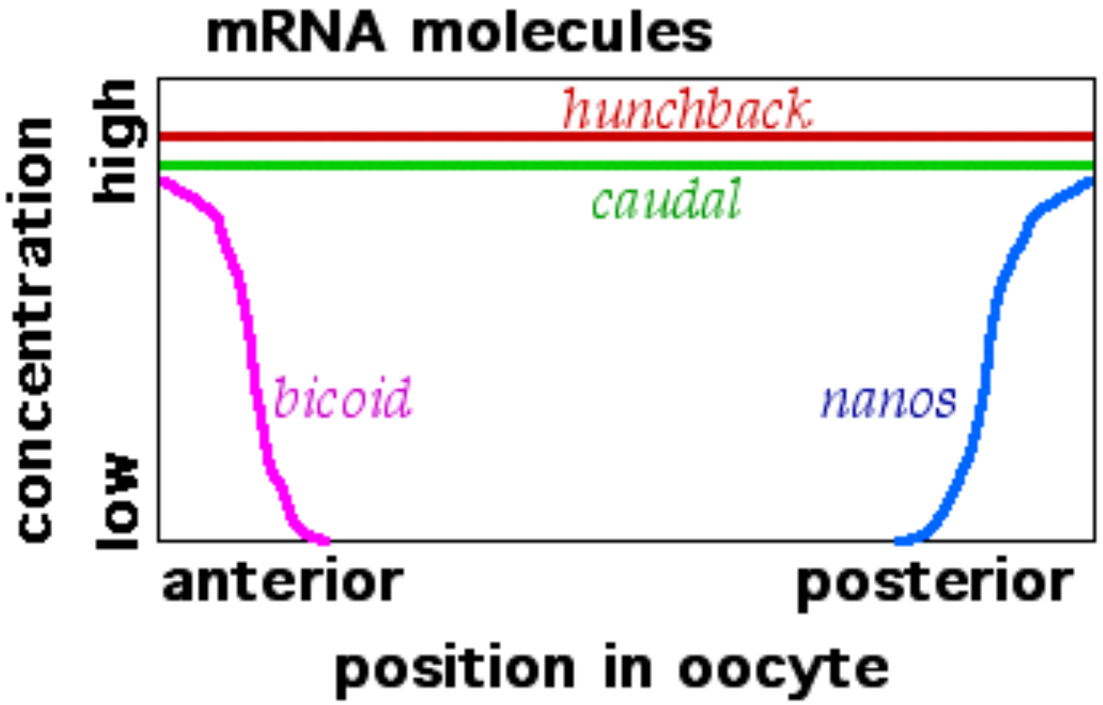
9 days



Screenshot of Edward B. Lewis, Christiane Nüsslein-Volhard, Eric F. Wieschaus, The Nobel Prize in Physiology or Medicine 1995 winners removed due to copyright restrictions.

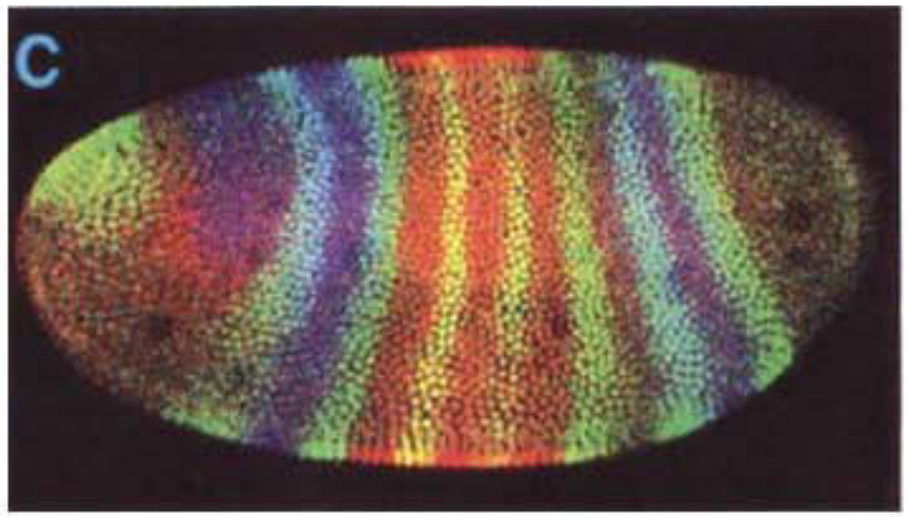
Materials:

