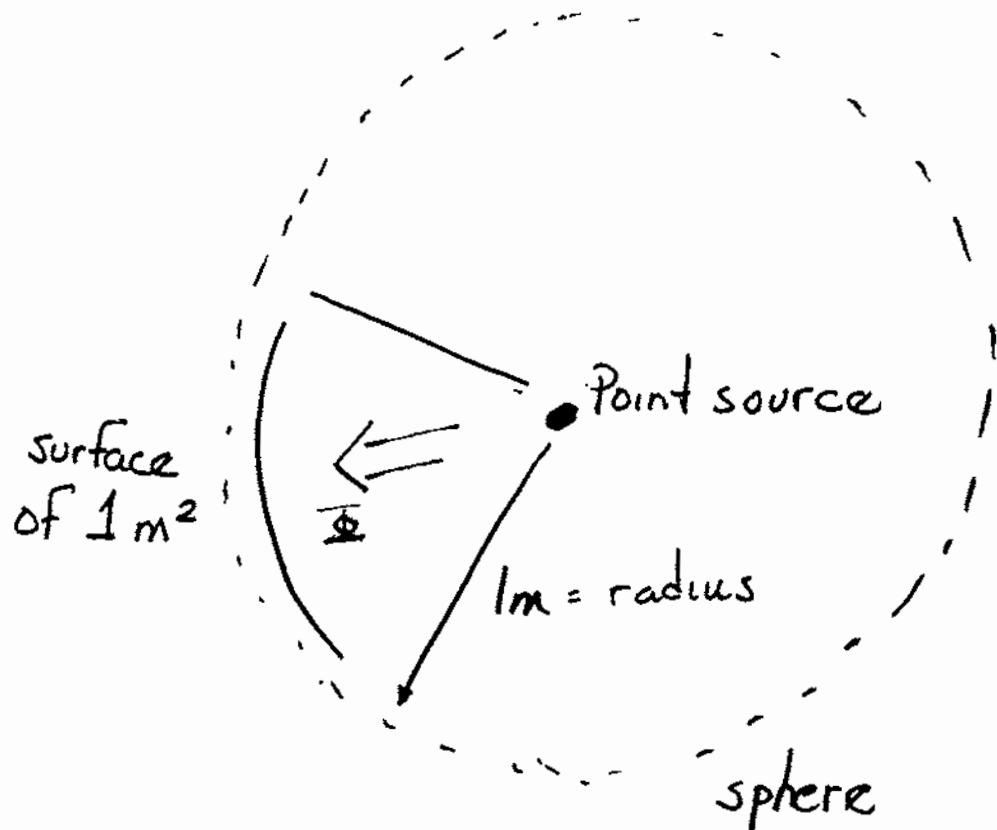


# Lighting

1. Definitions
2. Inverse Square Law & Applications
3. Indoor (electric) Lighting Design
4. Daylighting Fundamentals
5. Glare

## Definitions



I. luminous intensity. Consider a point source of 1 candela (cd)

$\Phi$  - luminous flux. A point source of 1 cd emits 1 lumen (lm) of luminous flux over 1 m<sup>2</sup> at a distance of 1 m

How many lumens in all?  $4\pi$ , because the surface area of a sphere is  $4\pi r^2$

E - illuminance      Luminous flux per unit  
surface area       $\text{lm}/\text{m}^2$

This is incident luminous power

$$1 \text{ lm}/\text{m}^2 = 1 \text{ lux}$$

$$1 \text{ lm}/\text{ft}^2 = 1 \text{ footcandle (fc)}$$

There are two measures of light leaving a surface: the total amount and the amount in a given direction

M. luminous exitance      Total luminous flux  
per unit surface area      leaving the surface  
 $\text{lm}/\text{m}^2$  (but is NOT called lux)

Consider the sphere to be translucent, with a transmissivity  $\hat{\tau}$  of 0.8. Then the luminous exitance is  $0.8 \text{ lm}/\text{m}^2$ .

L · luminance      Luminous Flux leaving  
a surface, as seen from one viewpoint

Consider the sphere, made of translucent material  
and glowing. It appears as a disk of surface  
area  $\pi r^2 = \pi 1^2 = \pi \text{ m}^2$



Its apparent luminous intensity is  $0.8 \text{ cd}$   
( $\hat{I} \cdot I$ )

$$L = \frac{0.8 \text{ cd}}{\pi \text{ m}^2} = \frac{0.8}{\pi} \frac{\text{cd}}{\text{m}^2}$$

Note!! a) Units are  $\frac{\text{cd}}{\text{m}^2}$       b)  $M = \pi L$

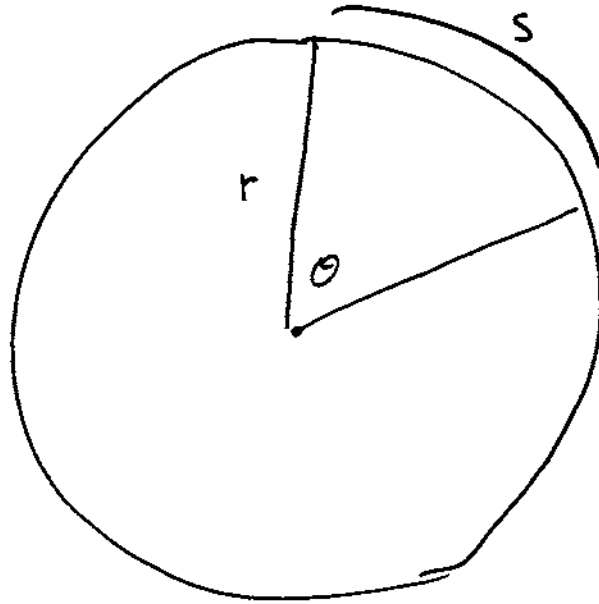
Not to worry! The candela and the lumen  
are intimately linked, by definition

A detour: what are the fundamental SI units? There are seven

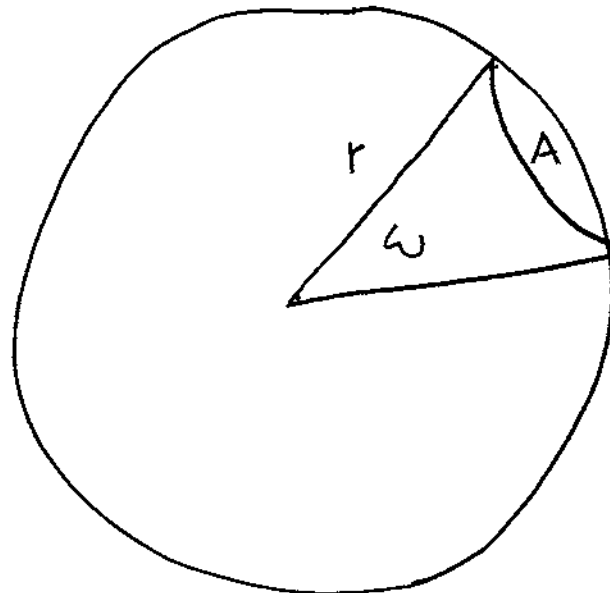
time	sec
length	meter
mass	kilogram
temperature	Kelvin
electric current	Ampere
amount of substance	moles
luminous intensity	candela

There are also two supplemental units:

angle	radian ( $2\pi$ radians = $360^\circ$ ) circumferential length = radius
solid angle	steradian ( $4\pi$ sr = <del>surf</del> total solid angle for sphere) surface area = radius <sup>2</sup>



When arc length  $s = r$ ,  $\theta = 1$  radian  
 $2\pi$  radians per circle



When  $A = r^2$ ,  $\omega$  (solid angle) = 1 steradian  
 $4\pi$  sr per sphere  
 Sa.

On the basis of the careful definitions of solid angle and solid angle, we can revisit the relation between candelas and lumens. A point source of 1 cd produces  $4\pi$  lm over  $4\pi$  sr. It is therefore appropriate to consider the candela as being dimensionally equivalent to  $\text{lm}/\text{sr}$ .

With this in mind, we can construct the following table

[Cohen p 27, Table 2-1, w/ units corrected for illuminance and luminosity or luminous exitance] Note that the official term for radiosity is radiant exitance.

Physics	Radiometry	Radiometric Units
	Radiant energy	joules [ $J = \text{kg m}^2/\text{s}^2$ ]
Flux	Radiant power	watts [ $W = \text{joules}/\text{s}$ ]
Angular flux density	Radiance	$[\text{W}/\text{m}^2 \text{sr}]$
Flux density	Irradiance	$[\text{W}/\text{m}^2]$
Flux density	Radiosity	$[\text{W}/\text{m}^2]$
	Radiant intensity	$[\text{W}/\text{sr}]$

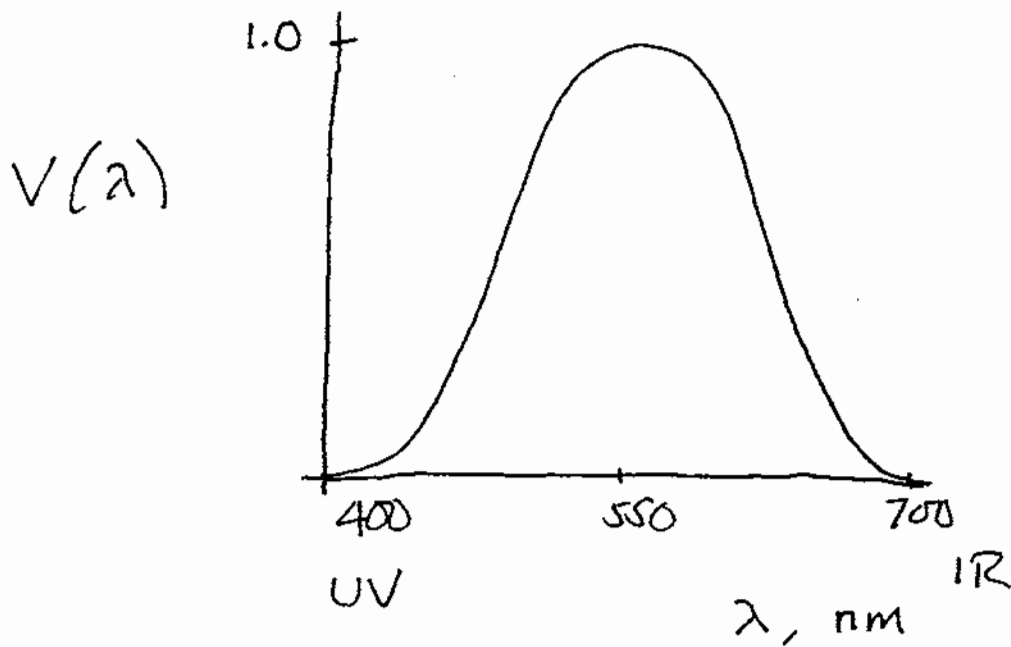
  

Physics	Photometry	Photometric Units
	Luminous energy	talbot
Flux	Luminous power	lumens [ $\text{talbots}/\text{second}$ ]
Angular flux density	Luminance	Nit [ $\text{lumens}/\text{m}^2 \text{sr}$ ]
Flux density	Illuminance	Lux [ $\text{lumens}/\text{m}^2 \text{sr}$ ]
Flux density	Luminosity	Lux [ $\text{lumens}/\text{m}^2 \text{sr}$ ]
	Luminous intensity	Candela [ $\text{lumens}/\text{sr}$ ]

## Examples

- "His face was radiant...."
- "The moon was luminous...."
- "The materials sample was irradiated in the reactor...."
- "The page of the book was illuminated...."

