16.881 – Robust System Design

Solution to Quiz #7
Design of Dynamic Systems

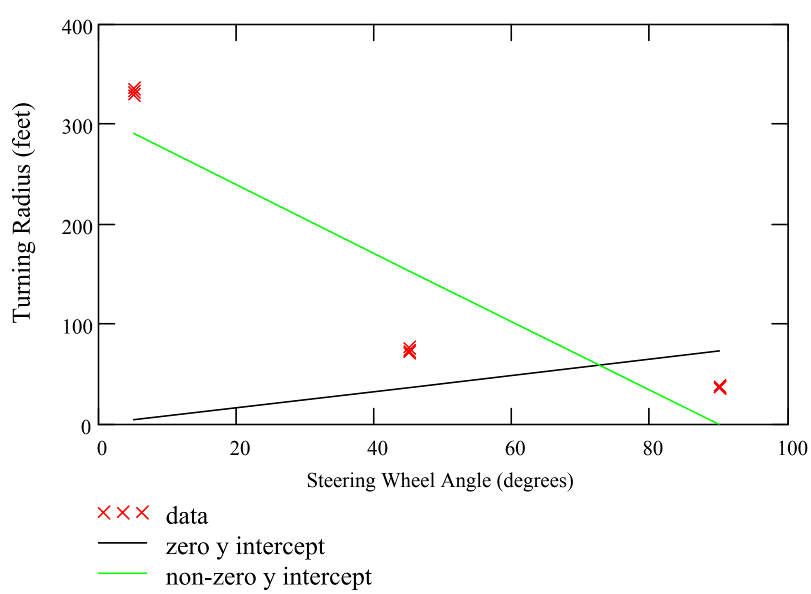
You are designing a light truck steering system. Customer surveys indicate that drivers prefer a consistent response of the steering system under a wide range of driving conditions (speeds, road surfaces, weather conditions, etc). The following data were collected to evaluate a baseline design:

	Turning radius (feet)		
Steering wheel angle (degrees)	Noise condition level 1	Noise condition level 2	Noise condition level 3
5	330	336	333
45	72	74	77
90	37	38	36

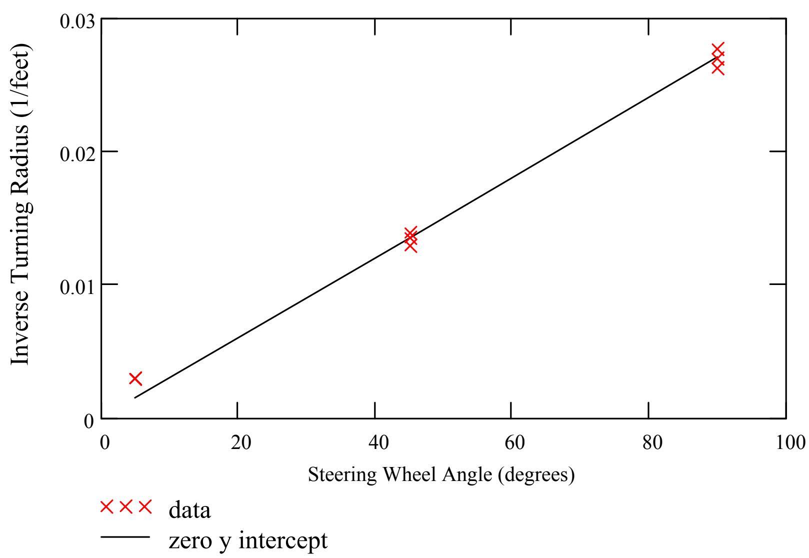
1) What problems might you run into if you performed a continuous-continuous dynamic signal to noise analysis using steering wheel angle as the signal and turning radius as the response? What would you choose as the signal factor and response of the system to make a continuous-continuous dynamic signal to noise analysis feasible? Sketch a graph (or a few graphs) to support your choice.