- small glass bowl
- wine or juice
- dish soap
- plastic wrap
- bamboo skewer

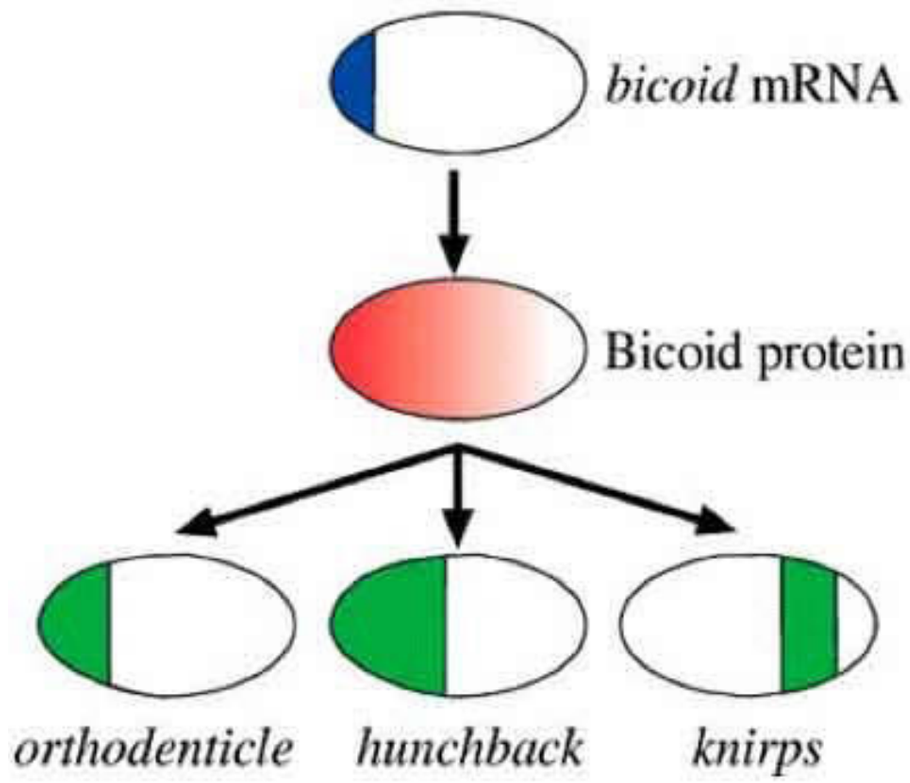


Courtesy of Fred the Oyster; figure in the public domain.

Figure of *Drosophila* early embryo protein gradients removed due to copyright restrictions.



Hughes & Krause, Methods Mol Biol, 2008



Diffusion and scaling during early embryonic pattern formation

Thomas Gregor*†‡§¶, William Bialek*†, Rob R. de Ruyter van Steveninck||, David W. Tank*†‡, and Eric F. Wieschaus‡§¶

¶Howard Hughes Medical Institute, *Lewis-Sigler Institute for Integrative Genomics, †Joseph Henry Laboratories of Physics, and ‡Department of Molecular Biology, Princeton University, Princeton, NJ 08544; and ||Department of Physics, Indiana University, Bloomington, IN 47405

