Course Introduction

Probability, Statistics and Quality Loss

HW#1 Presentations

Background

- SDM split-summer format
  - Heavily front-loaded
  - Systems perspective
  - Concern with scaling
- Product development track, CIPD
  - Place in wider context of product realization
- Joint 16 (Aero/Astro) and 2 (Mech E)

Course Learning Objectives

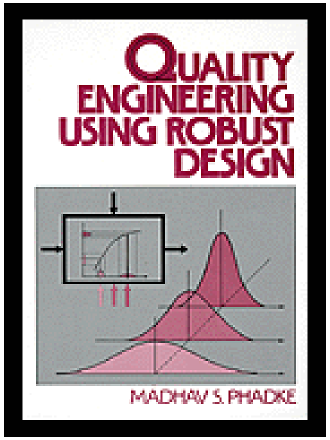
- Formulate measures of performance
- Synthesize and select design concepts
- Identify noise factors
- Estimate the robustness
- Reduce the effects of noise
- Select rational tolerances
- Understand the context of RD in the end-to-end business process of product realization.

Instructors

- Dan Frey, Aero/Astro
- Don Clausing, Xerox Fellow
- Joe Saleh, TA
- Skip Crevelling, Guest from RIT
- Dave Miller, Guest from MIT Aero/Astro
- Others ...

Primary Text

Phadke, Madhav S., *Quality Engineering Using Robust Design*. Prentice Hall, 1989.



Computer Hardware & Software

- Required
  - Access to a PC running Windows 95 or NT
  - Office 95 or later
  - Reasonable proficiency with Excel
- Provided
  - MathCad 7 Professional (for duration of course only)

Learning Approach

- Constructivism (Jean Piaget)
  - Knowledge is not simply *transmitted*
  - Knowledge is actively *constructed* in the mind of the learner (*critical* thought is required)
- Constructionism (Seymour Papert)
  - People learn with particular effectiveness when they are engaged in building things, writing software, etc.
  - http://el.www.media.mit.edu/groups/el/

Format of a Typical Session

- ~~**Lecture**~~ Well, almost
- Reading assignment
- Quiz
- Labs, case discussions, design projects
- Homework

Grading

- Breakdown
  - 40% Term project
  - 30% Final exam
  - 20% Homework (~15 assignments)
  - 10% Quizzes (~15 quizzes)
- No curve anticipated

Grading Standards

Grade Range	Letter Equivalent	Meaning
97-100	A+	Exceptionally good performance demonstrating superior understanding of the subject matter.
94-96	A	
90-93	A-	
87-90	B+	Good performance demonstrating capacity to use appropriate concepts, a good understanding of the subject matter and ability to handle problems.
84-86	B	
80-83	B-	
77-80	C+	Adequate performance demonstrating an adequate understanding of the subject matter, an ability to handle relatively simple problems, and adequate preparation.
74-76	C	
70-73	C-	
67-70	D+	Minimally acceptable performance demonstrating at least partial familiarity with the subject matter and some capacity to deal with relatively simple problems.
64-66	D	
60-63	D-	
<60	F	Unacceptable performance.

Reading Assignment

- Taguchi and Clausing, "Robust Quality"
- Major Points
  - Quality loss functions (Lecture 1)
  - Overall context of RD (Lecture 2)
  - Orthogonal array based experiments (Lecture 3)
  - Two-step optimization for robustness
- Questions?

Learning Objectives

- Review some fundamentals of probability and statistics
- Introduce the quality loss function
- Tie the two together
- Discuss in the context of engineering problems

Probability Definitions

- Sample space - List all possible outcomes of an experiment
  - Finest grained
  - Mutually exclusive
  - Collectively exhaustive
- Event - A collection of points or areas in the sample space

Probability Measure

- Axioms
  - For any event A, $P(A) \geq 0$
  - $P(U)=1$
  - If $AB=\phi$, then $P(A+B)=\mathrm{P}(A)+\mathrm{P}(B)$