A sketch of the data shows that there is very little variance due to the induced noise as compared to the variation caused by the signal factor. In other words, this system seems to be very repeatable. However, if the turning radius is taken as the response, the relationship is highly non-linear and has a non-zero y intercept.

If you use the Taguchi definition of β which is essentially a regression with a zero y intercept, you get a very poor fit to the data. Error variance will be extremely high. You can get a better fit by allowing for a non-zero y intercept. However, the clear non-linearity in the relationship will still contribute to error variance. It is clearly not error due to noise, so perhaps another approach should be taken.



One possibility is to change the definition of the response. The shape of the original data in the sketch should suggest to you an inverse relationship. Or, you can make a sketch of a car performing a turn and you'll see that the angle of the front tires should be proportional to the *inverse* of the turning radius. Making this change, you get very nicely linear behavior with a zero y intercept. Now, a Taguchi style continuous-continuous signal to noise ratio analysis should be able to help you reduce sensitivity to the induced noises without being obscured by non-linearity.



Some students performed other transformations to improve the linearity of the response such as taking the log of R or log of angle. These can work adequately as well. However, a choice based on the physics of the process is more likely to give good results.

Some students chose angle of the tire as the response. That is a creative idea, however, the goal is to become robust to noises like road conditions, weather, velocity of the car, etc. Since the angle of the tire will not likely be affected by these noises, this strategy is not likely to work. I'd say the response you chose was incomplete in that it doesn't fully reflect the needs of the customer as defined in the problem statement.

2) **Estimate** the slope of the least squares regression line (β) between your chosen signal and response.

With 1/R as the response, the slope is about (1/37ft)/90deg=0.0003 (1/ft*deg).

With R as the response and a non-zero intercept, the slope is (37ft-333ft)/(90deg-5deg)= -3.5 (ft/deg).

Any reasonable estimate based on a sketch would earn you full credit. Just writing the formula got you partial credit. Some students spent too much time trying to work through the entire formula to get an exact answer. I wanted you to think about the *concept* of β which is just the slope of a line fit to the response. This allows for a very quick estimate of its value.

3) **Estimate** the error variance (σ_e^2) for your chosen signal and response.

Looking at the sketch on the previous page, the data seem to deviate from the regression line by about 0.001 or so. That would make the error variance about 10E-6.

For this problem, any estimate that reflected your understanding that the error variance is the average of the squared deviation from the regression line was accepted for full credit. Writing down the formula got you some credit, but I was really looking for a feel of the *concept*. This understanding should allow you to make a rough estimate without going through all the mechanics of the calculation. These estimates are important even when you DO go through the mechanics of the equation because they serve as a check against miscalculation.

4) By parameter design, the signal to noise ratio of the system was increased by 12 dB while the error variance remained constant. What is your new estimate of β?

You might remember the 6dB rule from quiz #4. A 6dB gain increases the response to standard deviation ratio by a factor of two. It should be clear a 12dB gain is like two consecutive gains of 6dB which will increase β by a factor of four.

Alternately, you could right the equation for C-C S/N ratio and perform the necessary manipulations.

5) Give an example of a scaling factor that can be used to return the value of β back to its original value prior to the parameter design.

I wanted you to think about the *concept* of a scaling factor *in the context of dynamic problems*. You want a scaling factor to change β, the slope of the response with changes in signal, while not significantly affecting the S/N ratio.

In this problem, you were to imagine a parameter design problem wherein you seek to make a truck turn with a constant radius despite noises such as road conditions, weather, and velocity. You might imagine that control parameters like tire radius, tire width, suspension stiffness, steering linkage geometry, etc might allow you to improve the repeatability of steering performance. However, some of these parameters (particularly steering linkage geometry) might cause the sensitivity of radius to wheel angle to go off target. In this case, the diameter of the pinion in rack and pinion steering or the gear ratios in a worm gear steering box would be good scaling factors. I'd accept any answer that reflected an understanding of what scaling factors should do in a dynamic problem.