Contributed by Eric F. Wieschaus, November 1, 2005

PNAS | December 20, 2005 | vol. 102 | no. 51 | 18403–18407

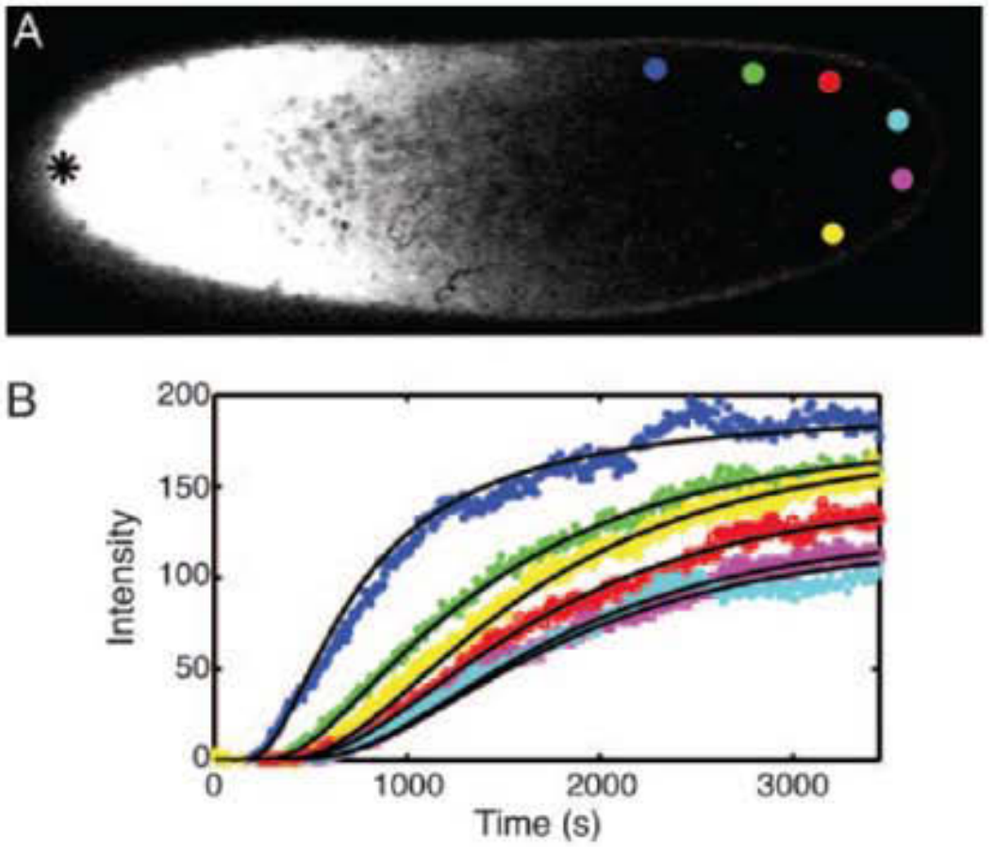


Fig. 1. Diffusion of inert molecules in the *Drosophila* embryo. (*A*) Two-photon image of a wild-type *D. melanogaster* embryo 8 min after injection of fluorescently labeled dextran molecules at the mid-plane of the embryo. The tip of the glass micropipette used for the injection is located at the anterior pole (black asterisk on the left side of the image). Colored discs show areas where fluorescence intensity was analyzed. (*B*) Changes in the fluorescence intensity with time for the six color-corresponding discs in *A*, extracted from a time series of images taken with a frame rate of 8 s. Solid lines represent the time courses computed from the best fit of a numerical 3D diffusion model. Note that 18 curves (6 per focal plane) are fit by the solutions of the same diffusion equation, with only a single free parameter, the diffusion constant *D*. (*C*) Diffusion coefficients of dextran molecules of different hydrodynamic radii (red dots). The solid line represents diffusion coefficients expected from the modified Stokes–Einstein relation (10), $D = k_B T/(6\pi\eta R) + b$, with a viscosity $\eta = 4.1 \pm 0.4$ cP and $b = 6.2 \pm 1.0$ $\mu m^2/s$; dashed line is at the value of *b*.