Discrete Random Variables

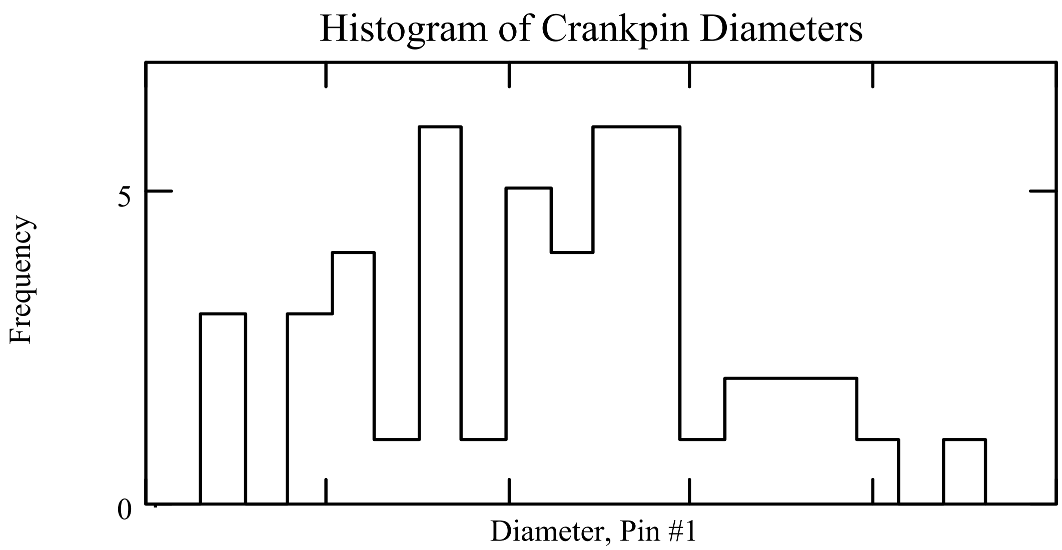
- A random variable that can assume any of a set of discrete values
- Probability mass function
  - $p_x(x_o)$ = probability that the random variable x will take the value x_o
  - Let's build a pmf for one of the examples
- Event probabilities computed by summation

Continuous Random Variables

- Can take values anywhere within continuous ranges
- Probability density function
  - $P\{a < x \le b\} = \int_a^b f_x(x)\mathrm{d}x$
  - $0 \le f_x(x)$ for all x
  - $\int_{-\infty}^{\infty} f_x(x)\mathrm{d}x = 1$

Histograms

- A graph of continuous data
- Approximates a pdf in the limit of large *n*



Measures of Central Tendency

- Expected value $$E(g(x)) = \int_a^b g(x) f_x(x) \mathrm{d}x$$
- Mean $$\mu = E(x)$$
- Arithmetic average $$\frac{1}{n}\sum_{i=1}^{n} x_i$$

Measures of Dispersion

- Variance $$VAR(x) = \sigma^2 = E((x - E(x))^2)$$
- Standard deviation $$\sigma = \sqrt{E((x - E(x))^2)}$$
- Sample variance $$S^2 = \frac{1}{n-1}\sum_{i-1}^{n}(x_i - \overline{x})^2$$
- n^{th} central moment $$E((x - E(x))^n)$$
- n^{th} moment about m $$E((x - m)^n)$$

Sums of Random Variables

- Average of the sum is the sum of the average (regardless of distribution and independence)

$$E(x+y)=E(x)+E(y)$$

- Variance also sums iff independent

$$\sigma^2(x+y)=\sigma(x)^2+\sigma(y)^2$$

- This is the origin of the RSS rule
  - Beware of the independence restriction!