The difference comes from the response of the human eye

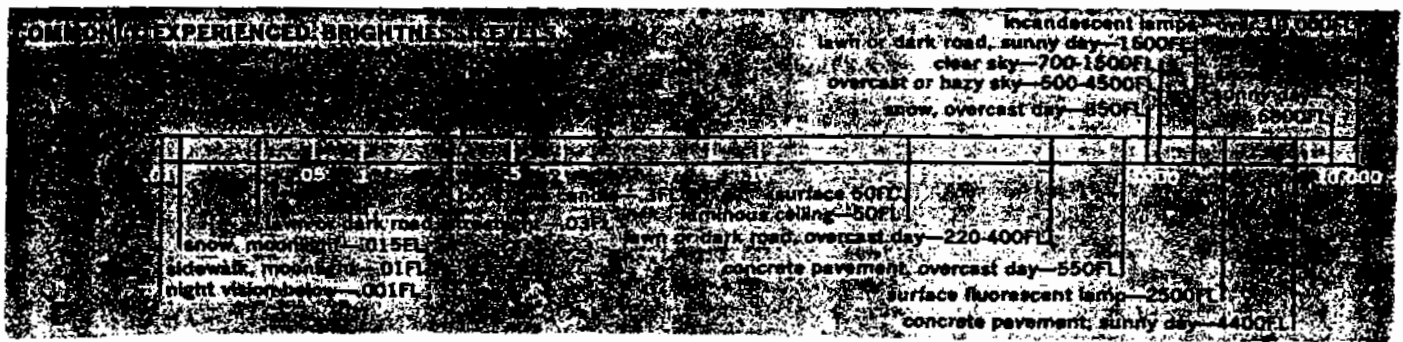


Our skin is sensitive to the entire spectrum of the sun. Our eyes are sensitive only over a limited part of the spectrum, and not uniformly.



# Luminance and the perception of brightness

From William Lam's Perception and Lighting as Formgivers for Architecture



Note:

1. Really talking luminance, not perceived brightness
2. FL?? Foot-lambert, the English unit for luminance.  
But it is not  $cd/m^2$  (!) Rather, it is  $\pi \cdot cd/m^2$ .

$$1 \text{ cd}/m^2 = 10.76 \text{ FL} \quad (10.76 \text{ FL} = 1 \text{ m}^2)$$

$$1 \text{ FL} = \frac{10.76}{\pi} \text{ cd}/m^2$$

Key point: eye functions over an enormous range,  $\sim 10^7$

Measurements along Charles River, 9/10/00

Moon  $\sim 1500 \text{ cd}/m^2$

Street light  $\sim 5000 \text{ cd}/m^2$

An aside on the moon

It has a reflectance of  $\sim 7\%$ .

Solar irradiance on the moon is  $\sim 1350 \text{ W/m}^2$

Luminous efficacy of the sun is  $\sim 90 \text{ lm/W}$

$1350 \text{ W/m}^2 \times 90 \text{ lm/W} = 121,500 \text{ lm/m}^2$  illuminance

Reflected luminosity is 7% of illuminance, or  $\sim 8400 \text{ lm/m}^2$

I measured  $\sim 1500 \text{ cd/m}^2$ . If we assume that luminosity =  $\pi$  · luminance, we get  $\sim 5,000 \text{ lm/m}^2$ , as measured along Charles. Difference is due to earth's atmosphere.

whatnow

## Perceived brightness - Lam's view

- Doubling luminance (or illuminance) results in just noticeable difference
- Perception of brightness depends on
  - context
  - expectation
  - adaptation

"Bright moonlit night"

"Dark overcast day"

A brilliantly illuminated crystal chandelier is a pleasure.

A room may appear gloomy and dimly lit during the day but bright and cheerful at night, when there are no bright windows or direct sunlight to which the eye will adapt, reducing the apparent brightness of other surfaces.

Lam asserts that it is wasteful to run interior lighting at full intensity at night, when artificial sources no longer compete with daylight illumination.

Who cares? The less-than-cheery state of lighting standards.

set the scene w/ thermal comfort - 6 variables, PMV/PPD

## Illuminance

Almost all lighting design today is done on the basis of illuminance. Why? It's easy.

Lighting designers once tried to be very scientific and relate light levels to specific aspects of the visual task:

size  \* size of interest

contrast

demo w/ booklet of examples

time

luminance

Too sensitive/complicated. So now we have illuminance categories and recommended categories for many, many tasks.

What about perceived brightness of surfaces?

Limits on luminance ratios (contrast ratios)

3:1 task and near surroundings  
10:1 task and far (but within room)  
surroundings

Problems with this approach:

- Lam's chandelier
- Lam's argument for lower illuminance at night
- Other aspects of perception (see Lam)
- Stultifying  
wall-to-wall uniformity  
would not produce a Menil

Alternatives / supplements to illuminance + contrast

relative visual performance (RVP) - ability to perform a task

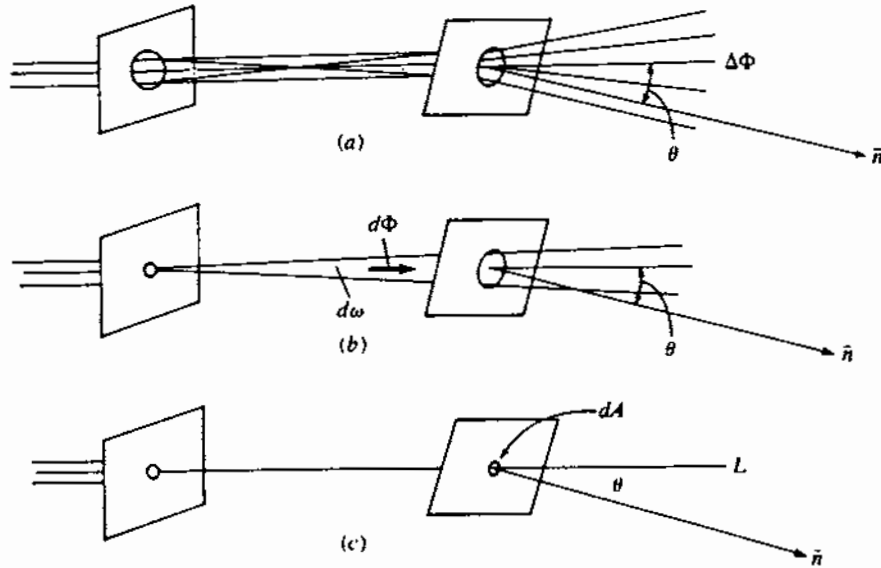
visual comfort probability (VCP) - glare

RVP.  $\frac{\text{correct answers}}{\text{time}}$  as  $F(\text{contrast, luminance})$

These can be computed in detailed simulations of visual environments,  
much like PMV/PPD

The lighting terms, revisited - because all sources are not points  
 Murdoch, chapter 2, is very similar to Cohen, chapter 2

Murdoch Figure 2.3



Define  $L$  of the rays by

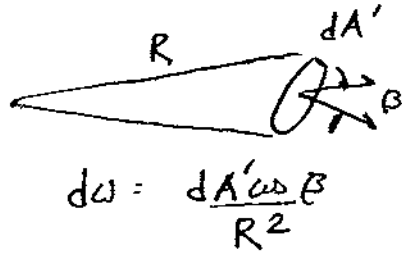
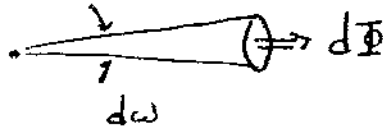
$$L = \frac{d^2\Phi}{d\omega dA \cos\theta}$$

Luminance at a point and in a direction is the differential luminous flux at a differential element of the surface surrounding the point and propagated in a differential cone containing the given direction, per differential solid angle and differential projected area

differential luminous flux may be arriving, leaving or passing through the differential surface element. Luminance is thus an inherent characteristic of a ray ( $\therefore \nabla L!$ )

# Differential definitions

$$I = \frac{d\Phi}{d\omega}$$

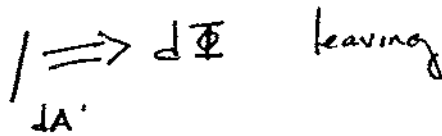


$dA'$  receiver area

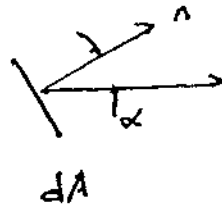
$$E = \frac{d\Phi}{dA'}$$



$$M = \frac{d\Phi}{dA'}$$



$$L = \frac{dI}{dA \cos \alpha}$$



$dA$  source area

$$\frac{d^2\Phi}{d\omega dA \cos \alpha}$$

$$\frac{dE}{d\omega \cos \alpha}$$



From these definitions we can derive two forms of the inverse-square law and relate  $L$  to  $M$  for a plane source and  $L$  to  $E$  for the sky

$$d\Phi = I d\omega = \frac{I dA' \cos \beta}{R^2}$$

$$E = \frac{d\Phi}{dA'} = \frac{I \cos \beta}{R^2}$$

This is the ISL;  $E$  decreases as  $\frac{1}{R^2}$  from a point source, as can be deduced from the sphere (a spatially uniform point source)

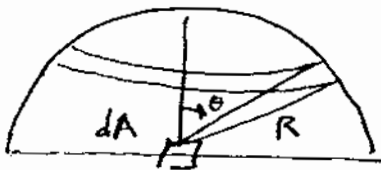
If  $dA$  is part of a finite area source

$$dE = \frac{dI \cos \beta}{R^2}$$

$$dI = L \cos \alpha dA$$

$$dE = \frac{L \cos \alpha \cos \beta dA}{R^2} \quad \text{ISL for finite area source}$$

Now consider the total flux from  $dA$



The illuminance on the ring is  $dE_r = \frac{L \cos \theta dA}{R^2}$

Multiply by area of ring to obtain luminous flux

$$\begin{aligned} d\Phi_r &= \frac{L \cos \theta dA}{R^2} 2\pi R^2 \sin \theta d\theta \\ &= 2\pi L dA \sin \theta \cos \theta d\theta \end{aligned}$$

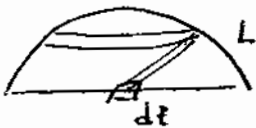
For a perfectly diffusing surface  $L$  is independent of  $\theta$  and the total luminous flux from  $dA$  is

$$\begin{aligned} d\Phi &= 2\pi L dA \int_0^{\pi/2} \sin \theta \cos \theta d\theta \\ &= \pi L dA \end{aligned}$$

