> Welcome to 16.90 iSession ...

➢ Instructor: Turn on Webex ,
and distribute MuddyCards ...

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...etc...

...etc...

Shock capturing of Finite Volume

$$\int_\Omega \frac{\partial \rho}{\partial t} + \nabla \cdot F(\rho) = 0$$

differential form

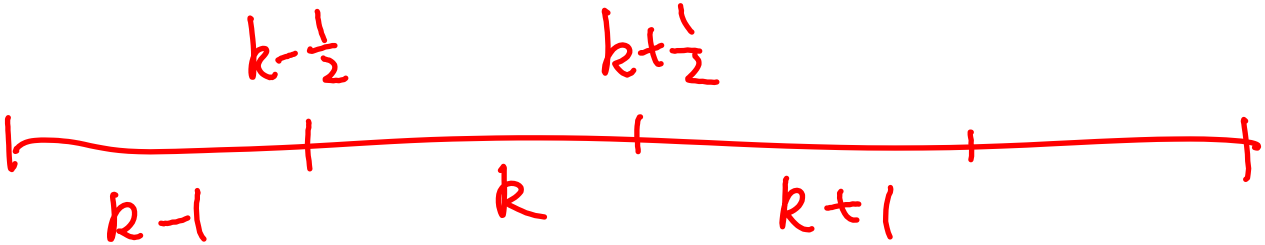
$$\frac{\partial \rho}{\partial t} + \frac{\partial F(\rho)}{\partial x} = 0$$

$$\frac{d}{dt}\int_\Omega \rho \, dx + \int_{\partial\Omega} \vec{n} \cdot F(\rho) = 0$$

integral form

$$\frac{d}{dt}\int_L^R \rho \, dx + F(\rho)\Big|_R - F(\rho)\Big|_L = 0$$

Upwind Scheme for finite volume



$$F_{k-\frac{1}{2}} = \begin{cases} F(\rho_{k-1}) & c > 0 \\ F(\rho_k) & c < 0 \end{cases}$$

C: $\quad \dfrac{\partial \rho}{\partial t} + c\dfrac{\partial \rho}{\partial x} = 0$

$c = \dfrac{dF}{d\rho} \qquad \dfrac{\partial \rho}{\partial t} + \dfrac{\partial F(\rho)}{\partial x} = 0$

Why Upwind Scheme?

Traffic jam simulation

$\rho(x,t)$

$\rho = 0$ empty

$\rho = 1$

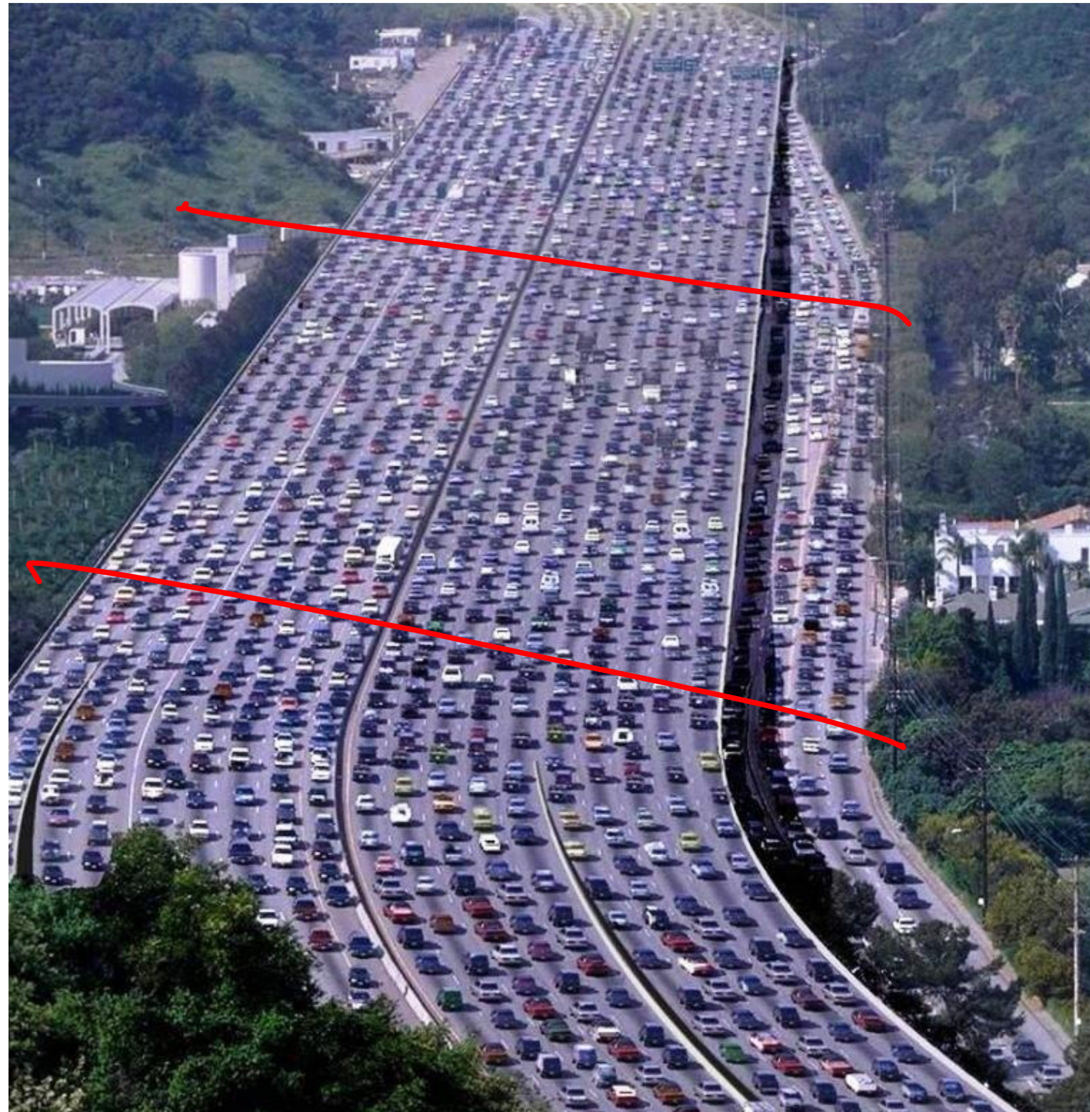
$u(\rho) = 0$

$u(\rho) = 1$; 85 mph

$u(\rho) = 1 - \rho$

What's $F(\rho)$

$= \rho \cdot u = \rho - \rho^2$



$$\frac{\partial \rho}{\partial t} + \frac{\partial F(\rho)}{\partial x} = 0 \quad \text{where } F(\rho) = \rho - \rho^2$$

$$c = 1 - 2\rho = \frac{dF}{d\rho}$$