20.180:FirstOrderDecay

First Order Decay (of anything)

Givens:

- A pile of some thing, **X**.
- A first-order chemical process by which **X** is destroyed (or transformed into something else).

Tasks:

- Compute amount of **X** remaining as a function of time.
- Compute amount of time until there is half as much **X** as there is now (this length of time is called the "half-life" of **X** or $t_{1/2}$).

Approach:

- Write differential equation for change in **X** over time.

$$\frac{dX}{dt} = -k_d * [X]$$

- Solve equation for [X] as a function of time, t.

$$\frac{dX}{[X]} = -k_d * dt$$

- Integrating from $X_{(t=0)}$ to $X_{(t=t)}$

$$[lnX]_{X_{(t=0)}}^{X_{(t=t)}} = [-k_d * t]_{(t=0)}^{(t=t)}$$

- Solving at the limits produces...

$$ln\left(\frac{X_{(t=t)}}{X_{(t=0)}}\right) = -k_d * t$$

- Which provides a general analytical solution for X as a function of time, t

$$X_{t=t} = X_{t=0} * e^{-k_d * t}$$

- Now, note that at $t_{1/2}$, $X_{(t=t)} / X_{(t=0)} = 0.5$ by definition. So we can substitute and get...

$$ln(0.5) = - k_d * t_{1/2}$$

- Which is the same as...

$$0.69 = k_d * t_{1/2}$$