Central Limit Theorem

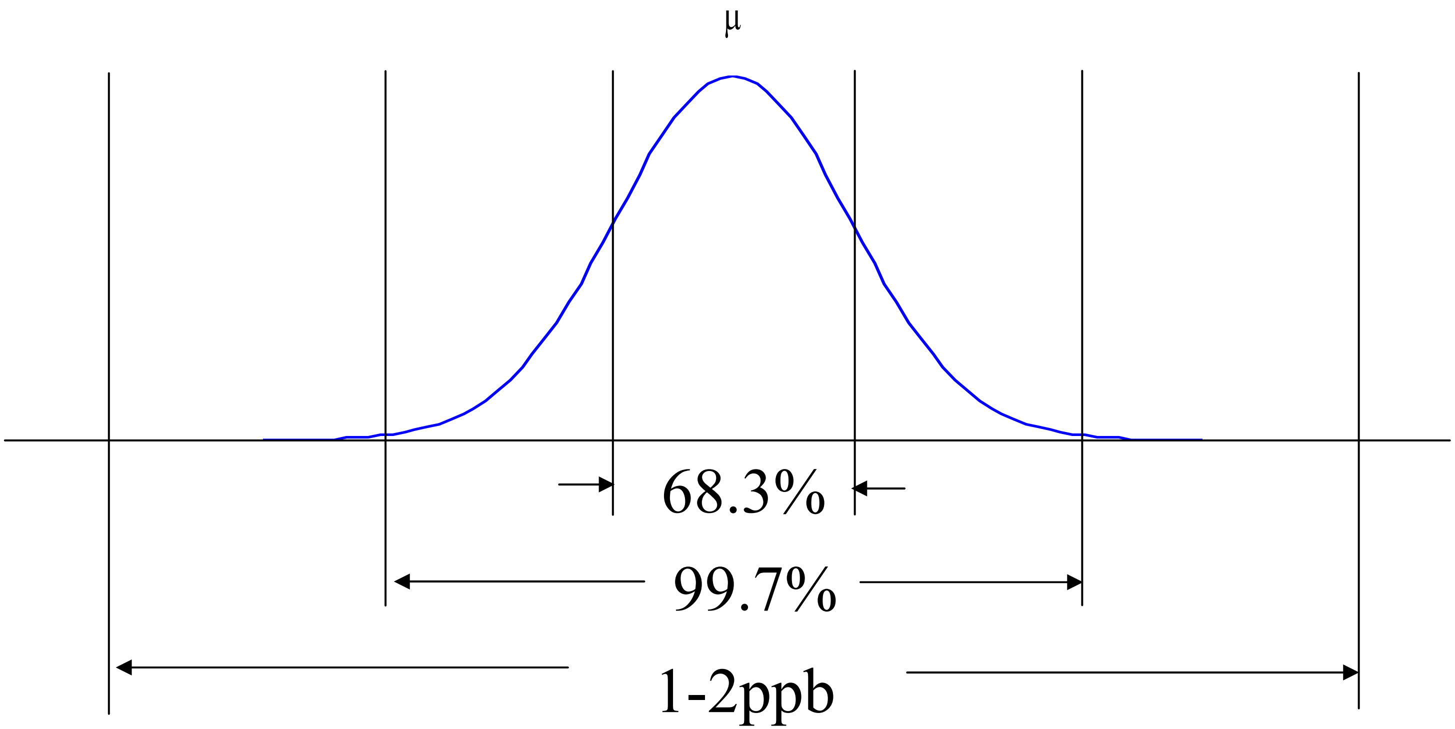
The mean of a sequence of n iid random variables with

- Finite μ
- $E\left(\left|x_i - E(x_i)\right|^{2+\delta}\right) < \infty \quad \delta > 0$

approximates a normal distribution in the limit of a large n.

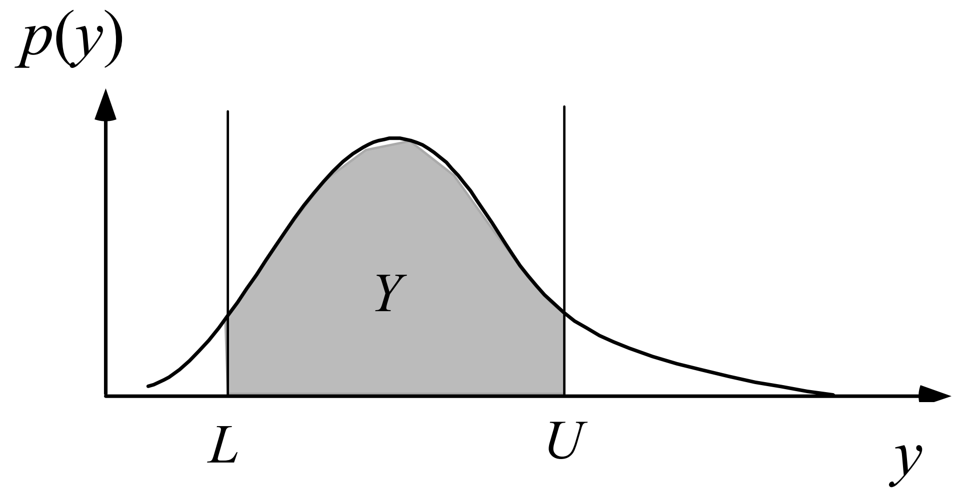
Normal Distribution

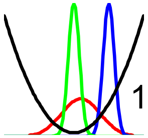
$$f_x(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