The luminous exitance (luminosity) is

$$M = \frac{d\Phi}{dA} = \pi L$$

Next, consider the sky as a source



$$dE = \frac{L(\theta) \cos \theta}{R^2} 2\pi R^2 \sin \theta d\theta$$

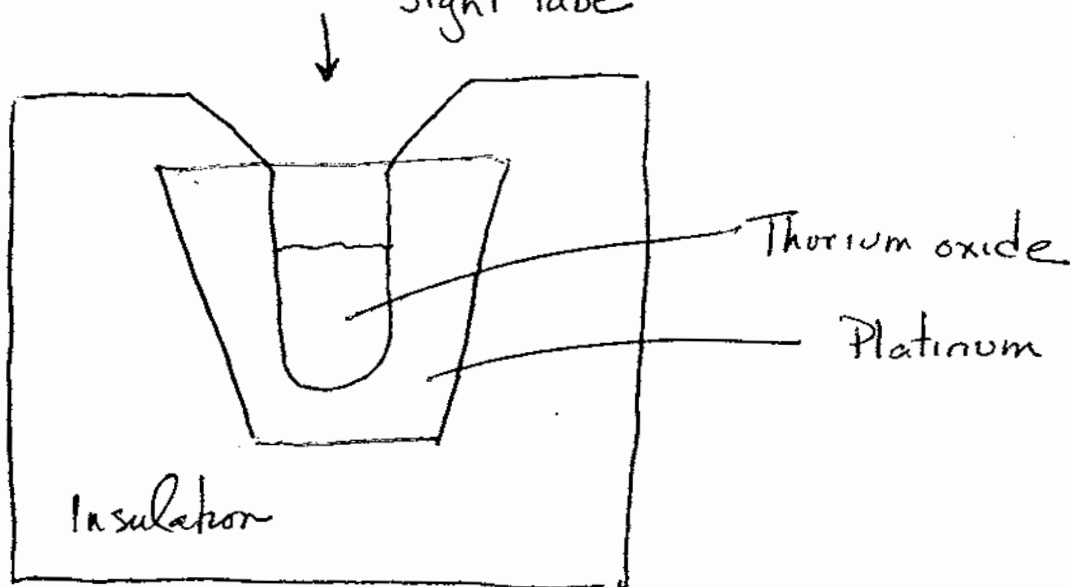
$$\begin{aligned} E_{\theta_1 \rightarrow \theta_2} &= 2\pi L_{\theta_1 \rightarrow \theta_2} \int_{\theta_1}^{\theta_2} \sin \theta \cos \theta d\theta \\ &= \pi L_{\theta_1 \rightarrow \theta_2} (\sin^2 \theta_2 - \sin^2 \theta_1) \end{aligned}$$

IF  $L(\theta)$  is constant over the hemisphere,

$$E_{0 \rightarrow 90} = \pi L$$

1948 definition of the candela  
Sight tube

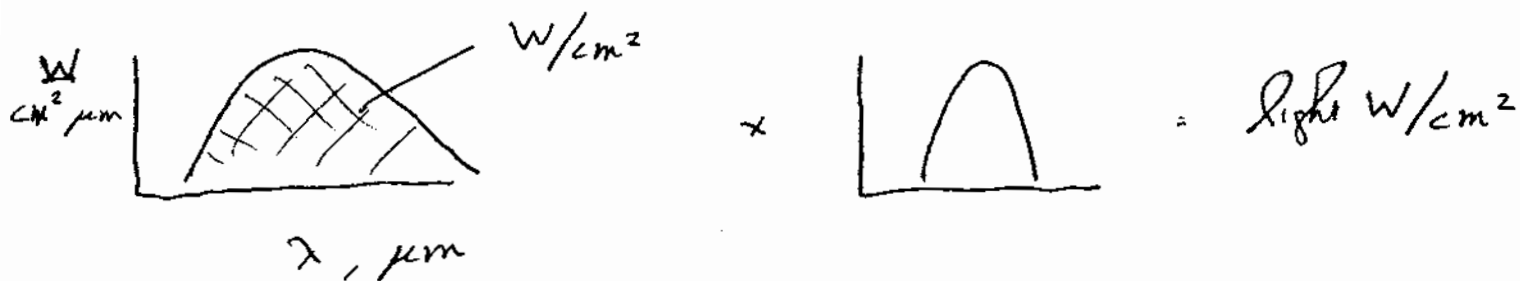
4/23/03



Platinum at point of solidification: 2042 K

Luminance of thorium oxide = 60 cd/cm<sup>2</sup> by definition

Perfectly diffusing, so  $M = 60\pi$  lm/cm<sup>2</sup>



$$0.27598 \text{ lW/cm}^2$$

60π lm/cm<sup>2</sup> must be equivalent to 0.27598 lW/cm<sup>2</sup>  
 Each light watt must generate  $\frac{60\pi}{0.27598} = 683$  lm

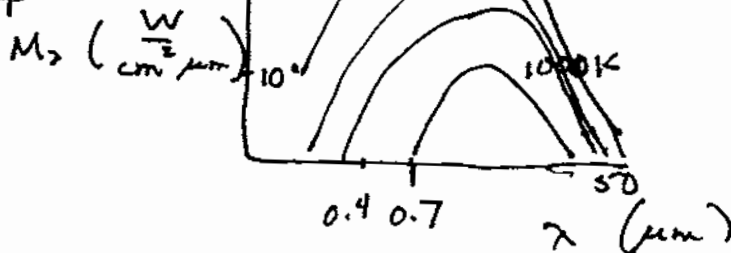
If the radiant power could be concentrated at 555 nm, each Watt would yield 1.0 light Watt and thus 683 lm  
 Therefore, the theoretical maximum luminous efficacy is 683 lm/W

4/13/04

Fuller explanation of relation between Lumen and Watt

1. Blackbody radiation

2. Shape



Murdoch 3.3

$$M_{\lambda} = \frac{2\pi^5 hc^2 \lambda^{-5}}{e^{hc/\lambda kT} - 1}$$

b. Peak

$$\lambda_{\max} T = 2878 \mu\text{m} \cdot \text{K}$$

$$M_{\lambda_{\max}} = 1.28 \times 10^{-15} T^5 \text{ W/cm}^2 \cdot \mu\text{m}$$

c. Power (area under curve)

$$M = \sigma T^4 \quad \sigma \text{ is Stefan-Boltzmann constant}$$

$$= 5.6032 \times 10^{-12} \text{ W/cm}^2 \text{ K}^4$$

2. Definition of Lumen

$$L = 60 \text{ cd/cm}^2 \text{ blackbody radiator at } 2042 \text{ K}$$

$$M = \pi L = 60\pi \text{ lm/cm}^2$$

3. Luminous efficacy of a 2042 K blackbody

$$M = 97.42 \text{ W/cm}^2 \quad (M \text{ for radiometric and photometric units})$$

$$\text{luminosity} = M_p (?) = 60\pi = 188.50 \text{ lm/cm}^2$$

$$\text{Efficacy} = \frac{188.50}{97.42} \approx 0(2) \text{ lm/W ? quite - but reasonable}$$

W filament melts at 3650 K, but most filaments operate at 2800-3000K

Blackbody at 3000K - higher power, more in visible

$$\text{Sun is blackbody } 92 \text{ lm/W} \quad T_{\text{surface}} = 5700 \text{ K} \quad M = 6.25 \text{ kW/cm}^2$$

A. Relating lm to W in a way that determines maximum efficacy

Need concept of light Watt



Do this at 2042 K and get not 97.4 W/cm but 0.27598 kW/cm<sup>2</sup>

$$\therefore \text{each kW must generate } \frac{60\pi}{0.27598} = 683 \text{ lm.}$$

If a source could be concentrated at 555 nm (0.555  $\mu\text{m}$ ), it would produce 683 lm/W.

This relationship is now used to define the candela:

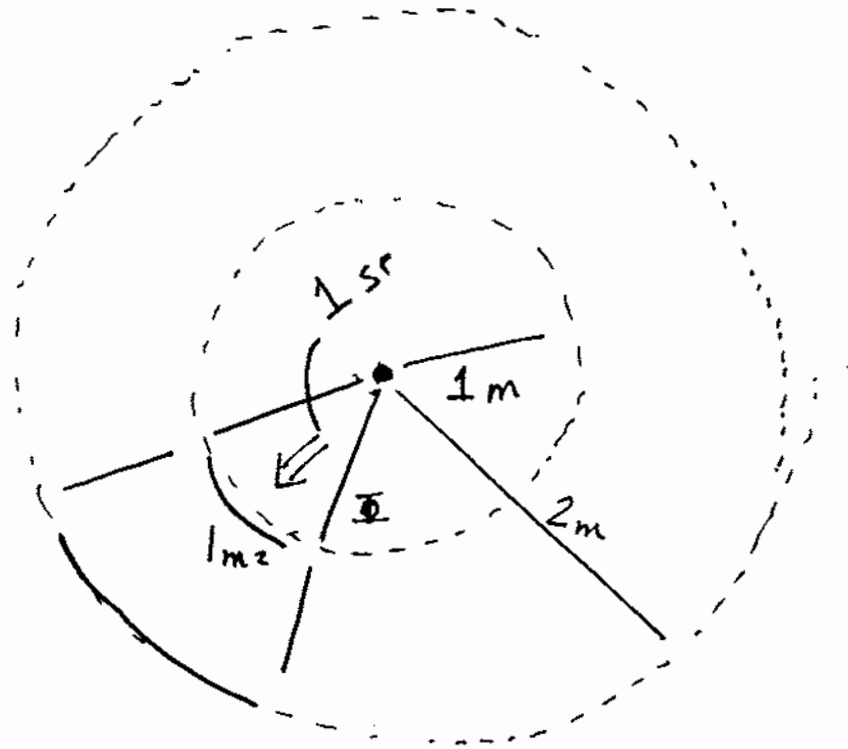
"The candela is the luminous intensity in a given direction of a source which is emitting monochromatic radiation of frequency  $540 \times 10^{12}$  Hz (555 nm) and whose radiant intensity in that direction is  $\frac{1}{683}$  W/sr."

For comparison of luminous efficacies

Incandescent	9-22 lm/W
Fluorescent	45-95
Metal Halide	80-115
High-pressure sodium	80-140
Sunlight	92

## 2. Inverse Square Law

Revisit our sphere



Double the radius

- surface area bounded by a solid angle of 1 sr now increases to 4 m<sup>2</sup>
- luminous flux remains  $\Phi = 1 \text{ lm}$
- illuminance  $E$  drops from 1 lux to  $\frac{1}{4}$  lux

Generalize:

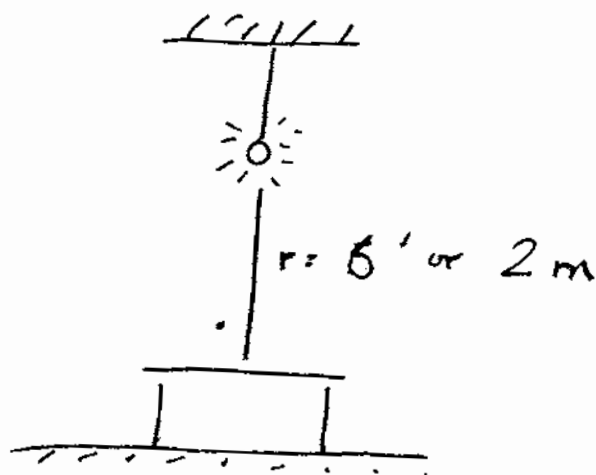
$$E = \frac{I}{R^2}$$

$$\frac{\text{lm}}{\text{m}^2} \text{ or lux} \quad \frac{\text{cd}}{\text{m}^2}$$

This is appropriate when a receiving surface is directly facing a point source

What can we do with this? It's the basic building block of all lighting calculations.

