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PDE vs ODE

$u(t)$ ODE $\frac{d}{dt}$

$U(x,t)$ PDE $\frac{\partial U}{\partial x}$, $\frac{\partial U}{\partial t}$

$U(x,y)$ $\frac{\partial U}{\partial x}$ $\frac{\partial U}{\partial y}$

$U(x,y,z,t)$

Conservation of "U"

U : unit amount of mass, momentum, energy

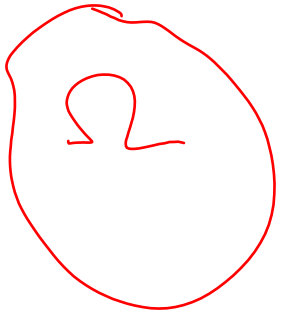
per volume

area

length

$U(x,t)$

U is ρ



$$\frac{d}{dt}\iiint_{\Omega} \rho \, dx + \oiint_{\partial\Omega} \vec{F}(\rho) \, dS = 0$$

In fluid flow $\vec{F}(\rho) = \rho \cdot \vec{u}$

Div. theorem.

$$\oint_{\partial\Omega} \vec{F}(\rho)\, dS = \iiint_{\Omega} \nabla \cdot \vec{F}(\rho)\, dx$$

$$\iiint_{\Omega} \left(\frac{d\rho}{dt} + \nabla \cdot \vec{F}(\rho) \right) dx = 0$$

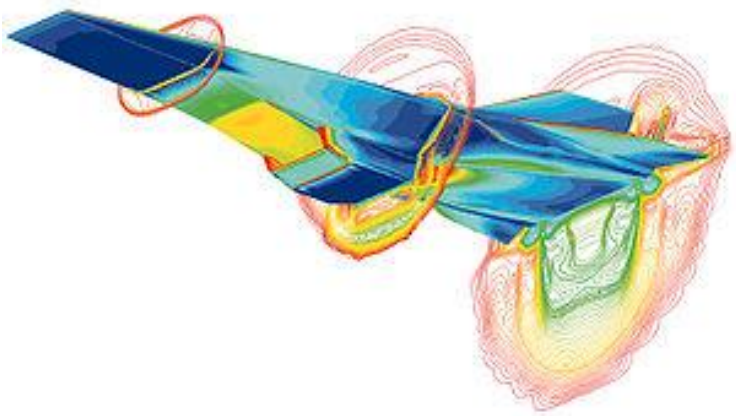
for any Ω

$$\Longrightarrow \quad \frac{d\rho}{dt} + \nabla \cdot \vec{F}(\rho) = 0 \quad : \text{PDE}$$

Differential Conservation Law of U

$$\frac{\partial U}{\partial t} + \nabla \cdot \vec{F}(U) = S$$

Examples of Conservation Law



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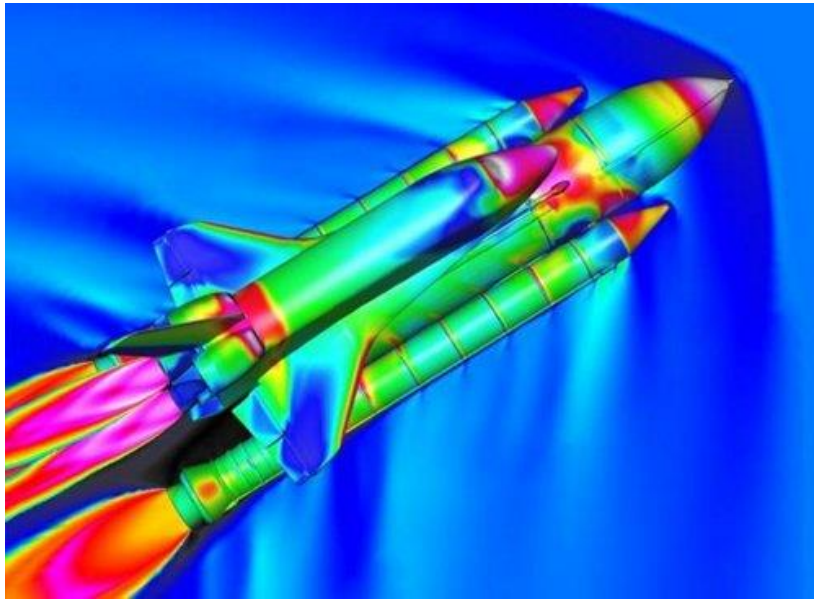

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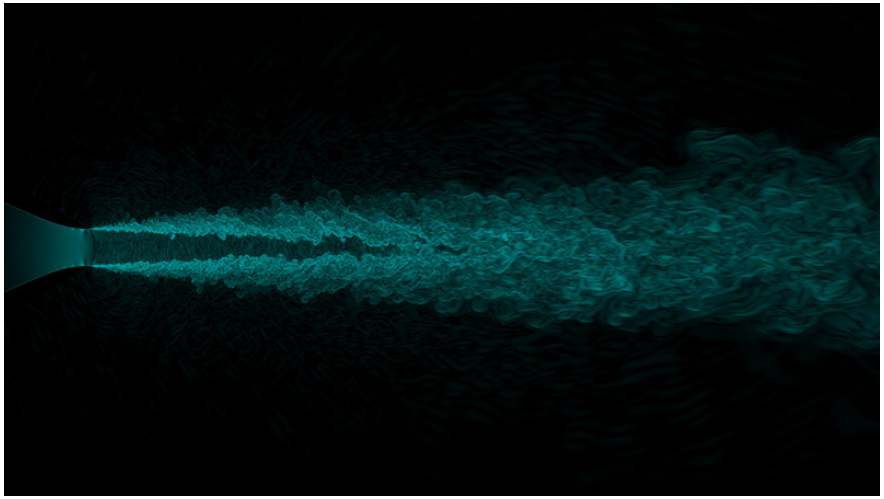