Engineering Tolerances

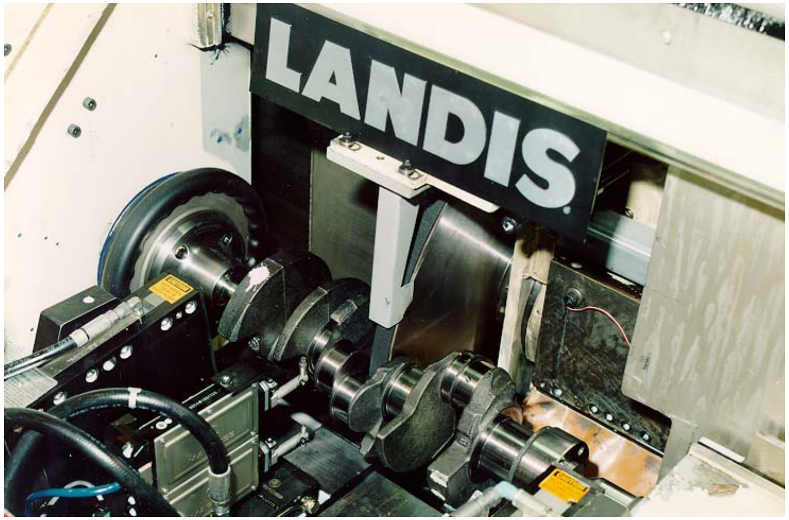
- Tolerance --The total amount by which a specified dimension is *permitted to vary* (ANSI Y14.5M)
- Every component within spec adds to the yield (*Y*)





Crankshafts

- What does a crankshaft do?
- How would you define the tolerances?
- How does variation affect performance?



GD&T Symbols

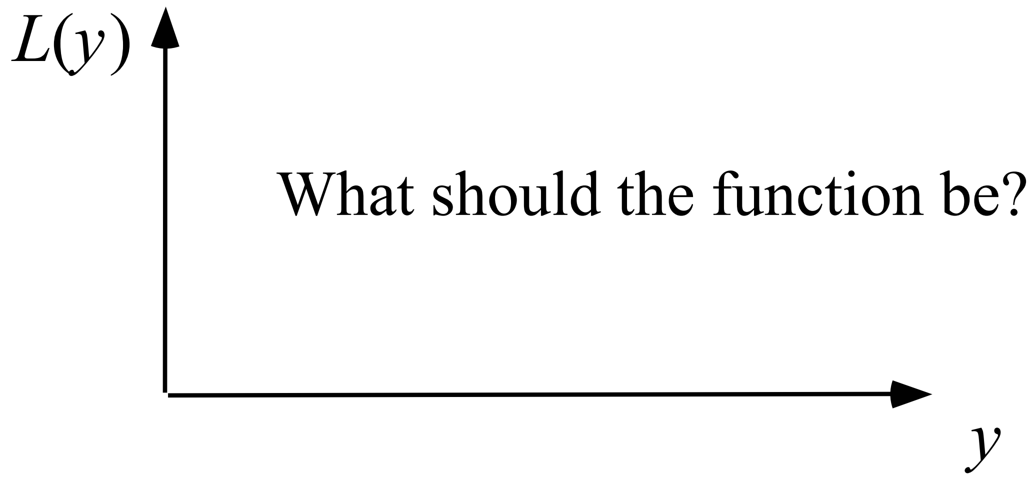
	TYPE OF TOLERANCE	CHARACTERISTIC	SYMBOL	SEE:
FOR INDIVIDUAL FEATURES	FORM	STRAIGHTNESS	⏤	6.4.1
		FLATNESS	⏥	6.4.2
		CIRCULARITY (ROUNDNESS)	○	6.4.3
		CYLINDRICITY	⌭	6.4.4
FOR INDIVIDUAL OR RELATED FEATURES	PROFILE	PROFILE OF A LINE	⌒	6.5.2 (b)
		PROFILE OF A SURFACE	⌓	6.5.2 (a)
FOR RELATED FEATURES	ORIENTATION	ANGULARITY	∠	6.6.2
		PERPENDICULARITY	⊥	6.6.4
		PARALLELISM	//	6.6.3
	LOCATION	POSITION	⌖	5.2
		CONCENTRICITY	◎	5.11.3
	RUNOUT	CIRCULAR RUNOUT	↗ *	6.7.2.1
		TOTAL RUNOUT	⌰ *	6.7.2.2

*Arrowhead(s) may be filled in.