We can find the illuminance on a desk from a bare incandescent light bulb dangling overhead



Bulb:  $60 \text{ W} \times 20 \text{ lm/W} = 1200 \text{ lm}$

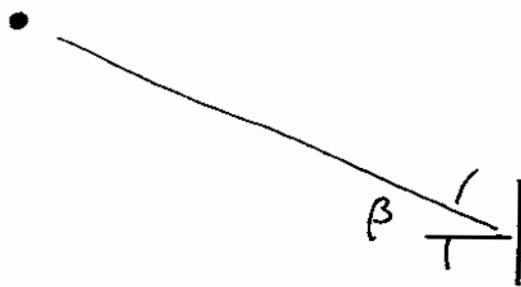
$$1200/4\pi \approx 100 \text{ cd if point source}$$

Desk:  $E = 100/2^2 = 25 \text{ lux}$



Extension # 1:

Receiver is not directly facing the source of light

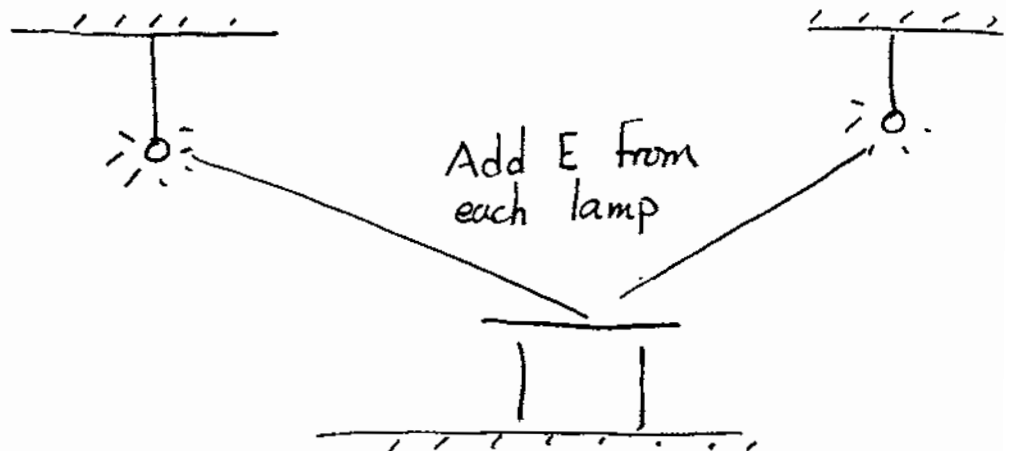


$$E = \frac{I \cos \beta}{R^2}$$

Good for:

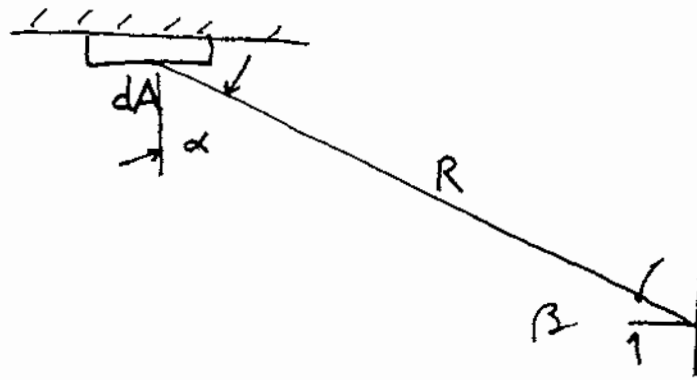
1. street lights shining on road signs (or road!)

2.



## Extension #2

Source of finite size not directly facing the line of sight between source and receiver




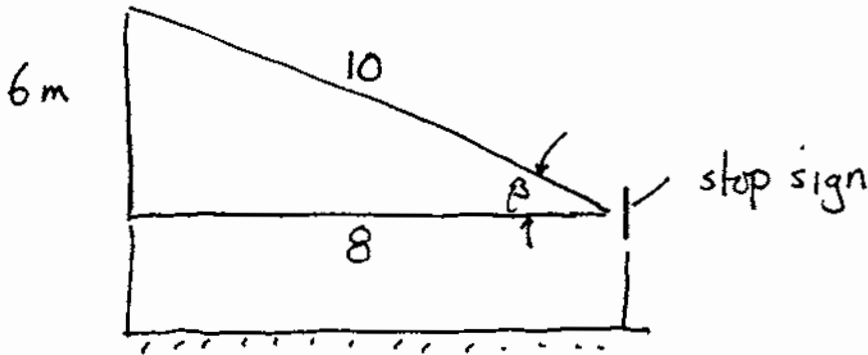
$$dE = \frac{L dA \cos \alpha \cos \beta}{R^2}$$

Application of I.S.R. in the lighting industry:

Characterize the luminous intensity, as a function of transverse and longitudinal angles, for light fixtures.

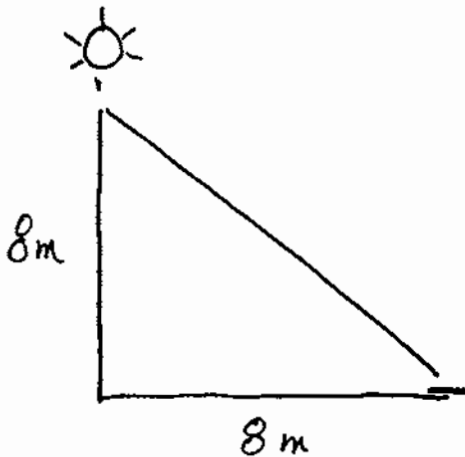
# Lighting calculations- practice

1.  2000 cd street lamp



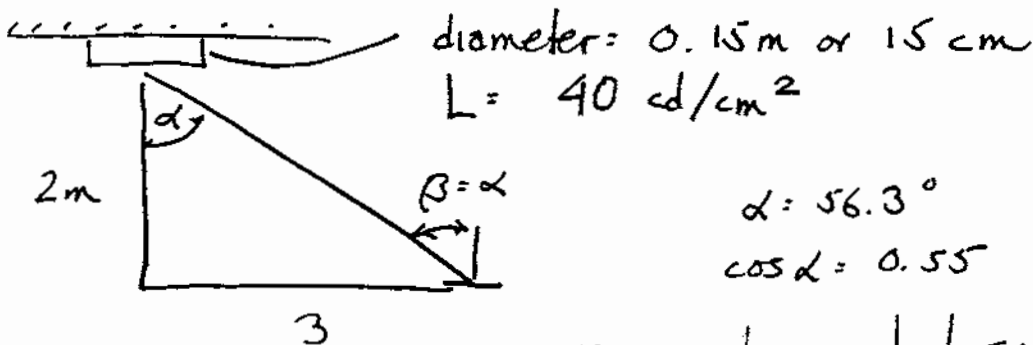
$E$  on sign = ?

2.



$M$  from the horizontal surface  
if  $\rho = 0.3 = ?$

3.



$$\alpha = 56.3^\circ$$

$$\cos \alpha = 0.55$$

$E$  on horizontal surface = ?

### 3. Lighting Design with Electric Lights

#### a. Caveat:

ignore glare

ignore brightness (contrast)  
ratios

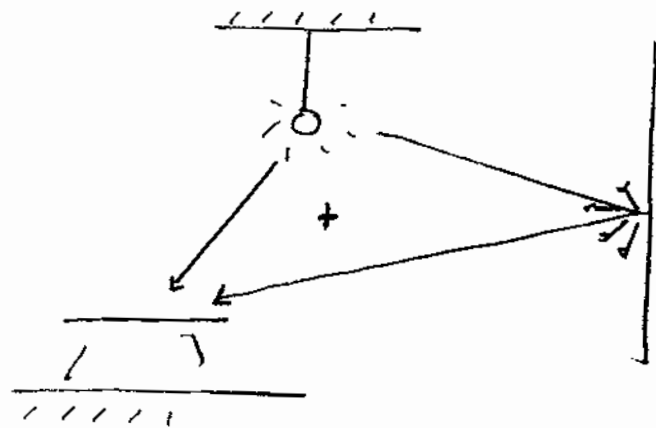
ignore "biological needs"

concentrate solely on illuminance

b. IES specifies illuminance targets for different tasks, modified by age of users and criticality of work and task background reflectance

c. What does I.S.R. NOT account for?

Inter-reflections



There are two types of reflection:

i Diffuse

ii Specular

A perfectly diffuse surface can be characterized by a single reflectance  $\rho$  (or three RGB values) and is called Lambertian.

For a Lambertian surface,

$$M = \pi L$$

d. The basic lumen-method calculation

$$E_{\text{workplane}} = \frac{\Phi_{\text{lamps}}}{A_{\text{workplane}}} \times CU \times LLF$$

lux  $\frac{\text{lm}}{\text{m}^2}$

CU = coefficient of utilization

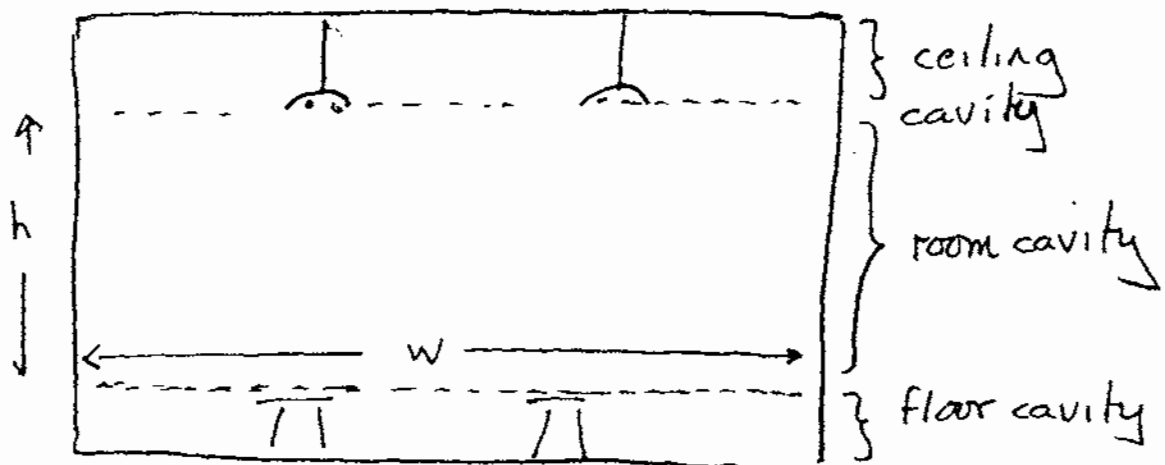
LLF = light loss factor

The coefficient of utilization accounts for the distribution of luminous flux, including:

1. Absorption
2. Multiple passes of photons through the work plane

$$CU = f \text{ (surface properties, room dimensions, lamp fixture type)}$$

Room dimensions are specified by a room cavity ratio (RCR)



$$RCR = \frac{5 h (1 + w)}{l \cdot w}$$

$$= 5 \times \frac{\text{vertical surface area}}{\text{horizontal surface area}}$$

The equivalent reflectances of the imaginary surfaces depend on the reflectances of the physical surfaces in the floor and ceiling cavities and the geometry of these cavities.

