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Fields, Forces, and Flows in Biological Systems
Fall 2015

Problem Set # 6 (Mechanical Subsystem) Issued: Friday 11/6/15
Due: Friday, 5pm – 11/13/15

Reading Assignment: Textbook, Ch5, sections 5.1-5.4 and 5.6

Turn in Problem set in dropbox to the right of elevators on the 2nd floor of Building 16. Please turn in Problems 1 & 2 into Box 1 and 3 into Box 2.

Problem 1: The Couette Viscometer (Prob 5.4 in FFF)

Do Problem 5.4 in FFF.

Problem 2: Cell Adhesion Assay

One method used to test the adhesion strength of a cell population is a radial flow assay (see figure). Cell culture medium enters the system at the center of a disk-shaped chamber in which cells or functionalized microbeads are adherent to the bottom surface[1]. As the flow moves radially outward from the center, the flow speed, and therefore the shear force exerted on each cell, decreases. The cells closest to the center experience the highest levels of force and are torn loose from the substrate. Beyond some critical radial distance r_c, the adhesion strength is sufficient so that the cells remain attached. Here we consider both the flow of medium and the effect it has on the individual cells. (Note that the cells in the drawing are much larger than actual size). For parts (a)-(d) below, you may assume viscous-dominated flow.

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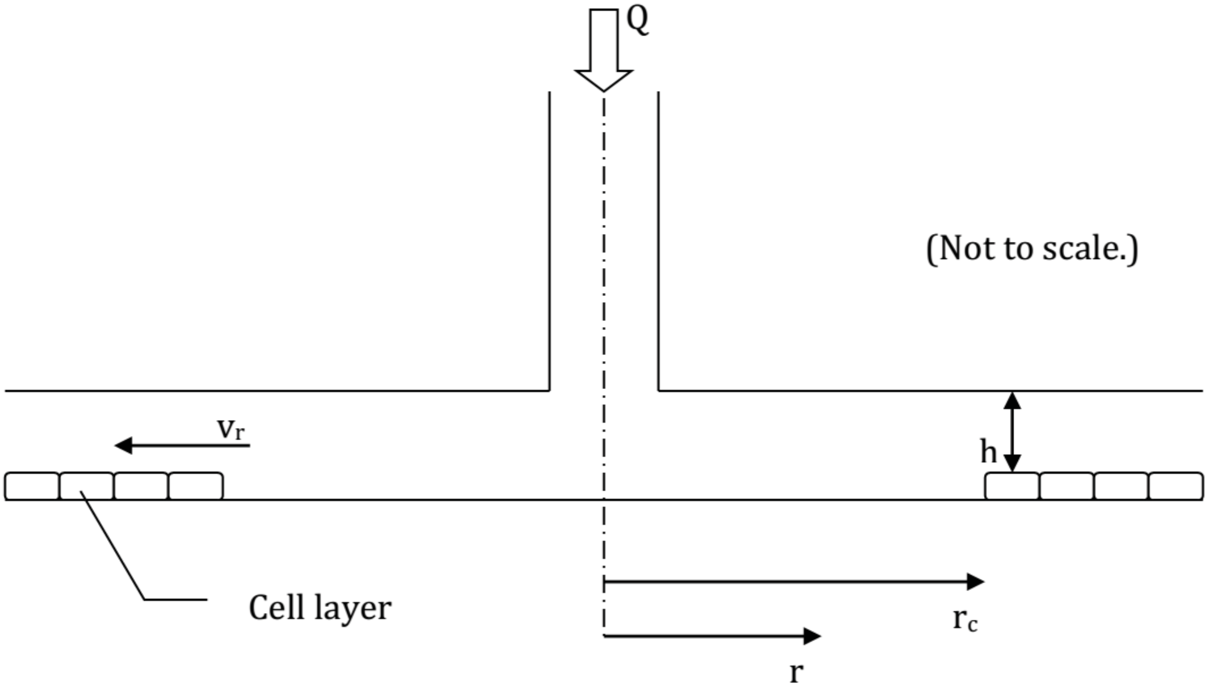
[1] Cozens-Roberts, C., Lauffenburger, D. A., & Quinn, J. A. (1990). Receptor-mediated cell attachment and detachment kinetics. *Biophys J*, *58*(October), 841–856.



a. Obtain an expression for the mean speed of the fluid as a function of the volume flow rate entering the chamber Q, distance measured from the axis r, and the height of the flow region h.
b. Calculate and sketch the velocity profile inside the gap $v_r(z)$ where z is measured vertically upward from the cell surface (at a position $r >> h$).
c. Obtain an expression for the shear stress acting on the cell that shows the dependence of shear stress on the viscosity of the medium μ, the volume flow rate Q, radial distance r, and the channel height h.
d. Using a force balance on a single cell, obtain an approximate (scaling) expression for the maximum adhesion strength f (force per unit area), again in terms of μ, Q, h, and r_c, as well as the surface area of the cell exposed to shear stress A.
e. Give a quantitative criterion in terms of given quantities that would allow you to evaluate whether or not the assumption of viscous flow is justified.

Problem 3: Flow between Parallel Plates

Consider an initially quiescent layer of an incompressible Newtonian fluid of thickness H that is sandwiched between two plates. The plates are wide compared to the thickness of the fluid. At time $t = 0^+$ we apply a constant velocity in the z-direction to the top plate while the bottom plate remains fixed.

a) Write down the general form of the Navier–Stokes equation for the transport of momentum in the z direction. Use scaling and order of magnitude estimates to identify terms that may be neglected. *Hint: The simplified PDE should resemble the diffusion problem we solved for an earlier homework.*

b) Write down the relevant boundary conditions for your now simplified momentum balance.

c) The steady-state velocity profile ($t \to \infty$) between the plates is given by

$$v_z(y) = V\frac{y}{H}.$$

Use your intuition to sketch how the z velocity profile $v_z(y,t)$ evolves over time.

d) Without solving the governing differential equation in part (a), use scaling analysis to estimate the characteristic time required to reach steady-state.