FIG. 68 GEOMETRIC CHARACTERISTIC SYMBOLS

Loss Function Concept

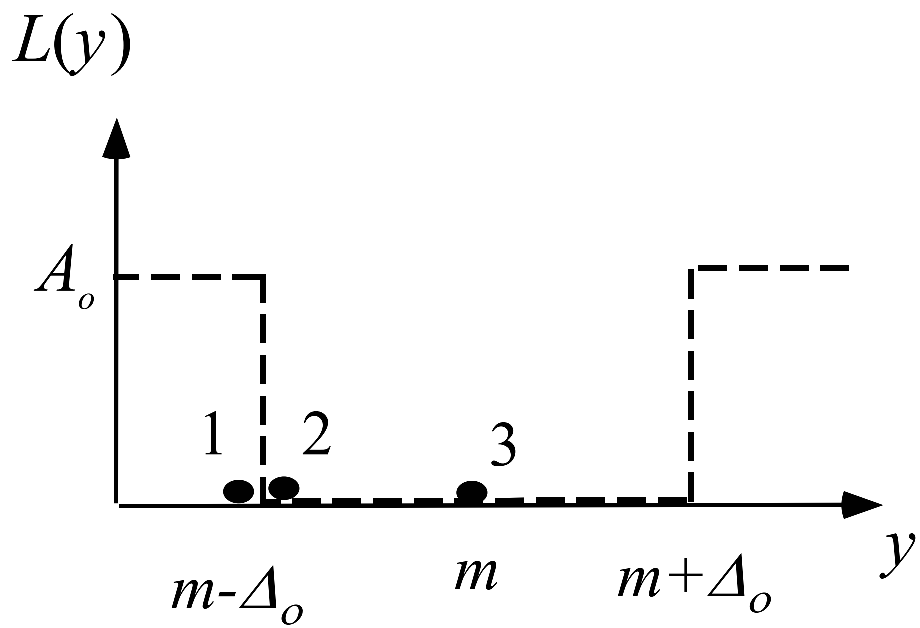
- *Quantify* the economic consequences of performance degradation due to variation

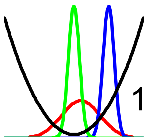


Fraction Defective Fallacy

- ANSI seems to imply a "goalpost" mentality
- But, what is the difference between
  – 1 and 2?
  – 2 and 3?

  Isn't a continuous function more appropriate?

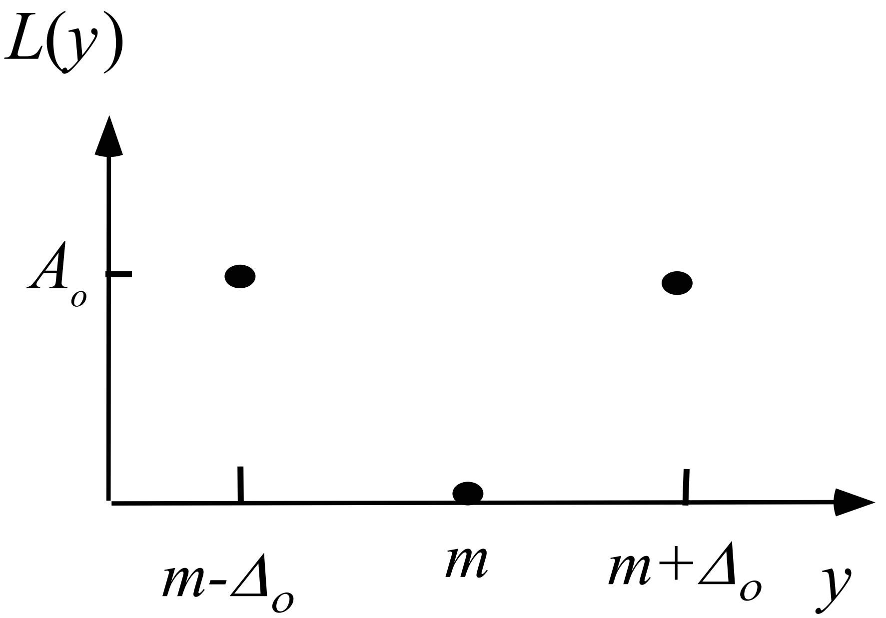




A Generic Loss Function

- Desired properties
  - Zero at nominal value
  - Equal to cost at specification limit
  - C1 continuous
- Taylor series