The light loss factor LLF depends on:

lamp output decreases over time

light fixture dirt

room surface dirt

A typical LLF might be  $\approx 0.7$

## Example

Design the lighting for a classroom

$$15 \text{ m} \times 6 \text{ m} \times 3 \text{ m}$$

$$\rho_c = 80\%$$

$$\rho_w = 50\%$$

$$\rho_f = 30\%$$

Use luminaire # 6, with 2 40-W fluorescent lamps rated at 3150 initial lumens each ( $\sim 80 \text{ lm/w}$ )

Take 500 lux as the target illuminance and assume the workplane is at 80 cm

$$\text{RCR} = \frac{5 \times (3 - 0.8) \times (15 + 6)}{15 \times 6} = 2.6$$

$$\text{FCR} = \text{RCR} \times \frac{0.8}{2.2} = 0.93$$

From the table, RCR for  $\rho_f = 20\%$  is  $\sim 0.59$

The correction for  $\rho_f = 30\%$  is small, a multiplier of 1.04, giving  $\text{CU} = 0.61$



LLF can be calculated from detailed tables.  
Ignore them (but see Chapter 7 of Murdoch)  
and go with  $LLF = 0.7$

Use the basic equation, solving for  $\Phi$

$$\begin{aligned}\Phi &= \frac{E \cdot A}{CU \cdot LLF} = \frac{500 \text{ lux} \cdot 90 \text{ m}^2}{0.61 \times 0.7} \\ &= \frac{45000 \text{ lm}}{0.43} = 105,400 \text{ lm}\end{aligned}$$

Number of luminaires:

$$\frac{105,400}{2 \times 3150} = 16.7$$

Go with 16-17 fixtures, spaced evenly

# 4. Daylighting Fundamentals

(Murdoch chpt 9)

## a. Light from the sun and sky

i. Numbers to remember - rough - on Earth's surface

Sun:  $\sim 1 \text{ kW/m}^2$  "direct normal" irradiance  
(pointing directly toward the sun)

$\sim 100 \text{ lm/W}$  luminous efficacy

$\therefore \sim 100,000 \text{ lux}$

ii. In more detail....

Start outside Earth's atmosphere

$1377 \text{ W/m}^2$

$127.5 \text{ klux}$  ( $127,500 \text{ lux}$ ) call it  $E_0$

$93 \text{ lm/W}$

Now drop down to Earth's surface. Must account for attenuation of light through atmosphere

$$E_{\text{direct normal Earth's surface}} = E_p$$

$$= E_0 e^{-cm}$$

$$m = \frac{1}{\sin h}$$

$h$ : angle of elevation of sun above horizon

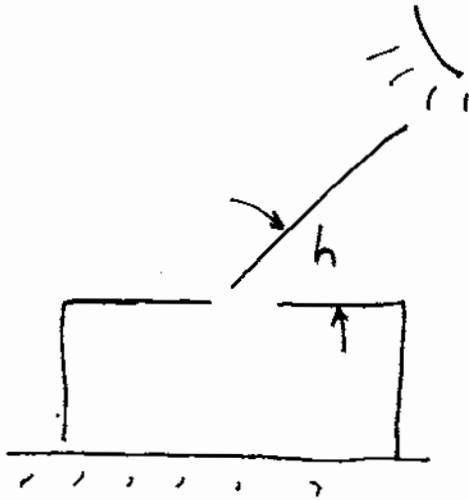
$c$ : atmospheric extinction coefficient = 0.210

Try a few elevation angles

$h$	$E_p$	$h$	$E_p$
90°	103 klux	40	92
80	103	30	84
70	101	20	69
60	100	10	38
50	97	0	0

## b. Illuminance from the Sun on building surfaces

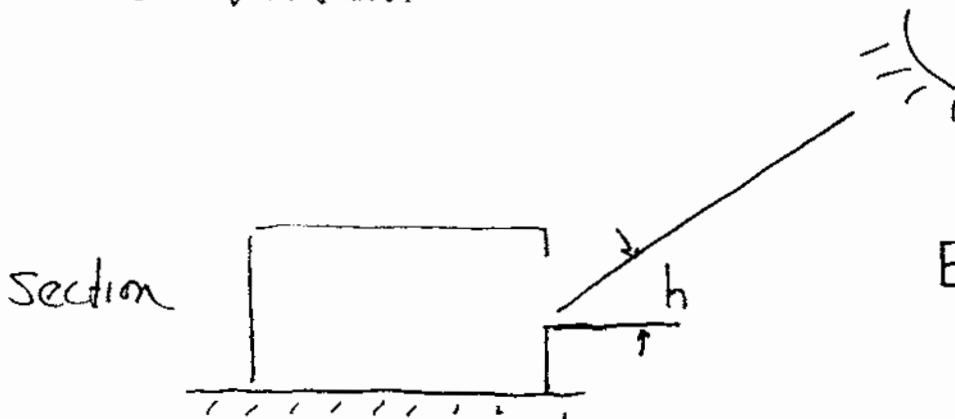
i. Horizontal



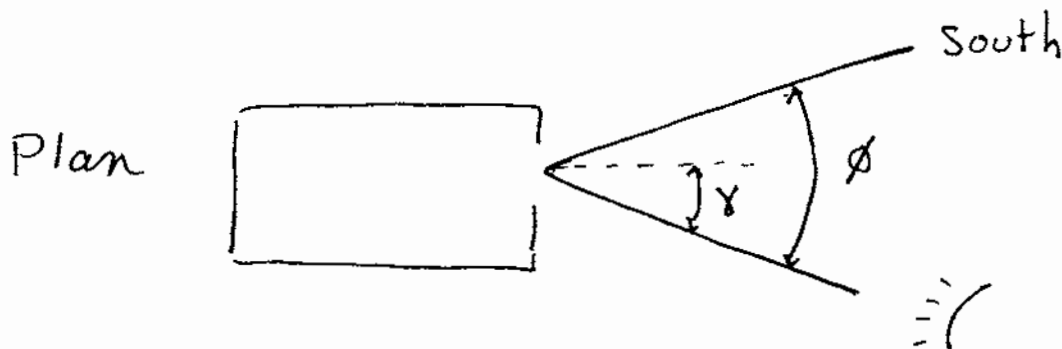
$$E_{\text{horz}} = E_{\text{sun}} \sin h$$

where  $h$  is solar altitude

ii. Vertical



$$E_{\text{vert}} = E_{\text{sun}} \cos h \cos \gamma$$



The altitude and azimuth of the Sun depend on latitude, date and time. See Murdoch chapter 9 (handout) and the web site in the climate handout (later)

At solar noon, the azimuth is  $0^\circ$  and the altitude is easily calculated for an equinox and the two solstices:

$$\text{Summer solstice } (\sim 6/21) \quad h = 90 - L + 23.5^\circ$$

$$\text{Equinox } (\sim 3/21, 9/21) \quad h = 90 - L$$

$$\text{Winter solstice } (\sim 12/21) \quad h = 90 - L - 23.5^\circ$$

$23.5^\circ$  is the tilt of the Earth's rotational axis with respect to the plane of the Earth's revolution around the Sun.

On-line sun-angle calculators:

<http://www.susdesign.com/sunangle/>  
[www.aie.org.au/melb/material/resource/solar.htm](http://www.aie.org.au/melb/material/resource/solar.htm)

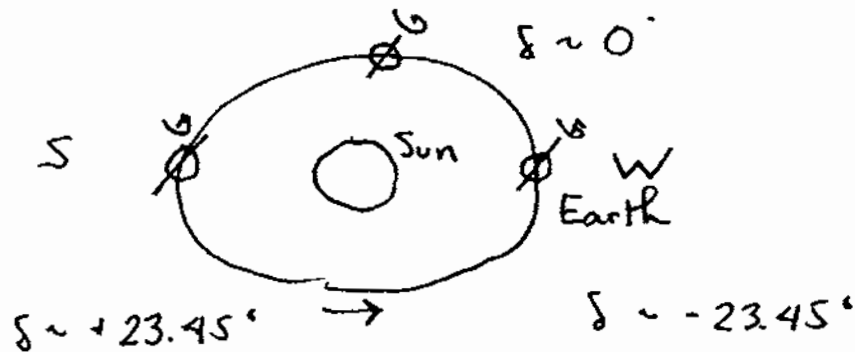
One last detail....

What is the angle  $h$ , you ask, for a given latitude, date and time - not just an equinox or solstice?

$$\sin h = \cos L \cos \delta \cos H + \sin L \sin \delta$$

$L$  - latitude

$\delta$  - Earth's declination (tilt of spin axis relative to orbital path, as measured in a plane that includes sun and Earth)



$$\delta = 23.45^\circ \sin \beta \quad \beta = \frac{360}{365} (n - 81) \text{ degrees}$$

$H$  = hour angle, the number of minutes before or after noon, multiplied by 0.25 (Can you guess why?)

And, for completeness, there is a simple formula for the sun's azimuthal angle  $\phi$

$$\sin \phi = \frac{\cos \delta \sin H}{\cos R}$$

We won't often need these equations. Instead, we'll use charts or computer programs

4/27/04. We need altitude and azimuth for window design

60 minutes =  $15^\circ$  of arc  
Work with solar time

$$ST = TZ + ET + \Delta T$$

TZ = time zone time

ET = correction for elliptical orbit

sun is behind local time by ~ 15 min in Feb  
ahead (\*) Oct/Nov

$\Delta T$  = time difference between center of time zone ( $75^\circ$  W for Eastern) and observer's location

Tutorial: [depace.palomar.edu/~jthorngren/tutorials.html](http://depace.palomar.edu/~jthorngren/tutorials.html)

To go from  $E_p$  (direct normal) to  $E_h$  or  $E_v$ , use formulas or graphs.

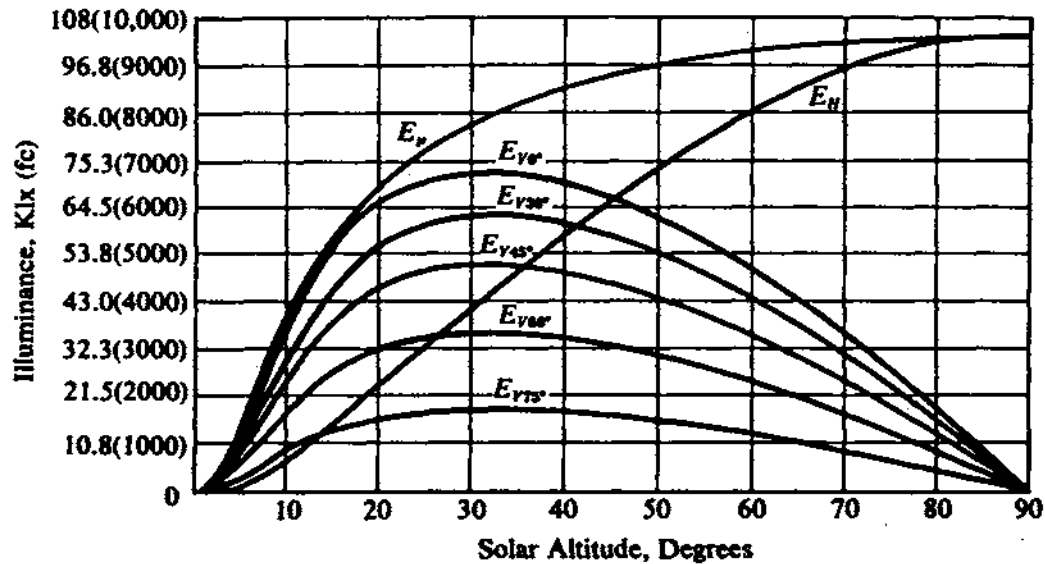


Figure 9.4 Solar illuminance. (Source: IES Daylighting Committee, *Recommended Practice of Daylighting*, IES, New York, 1979, fig. 37-A, with permission.)