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Examples of Conservation Law

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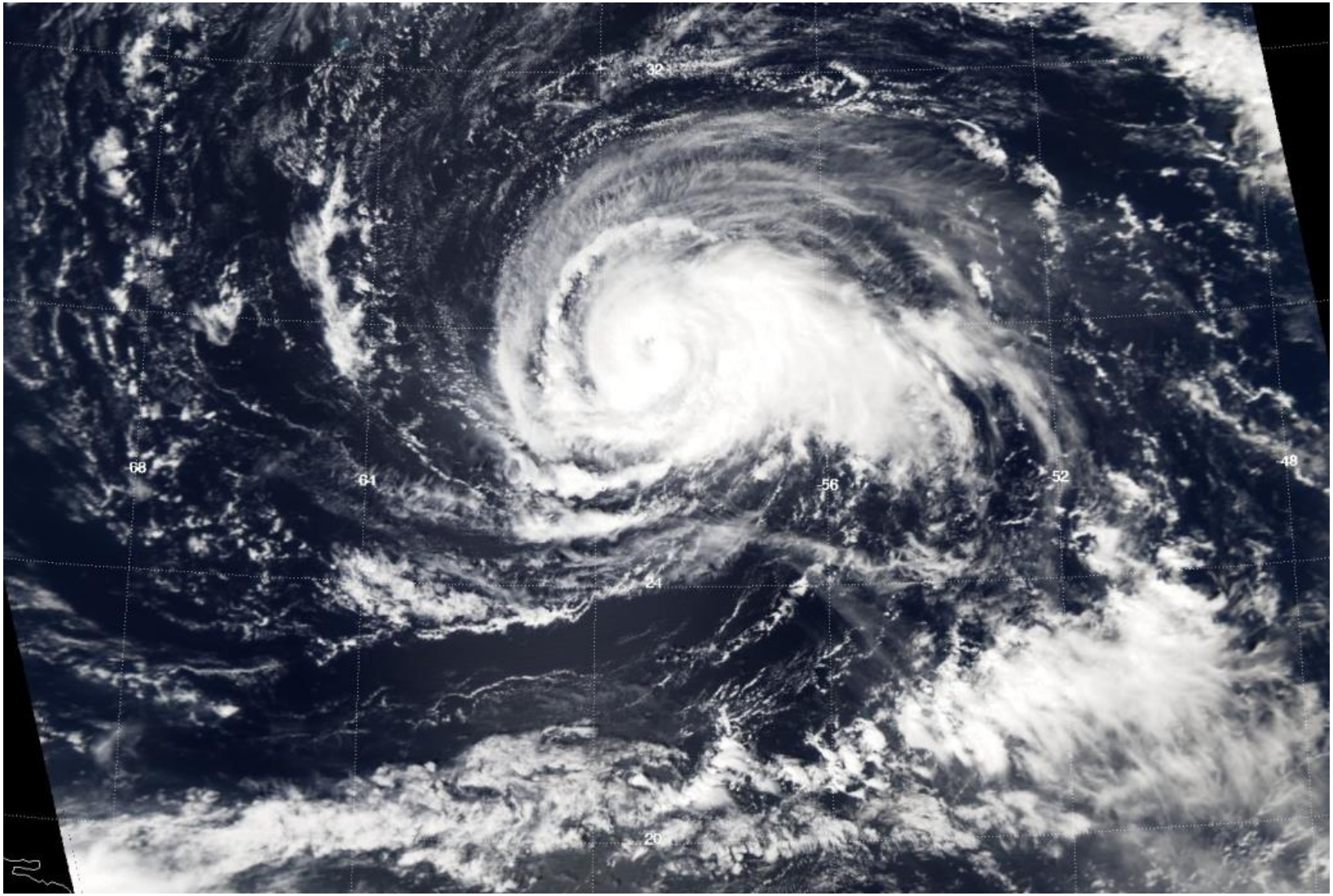

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An Example of Conservation Law
Advection

Convection

$$\frac{\partial u}{\partial t} + U\frac{\partial u}{\partial x} = 0$$

here $u(x,t)$, is the unknown

U is a constant

$$\frac{\partial u}{\partial t} + \frac{\partial F(u)}{\partial x} = 0 \quad \text{where } F(u) = Uu$$

An Example of Conservation Law Advection – Characteristics

$$u(x,t) = u_0(x - Ut)$$

An Example of Conservation Law Burgers Equation – Characteristics

An Example of Conservation Law Advection Diffusion

16.90 Computational Methods in Aerospace Engineering

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