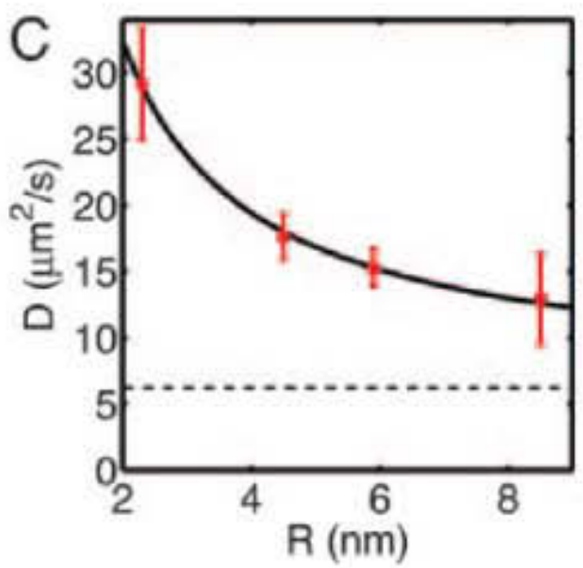
In the simplest model (14), Bcd protein diffuses through the embryo and decays with a lifetime τ. The spatiotemporal dynamics of the concentration profile are determined by

$$\frac{\partial c(\vec{r}, t)}{\partial t} = D\nabla^2 c(\vec{r}, t) - \frac{1}{\tau} c(\vec{r}, t), \qquad [1]$$

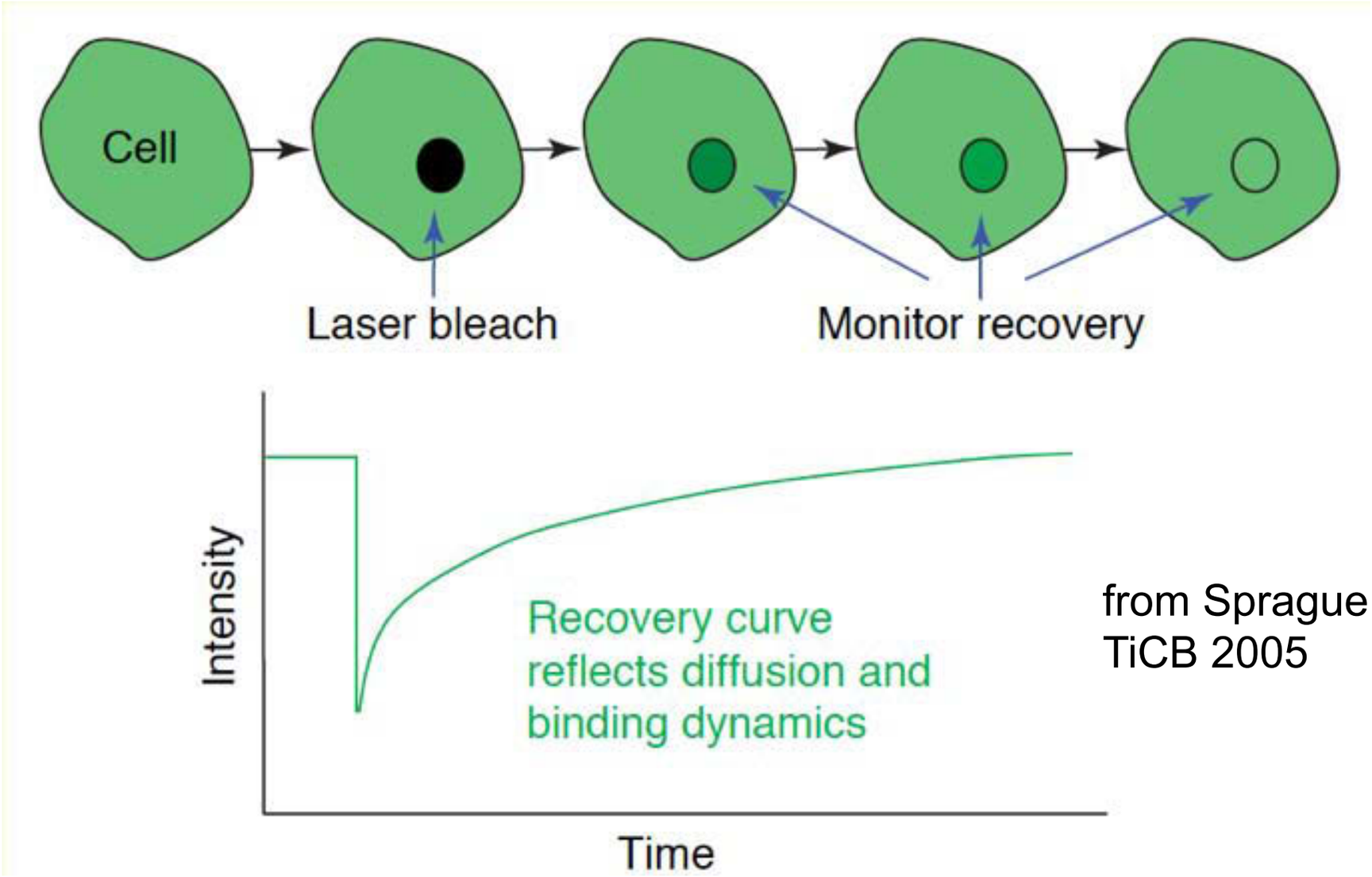
where D is the diffusion constant. The steady state therefore is determined by