Here  $E_{v0}$  means illuminance from the sun on a vertical surface (a wall!), oriented such that its normal is pointing toward the sun (i.e.,  $\gamma=0$ )



What about the sky - overcast or clear?

Atmospheric scientists have measured sky luminance.

For an overcast sky, the luminance varies with polar angle (the angle from the zenith).

$$L_{\text{zenith}} \sim 2.5 - 3.0 \times L_{\text{horizon}}$$

Luminance is relatively constant with azimuthal angle  $\phi$ .

It is customary in the U.S. to use a single value of overcast sky luminance for a given latitude, date, and time, which is called

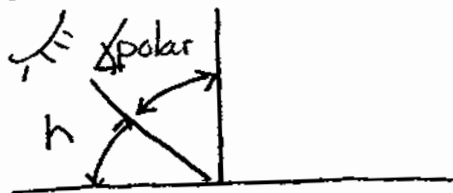
equivalent sky luminance

and is chosen to produce <sup>the</sup> same horizontal illuminance as is produced by the actual sky. Recall

$$E_h = \pi L$$

where  $L$  is a uniform, single value, the equivalent sky luminance.

Equivalent sky luminance can be plotted as a function of solar altitude  $h$



What about a clear sky?

It's defined as having less than 30% cloud cover.  
(Which 30%?!)

Clear-sky luminance varies with both polar and azimuth angles. The luminance near the sun and the horizon may be 12 x higher than the zenith luminance, and may be 6 x greater than the illuminance at the opposite horizon. Nevertheless, an equivalent sky luminance has been defined in the U.S. It is a function of compass direction, date, time, and latitude. Equivalent sky luminance can be plotted as a function of solar altitude alone. The curve-fit plot masks considerable data scatter.

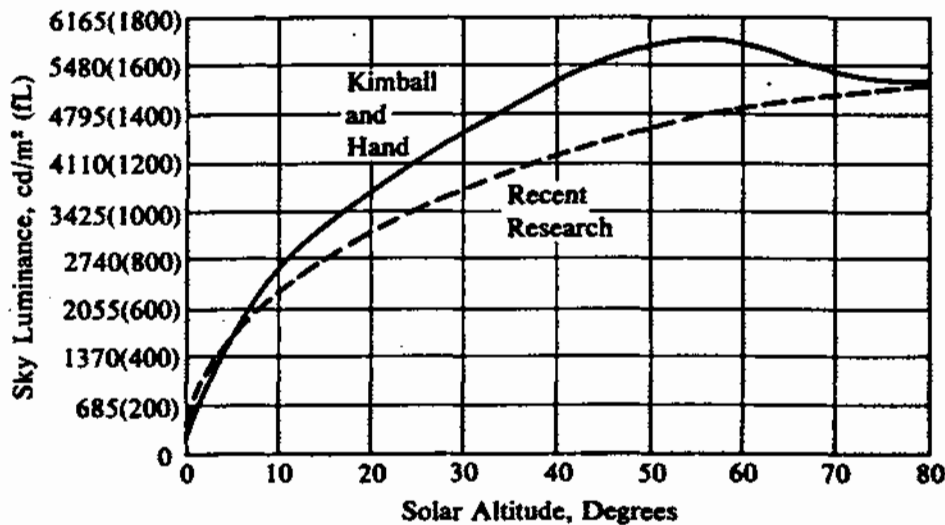


Figure 9.6 Equivalent sky luminance—clear sky (no direct sun).

Putting it together...

The horizontal illuminance for the sun, a clear sky, and their sum can be plotted as a function of solar altitude.

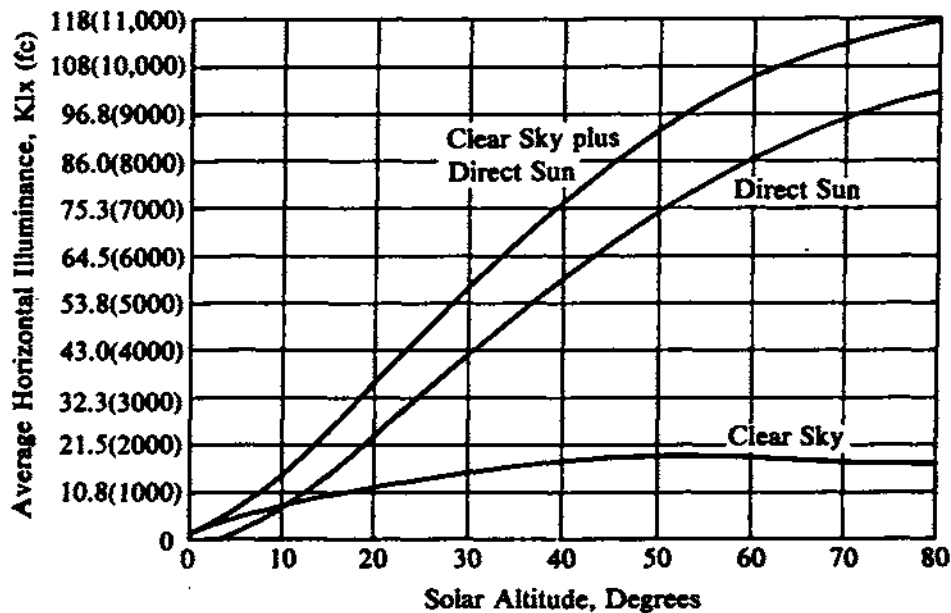


Figure 9.7 Horizontal luminance—clear sky plus direct sun.

c. Lumen method for skylights (openings in flat roofs)

The approach is (nearly) identical to the lumen method for electric lights

$$E_{\text{workplane}} = \frac{\Phi_{\text{sun+sky}}}{A_{\text{workplane}}} \times K_u \times K_m$$

$K_u$ : utilization coefficient

$K_m$ : light-loss factor

$$\Phi_{\text{sun+sky}} = (E_{\text{sun}} + E_{\text{sky}}) \times A_{\text{skylight}}$$

An aside: combine the above two equations

$$\frac{E_{\text{workplane}}}{E_{\text{sun}} + E_{\text{sky}}} = \frac{A_{\text{skylight}}}{A_{\text{workplane}}} \times K_u \times K_m$$

Say we want 500 lux and have 50,000 lux on our skylight. Then

$$\frac{E_{\text{workplane}}}{E_{\text{sun}, E_{\text{sky}}}} = \frac{500}{50,000} = 0.01$$

Depending on  $K_u$  and  $K_m$ , our skylights may not need to be very large relative to the floor area.

$$K_u = RCU \times T_n$$

RCU = room coefficient of utilization

$T_n$  = net skylight transmittance

Example from Murdoch:

$$K_u = 0.85 \times 0.34$$

↳ note very low

$$K_m = 0.71$$

$K_u \cdot K_m = 0.24$ , which effectively multiplies the required skylight area by a factor of 4 ( $1/0.24$ )

#### d. Lumen method for windows (side lighting)

- Empirical method good for only three points in a room
- Accounts for direct and diffuse light
- Requires separate calculation of light on a window from the sun and sky, and from the ground

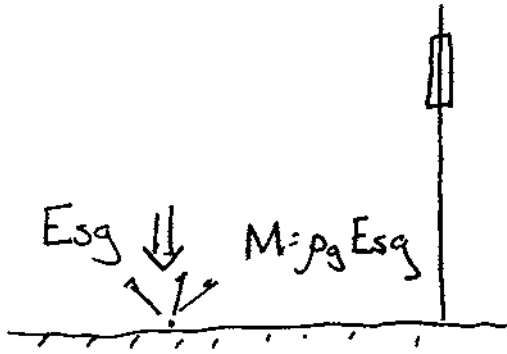
Light from the sky and sun - we've seen it!

Direct (from sun) - use formulas or figures

Diffuse (from sky) - use figures

Light from the ground

$$E_{gw} = \frac{\rho_{\text{ground}} E_{sg}}{2}$$



$$\Phi_{sw} = E_{sw} \hat{z} A_w$$

$$\Phi_{gw} = E_{gw} \hat{z} A_w$$

$$E_{sp} = \Phi_{sw} C_s K_s K_m$$

$$E_{gp} = \Phi_{gw} C_g K_g K_m$$

See Murdoch

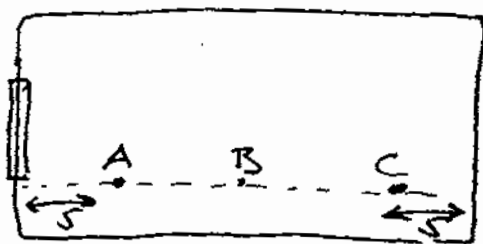
CAREFUL!!  $C_s \cdot K_s$  and  $C_g \cdot K_g$  are NOT dimensionless. Must work in English units.

C - utilization factor based on room length and width

K - utilization factor based on room height and width

$K_m$  accounts for dirt on room surfaces and windows

$C_s \cdot K_s$  and  $C_g \cdot K_g$  are SMALL.



Even for the point closest to the window

$$C_s \approx 0.01 - 0.02$$

K is independent of location (A, B or C) and is  $\sim 0.01$

Example from Murdoch:

$$\Phi_{sw} = 102,800 \text{ lm}$$

$$\Phi_{gw} = 51,100 \text{ lm}$$



$$E_{\max} (\text{Pt. A}) = 120 \text{ fc} (E_{\text{sp}}) + 34 \text{ fc} (E_{\text{gp}}) = 154$$

$$E_{\text{mid}} (\text{Pt. B}) = 30 + 22 = 52$$

$$E_{\min} (\text{Pt. C}) = 18 + 14 = 32$$

For a room  $40' \times 30' \times 12'$   
L      W      H.