$$f(x) \approx \sum_{n=0}^{\infty} \frac{1}{n!}(x-a)^n f^{(n)}(a)$$



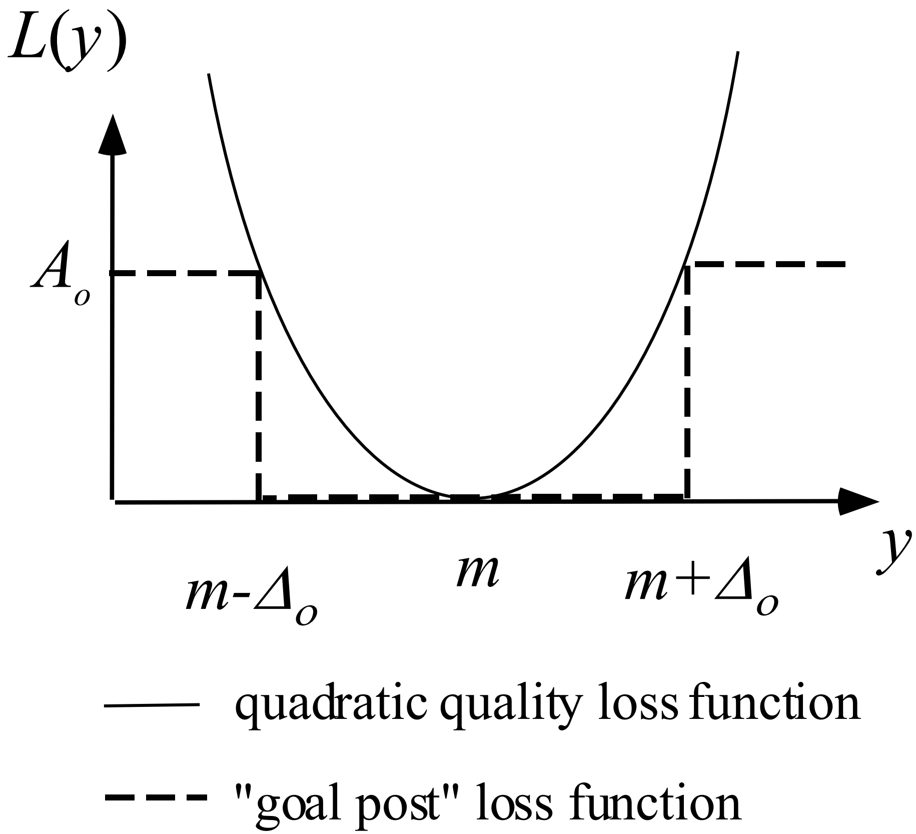
Nominal-the-best

- Defined as

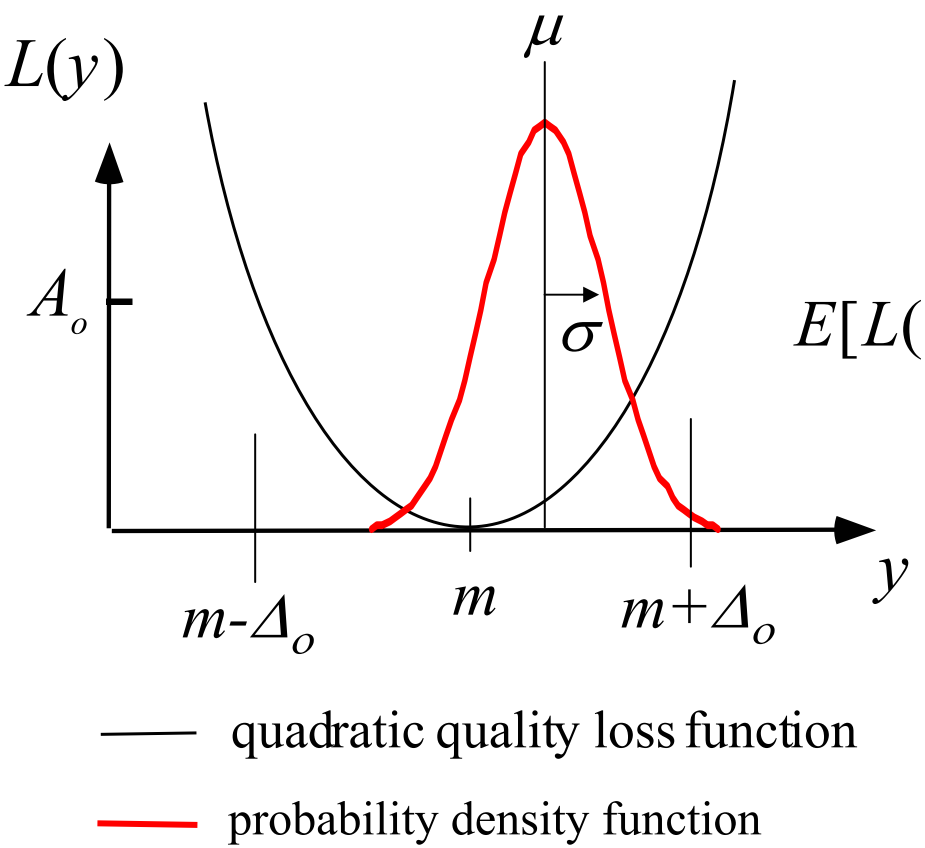
$$L(y) = \frac{A_o}{\Delta_o^{\ 2}}(y-m)^2$$