$$D\nabla^2 c_{ss}(\vec{r}) = \frac{1}{\tau} c_{ss}(\vec{r}). \qquad [2]$$

If there is a source (translation of maternal RNA) at $x = 0$ and no variations along the dorsal–ventral direction, then the solution, projected along the anteroposterior axis, is $c_{ss}(x) = A \exp(-x/\lambda)$, where $\lambda = \sqrt{D\tau}$, and the constant A is set to match the diffusive flux to the translation rate at $x = 0$; this solution is valid if $L/\lambda \gg 1$, as observed. Identifying staining intensity as proportional to concentration, and allowing for background fluorescence B, the raw data of Bcd immunofluorescence intensities were fitted by $I = A \exp(-x/\lambda) + B$ for abscissae $x \in 15$–85% egg length. A nonlinear Nelder–Mead fitting procedure was used to estimate the parameters A, B, and λ for each embryo (13).



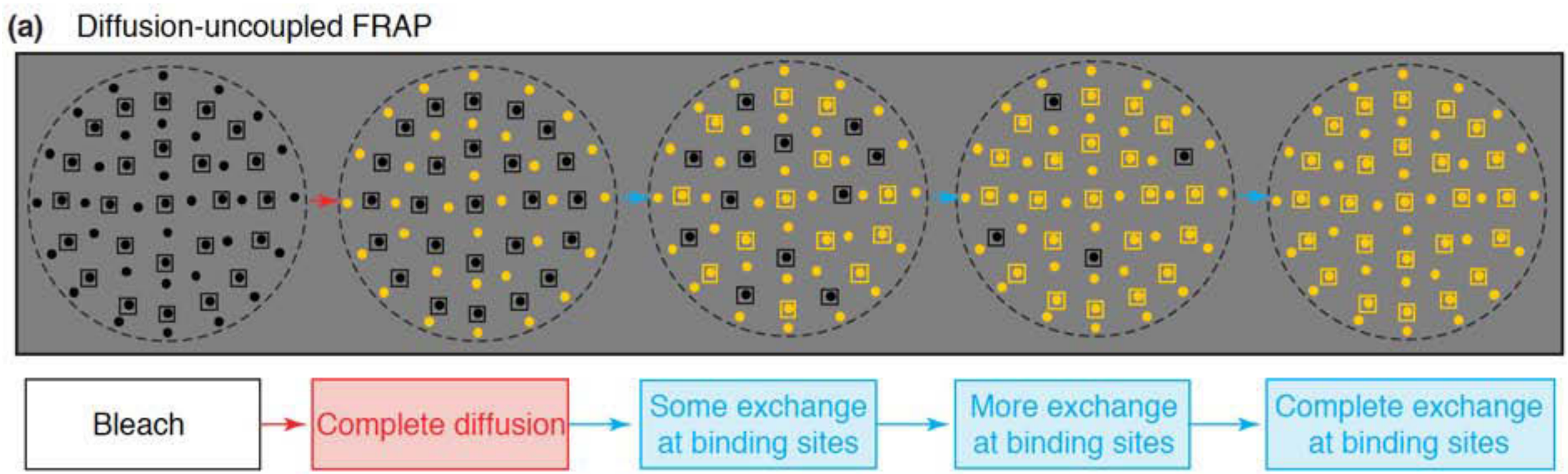
Fluorescence Recovery After Photobleaching (FRAP) to measure diffusion and reaction in situ