If the floor areas represented by A, B and C were equal, then  $E_{\text{avg}} \approx 75$  and  $\Phi_{\text{workplane}} =$

$$75 \text{ fc} \times 1200 \text{ ft}^2 = 90,000 \text{ lux}$$

Compare with 154,000 lux transmitted through windows.

$$\frac{90,000}{154,000} = 0.58 = 0.76 \times 0.77, \text{ where}$$

$K_m$  in the example is 0.77. Therefore 76% of luminous flux is utilized (roughly!)

$$E_{sp} = E_{sp} + E_{gp}$$

$$= (E_{sw} C_s K_s + E_{gw} C_g K_g) \vec{A}_w K_m$$

$C_s K_s$  and  $C_g K_g$  in  $\mathbb{R}^{-2}$

$A_w$  in  $\mathbb{R}^2$

## e Daylight factor method

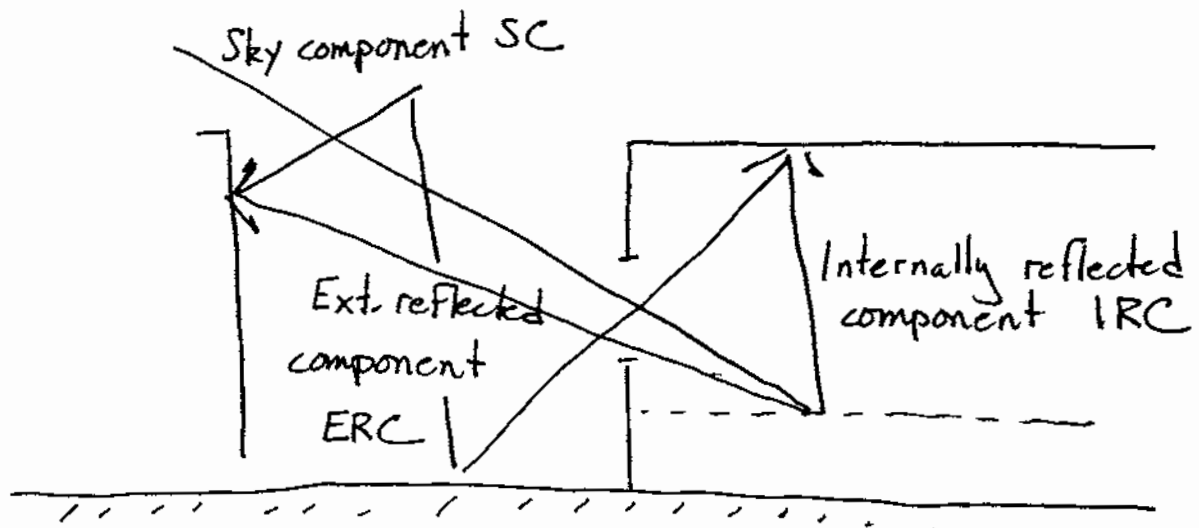
- analytic, not empirical
- good ONLY for cloudy skies (European origin)

Daylight factor DF =

$\frac{E_{\text{point indoors}}}{E_{\text{outdoors, on ground, under unobstructed hemisphere}}}$

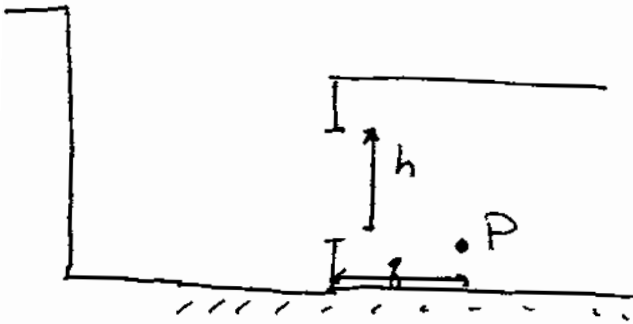
This ratio is intuitive and appealing.

The DF has three components



$$DF = SC + ERC + IRC$$

SC depends on  $\frac{\omega}{g}$  and  $\frac{h}{g}$



$\omega$  is width of the window

P is on a line perpendicular to the lower left corner of the window

SC varies from 0.01 to 15 %  
Must subtract SC due to obstruction

ERC is small. The luminance of the obstruction is taken as 10-20% that of the sky.

IRC. in Murdoch's example, IRC is  $\approx$  half of SC and is therefore significant.

# Glare

"Sensation produced by luminances within the visual field that are greater than the luminance to which the eyes are adapted to cause annoyance, discomfort, and/or decrease in visibility and visual performance"

- Murdoch

## ISSUES

Enlargement of object

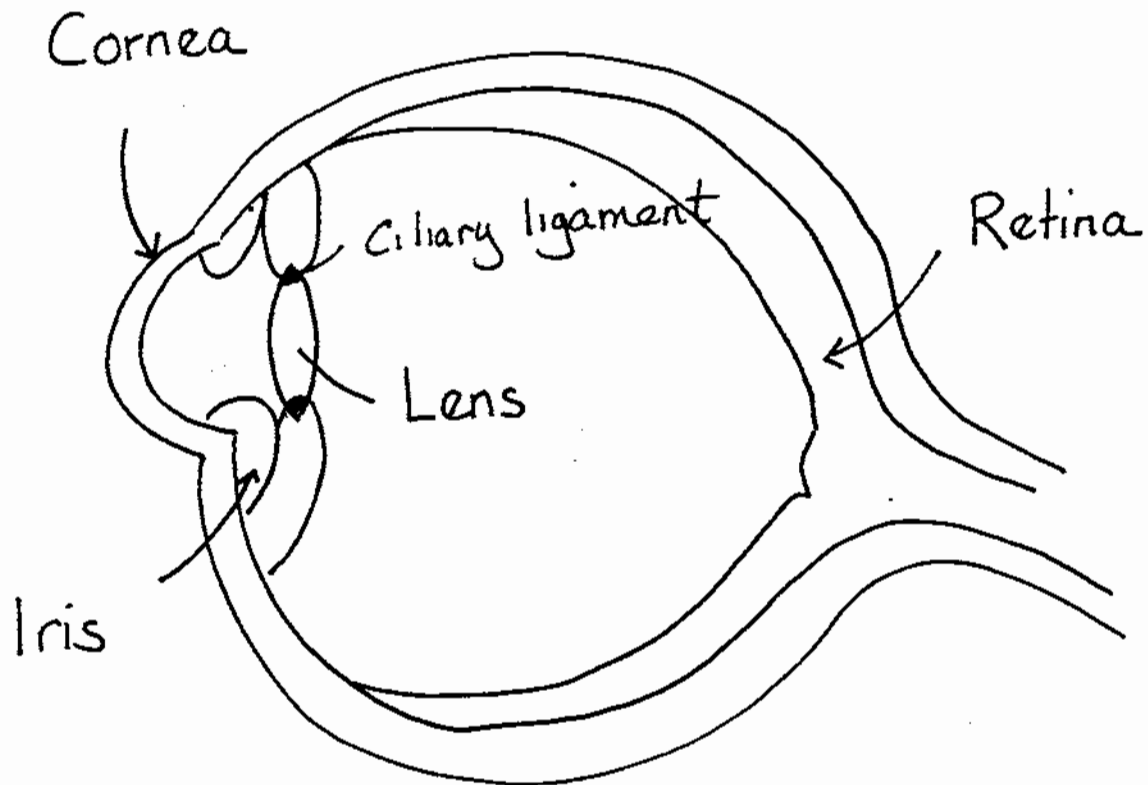
Disability - reduced visual performance

Discomfort

## Enlargement

### 1. Lenticular halo

- diffraction of light by radial fibers at the periphery of the lens
- only occurs when pupil diameter  $> 3\text{mm}$



Pupil diameter ranges from 2 - 8 mm

2 mm bright light

8 mm dim light

3 mm corresponds to a field luminance of  $10 \text{ cd/m}^2$ , a dimly lit interior

## 2. Ciliary corona

- scattering of light in the lens
- day or night, but more visible at night, when field luminance is low
- more pronounced for small sources

### 3. Bloom

- scattering of light in cornea, lens, and retina
- also known as disability glare or veiling luminance
- scattered light adds to luminance of task and background, decreasing contrast

### Disability

1. Reflected glare or veiling reflections -
  - due to strong light on task and background
  - predominant cause of reduced contrast
2. Direct glare
  - noted above
3. Transient adaptation

### Discomfort

1. Reflected - we will not consider
2. Direct

# Visual Comfort and Direct Glare

See Murdoch chapter 8

## Field luminance

Background luminance against which glare sources are viewed

$$F = \frac{L_w \omega_w + L_f \omega_f + L_c \omega_c + \sum L_s \omega_s}{5}$$

w - wall

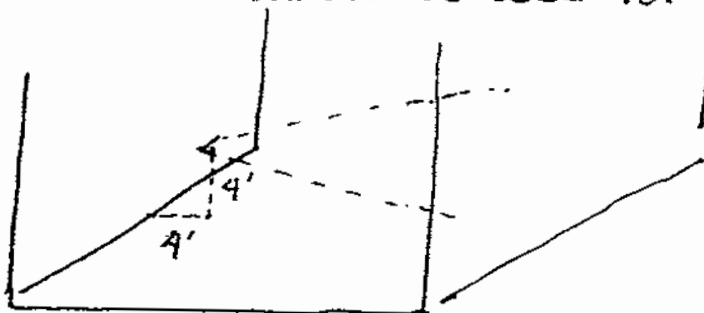
f - floor

c - ceiling

s - sources

5 steradians = total field of view, a cone of  $\sim 78^\circ$

For a "standard viewing position," the average luminance of the opposite wall is sometimes used for F





## Visual size

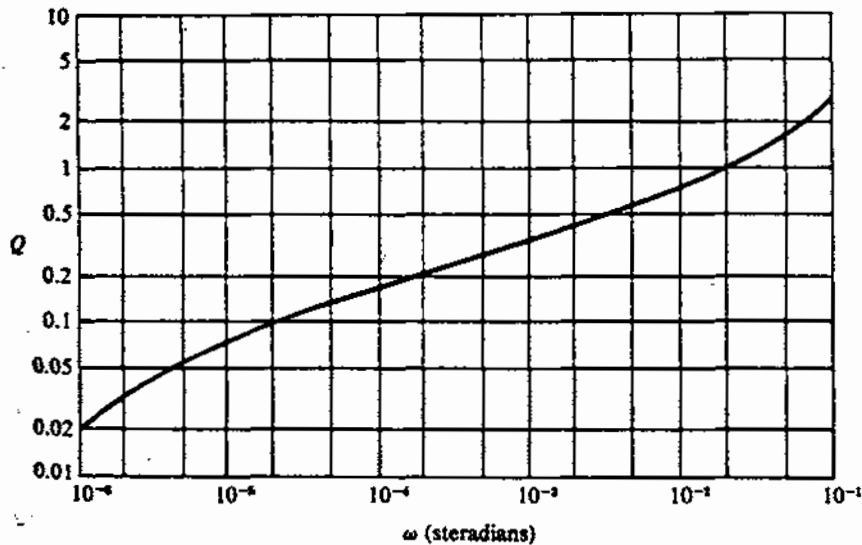
The size of a glare source

$$\omega = \frac{A_p}{H^2 + X^2 + Y^2}$$

$$A_p = A \cos \phi = A \frac{H}{\sqrt{H^2 + X^2 + Y^2}}$$

Empirically, the glare effect  $Q$  is related to the solid angle  $\omega$  in a complicated way

$$Q = 20.4\omega + 1.52\omega^{0.2} - 0.075$$



## Source luminance

Average luminance of the glare source in the direction of the observer.