- Average loss is proportional to the 2nd moment about *m*

(HW#2 prob. 1)



Average Quality Loss



$$E[L(y)] = \frac{A_o}{\Delta_o^{\;2}}\left[\sigma^2 + (\mu - m)^2\right]$$

Other Loss Functions

- Smaller the better (HW#3a)

$$L(y)=\frac{A_o}{\Delta_o^{\;2}}y^2$$

- Larger-the better (HW#3b)

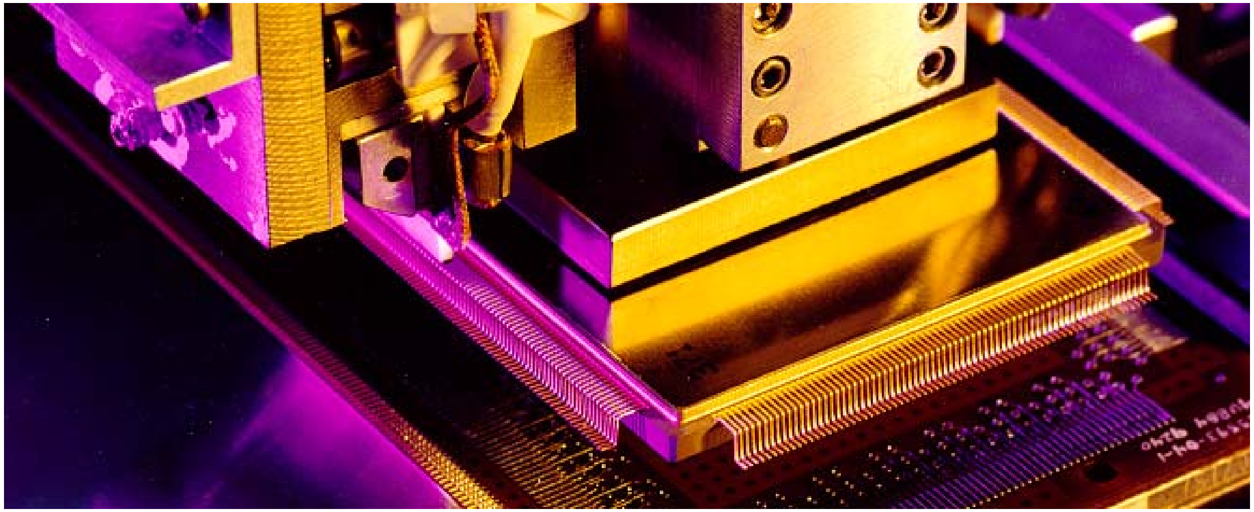
$$L(y)=A_o\Delta_o^{\;2}\frac{1}{y^2}$$

- Asymmetric (HW#2f&#3c)

$$L(y)=\begin{cases}\dfrac{A_o}{\Delta_{Upper}^{\;2}}(y-m)^2 \text{ if } y>m\\[2ex]\dfrac{A_o}{\Delta_{Lower}^{\;2}}(y-m)^2 \text{ if } y\le m\end{cases}$$

Printed Wiring Boards

- What does the second level connection do?
- How would you define the tolerances?
- How does variation affect performance?



Next Steps

- Load Mathcad (if you wish)
- Optional Mathcad tutoring session
  - 1hour Session
- Complete Homework #2
- Read Phadke ch. 1 & 2 and session #2 notes
- Next lecture
  - Don Clausing, Context of RD in PD