from Sprague & McNally TiCB 2005

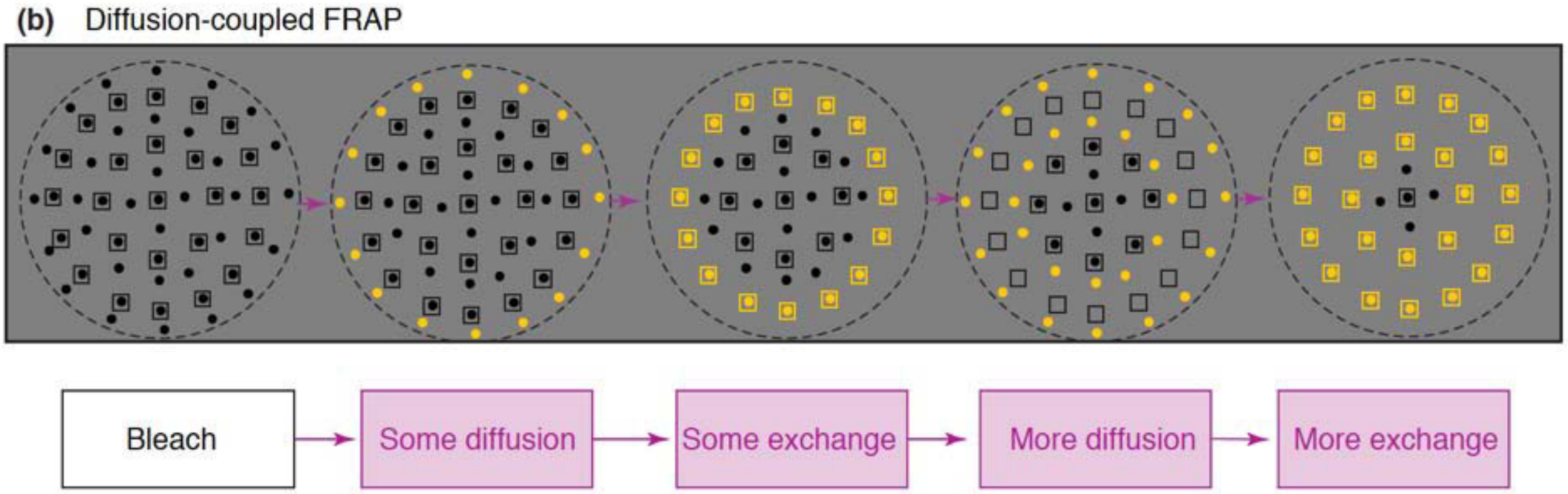
diffusion-dominated recovery:

$$\frac{\partial f}{\partial t} = D_f \nabla^2 f \qquad frap(t) = f(t) = e^{-\frac{\tau_D}{2t}}\left[I_0\left(\frac{\tau_D}{2t}\right) + I_1\left(\frac{\tau_D}{2t}\right)\right]$$



reaction-dominated (off-rate) recovery:

$$frap(t) = 1 - C_{eq} e^{-k_{off} t}$$



Courtesy of Elsevier, Inc., http://www.sciencedirect.com. Used with permission.
Source: Sprague, Brian L. and James G. McNally. "FRAP analysis of binding: proper and fitting." Trends in Cell Biology 15, no. 2 (2005): 84-91.

effective diffusion recovery:

$$\frac{\partial f}{\partial t} = D_{\mathrm{f}} \nabla^2 f - k_{\mathrm{on}}^{*} f + k_{\mathrm{off}} c \qquad D_{\mathrm{eff}} = \frac{D_{\mathrm{f}}}{\left(1 + \left(k_{\mathrm{on}}^{*} / k_{\mathrm{off}}\right)\right)}$$

20.430J / 2.795J / 6.561J / 10.539J Fields, Forces, and Flows in Biological Systems
Fall 2015