If  $I$  is the luminance intensity in candelas in the direction of the observer and  $A_p$  is the projected area of the luminance in that direction,

$$L = \frac{I}{A_p}$$

Fundamental glare formula for a single source

$$M = \frac{0.5 L Q}{P F 0.44}$$

where  $M$  = glare sensation

$L$  and  $F$  are in  $\text{cd}/\text{m}^2$

Meaningful glare measure : value of source illuminance which produces a sensation at the borderline between comfort and discomfort for the average observer

Borderline Comfort - Discomfort Luminance (BCD)

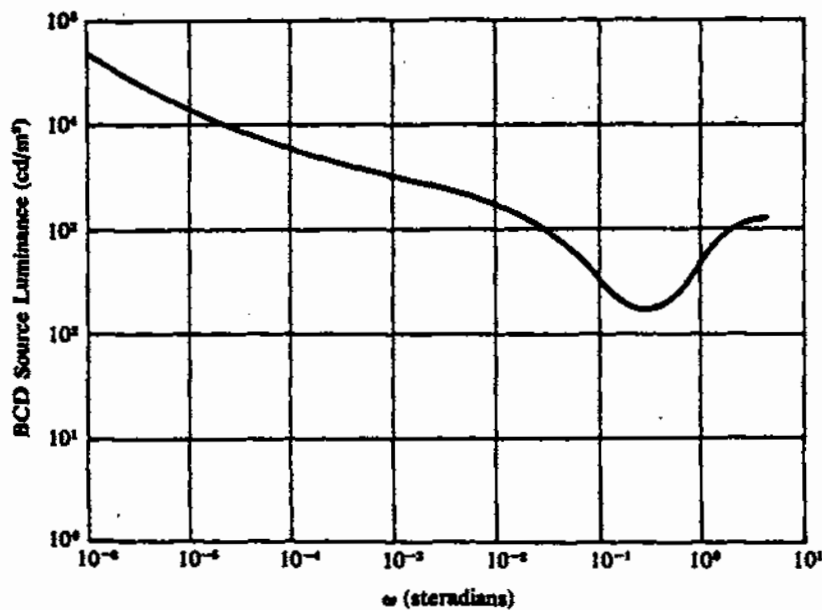


Figure 8.22 BCD luminance versus source solid angle.

Solid angle (steradians)	Cone angle (degrees)
5.0	78.2
3.0	58.5
2.0	47.0
1.0	32.7
0.5	23.0
0.2	14.5
0.1	10.2
0.05	7.2
0.02	4.6
0.01	3.2
0.005	2.3
0.002	1.4
0.001	1.0
0.0001	0.3

## Multiple sources

Experiments needed, due to non-linear relationship between glare sensation and source size  $\omega$ .

Study with 210 observers and 48 lighting conditions

$$M_t = M_1 + M_2 + \dots + M_n$$

$$\text{Discomfort glare rating DGR} = (M_t)^a$$

$$a = n^{-0.0914}$$

One more discomfort glare evaluation, this time for a group of observers :

Visual comfort probability (VCP)

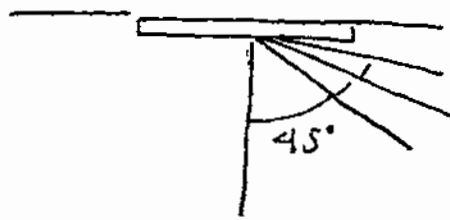
107 observers in 35 lighting conditions

$$VCP = \frac{100}{\sqrt{2\pi}} \int_{-\infty}^K e^{-k^2/2} dk$$

$$K = 6.374 - 1.3227 \ln DGR$$

IES glare criterion: meet the following three conditions

1. VCP  $\geq 70$
2. Ratio of maximum to average luminance  $\leq 5:1$  at angles of  $45^\circ$ ,  $55^\circ$ ,  $65^\circ$ ,  $75^\circ$  and  $85^\circ$  from the vertical for both lengthwise and crosswise viewing



3. Maximum luminance luminances lengthwise and crosswise do not exceed the following values:

Angle from vertical	Maximum L $\text{cd/m}^2$
$45^\circ$	7710
$55^\circ$	5500
$65^\circ$	3860
$75^\circ$	2570
$85^\circ$	1695

IES has a glare formulation slightly different

Guth (in Murdoch)

$$M = \frac{0.5 L Q}{P F^{0.44}}$$

glare sensation

IES

$$G = \frac{K P^* L^{1.6} \omega^{0.8}}{F} \quad \text{glare constant}$$

$P^*$  is large along line of sight.  
opposite of  $P$

$$GI = 10 \log_{10} G \quad \text{glare index}$$

Guth / Murdoch's formulation leads to

VCP - useful

incorporated in Radiance  
and Lumen-Micro

The IES formulation leads to a glare index for daylighting

The degree of discomfort glare due to the sky seen through a window can be predicted from a glare index based on the Cornell large source glare formula

$$G = K \cdot \frac{L^{1.6} \Omega^{0.8}}{F + 0.07 \omega^{0.5} L}$$

$\Omega$  is the solid angle of the source, modified to account for position in the field of view

$$GI = 10 \cdot \log_{10} G$$

Daylight Glare Index (DGI)

$$DGI = \frac{2}{3} (GI + 14) \quad \text{for } GI \leq 28$$

Observed: greater tolerance of glare from sky, as seen through a window, than from a comparable artificial lighting situation, provided the glare index is not too high

This result agrees with another, where the conclusion was that "discomfort glare from a single window (except a small one) is practically independent of size and distance from the observer, but is critically dependent on the sky luminance."

**TABLE VI - Comparison of Glare Indices for artificial light (IES GI) and daylight (DGI)**

<i>Glare Criterion</i>	<i>IES GI</i>	<i>DGI</i>
Just imperceptible	10	16
	13	18
Just acceptable	16	20
	19	22
Just uncomfortable	22	24
	25	26
Just intolerable	28	28



Consider a room

- $12 \times 12 \times 6$  m
- average internal reflectance of 0.4
- window is viewed from a perpendicular distance of 6 m

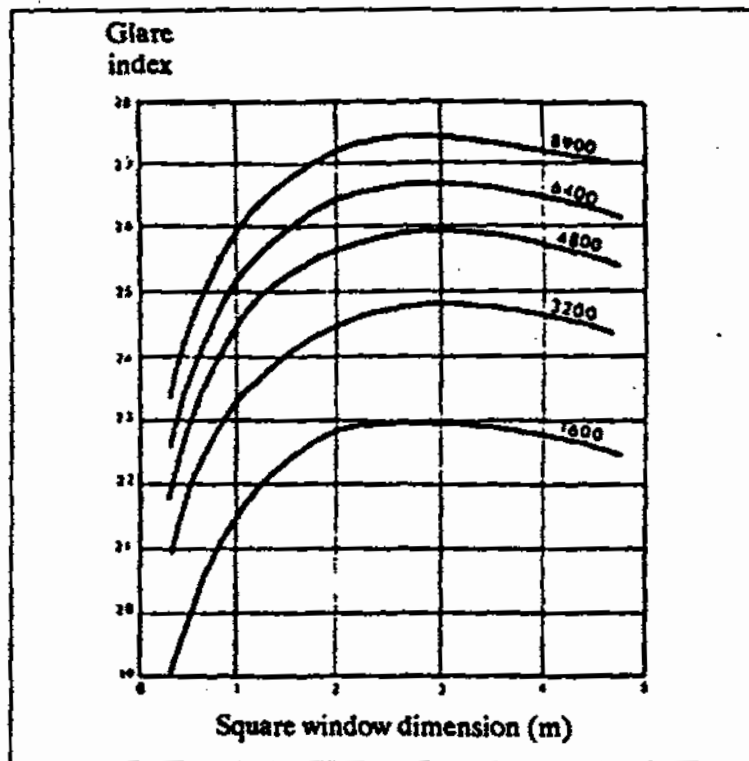


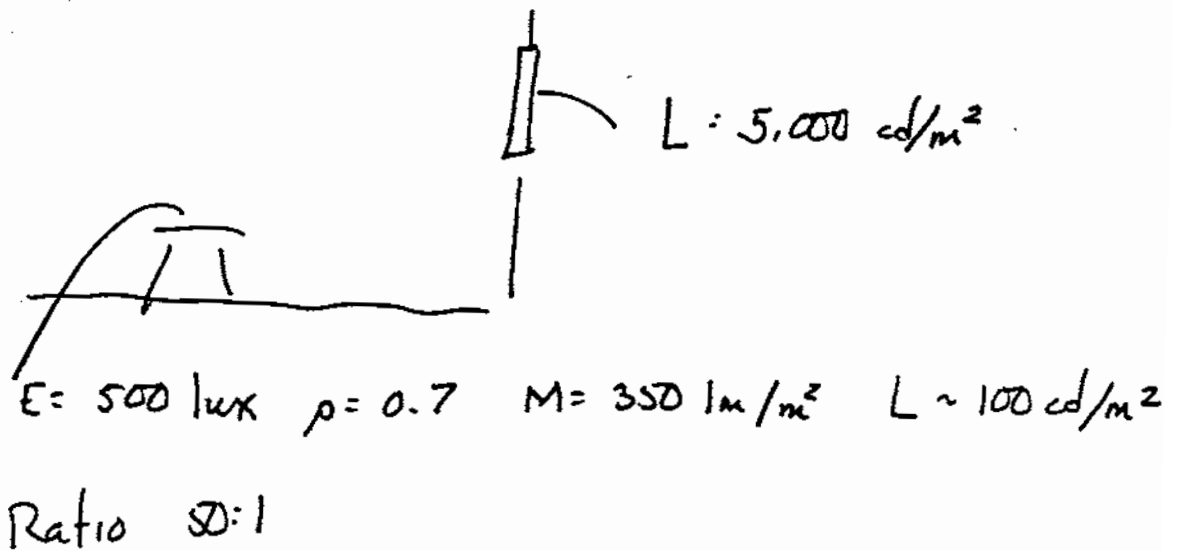
Figure 17 - The relationship between daylight glare index and window size for sky luminances between 1600 and 8900  $\text{cd/m}^2$ . The window is viewed from a perpendicular distance of 6m in a room of dimensions 12m x 12m x 6m. Results for average reflectance of 0.4 (17).

$$\text{DGI} = 26 \text{ for } L_{\text{sky}} = 8900 \text{ cd/m}^2$$

## Targets for luminance ratios

- Task and darker surroundings 3:1
- Task and remote darker surfaces 10:1
- Light sources and surroundings 20:1
- Maximum (except if decorative) 40:1
- Highlighting of objects for emphasis 50:1

1. If 10:1, then sky is a "problem" - maybe



2. Direct sunlight is a problem - at times

$$E = 100,000 \text{ lux} \quad \rho_{\text{floor}} = 0.3 \quad M = 30,000 \text{ lm/m}^2$$
$$L \sim 10,000 \text{ cd/m}^2$$

Ratio 100:1

Observations:

- Lighting rules/requirements are based on human-subject studies
- For a library, trust your judgment

- Typically need direct light to adequately illuminate spaces > 5m from window, in absence of skylights
- Contact with sun counts if done right
  - Lobby 7 late afternoon ✓
  - Studio 5 early morning X
- Privacy counts

Conclusion:

human needs, as a lighting design strategy, make sense