Darrell Cain, Zack Anderson

Below is an analysis of the parameter to be maximized along with the constraints on the system. Based on these equations and the fixed parameters it is possible to calculate the range of efficiencies of the overall rectenna panel based upon the efficiency of the climber. This gives the mathematical foundation to build a Matlab simulation which would calculate the range of efficiencies the rectenna would need to fall in based upon the speed of the climber, the size of the dish, the area of antennas and various other parameters.

Important parameter: $\varepsilon = \frac{P_{out}}{P_{in}}$

Estimate Efficiency: 70 percent

Constraints:
Input: Microwave Field
Size of Dish: 2 meter diameter
Base generator: 800 W 5.8 GHz generator
Beam: Circularly polarized and directional

The following observations can be made about Rectennas

- Two things cause microwave signals to weaken
  - Spreading over a larger sphere of influence
  - Attenuation due to the atmosphere
- Necessary to determine how the field will change in saturation based upon height
  - Since beam is directional, the beam does not spread and remains constant (same amount of flux)
  - Attenuation is minimal[1]
- Area of dish = A = pi * 1^2 = 3.14 m^2;
- ρ_μ = 800/3.14 = 254.7 W/m^3
- At first glance, $P_{in} = \rho_\mu$ * A = 800 Watts
- However
  - Specs of current design base unit = 46.915 mm X 47.26 mm
  - Area of current design base unit = .00221 m^2
  - Area of antenna's in a base unit = .000581 m^2
  - It's possible to calculate the area of antenna which is exposed to the radiation
    - Ratio of Area of Antenna Design Base to Area of Current Design = .2628
    - Useable area = $A_{useable}$ = Ratio * A = .825 m^2
    - Therefore $P_{in} = \rho_\mu * A_{useable}$ = 210.25 Watts

[1] http://www.profc.udec.cl/~gabriel/tutoriales/rsnote/cp3/cp3-2.htm Figure 3.2.1

- With a $\begin{matrix} \varepsilon = .7 \\ P_{out} = 147.18 \end{matrix}$
  - Moving at 2 m/s with a 25 kg structure total work done by the climber = .5 * 25 * 2^2 = 50 Watts
  - Therefore if everything works correctly the multiple of the other efficiencies must be greater than .3397
- Brainstormed Solutions
  - Multiple layers of rectenna, aligned so as to allow the climber to receive microwave on each level

- The following observation can be made about the constraints imposed by the capacitors on board
  - $P_c \le .25 * m_{\text{climber}} g h_{\text{total}}$
  - $m_{\text{climber}} g h_{\text{total}} = .25 * 25 * 9.8 * 100 = 6125J$
- Power of capacitors can also be represented by
  - $P_c = \int_0^t P_{out} - W_{\text{climber}} / \varepsilon_{\text{climber}} dt$
  - From t = 0 to t = 50
  - Therefore
    - $0 \le \int_0^t P_{out} - W_{\text{climber}} / \varepsilon_{\text{climber}} dt \le 6125J$
  - If we use
    - v=constant = 2 m/s
$W_{\text{climber}} = 50J$
    - $P_{out} = 147.18J$
    - In simplified version, neither is time dependent, therefore max is at when t = 50
    - $0 \le 7359J - 2500J / \varepsilon_{\text{climber}} \le 6125J$
$-7359 \le -2500 / \varepsilon_{\text{climber}} \le -1234$
$.4936 \le 1 / \varepsilon_{\text{climber}} \le 2.9436$
$.3397 \le \varepsilon_{\text{climber}} \le 2.0259$
with physical constraints
$.3397 \le \varepsilon_{\text{climber}} \le 1$
    - This was found earlier by examining a very simple equation
  - This is a simplified version. If we reintroduce the rectenna efficiency as a variable into the problem things become interesting
    - $P_{out} = 210.25 * \varepsilon_{rectenna}$
    - Therefore
      - $0 \le 10512.5J * \varepsilon_{\text{rectenna}} - 2500J / \varepsilon_{\text{climber}} \le 6125J$
    - Why this is important?

- By allowing both $\varepsilon_{\text{rectenna}}, \varepsilon_{\text{climber}}$ to be unfixed variables, you get an equation where one is driven by the other. More specifically by solving the following equations it can be shown

constraints

$0 \le \varepsilon_{\text{rectenna}} \le 1; 0 \le \varepsilon_{\text{climber}} \le 1$

$-10512.5J * \varepsilon_{\text{rectenna}} \le -2500J / \varepsilon_{\text{climber}} \le 6125J - 10512.5J * \varepsilon_{\text{rectenna}}$

$10512.5J * \varepsilon_{\text{rectenna}} - 6125J \le 2500J / \varepsilon_{\text{climber}} \le 10512.5J * \varepsilon_{\text{rectenna}}$

therefore

$$\frac{1}{4.205 * \varepsilon_{\text{rectenna}}} \le \varepsilon_{\text{climber}} \le \frac{1}{4.205 * \varepsilon_{\text{rectenna}} - 2.45}$$

for a $\varepsilon_{\text{rectenna}}$ that varies,

maximum efficiency of rectenna at

$\varepsilon_{\text{rectenna}} = 1$

$.2378 \le \varepsilon_{\text{climber}} \le .5698$

minimum efficiency of rectenna at

$\varepsilon_{\text{rectenna}} = .5826$

$.4082 \le \varepsilon_{\text{climber}} \le \infty; .4082 \le \varepsilon_{\text{climber}} \le 1$

By solving

$$\frac{1}{4.205 * \varepsilon_{\text{rectenna}} - 2.45} = 1$$

the point at which top possible climber efficiency reaches 1 is found

$\varepsilon_{\text{rectenna}} = .8205$

$.2898 \le \varepsilon_{\text{climber}} \le 1$

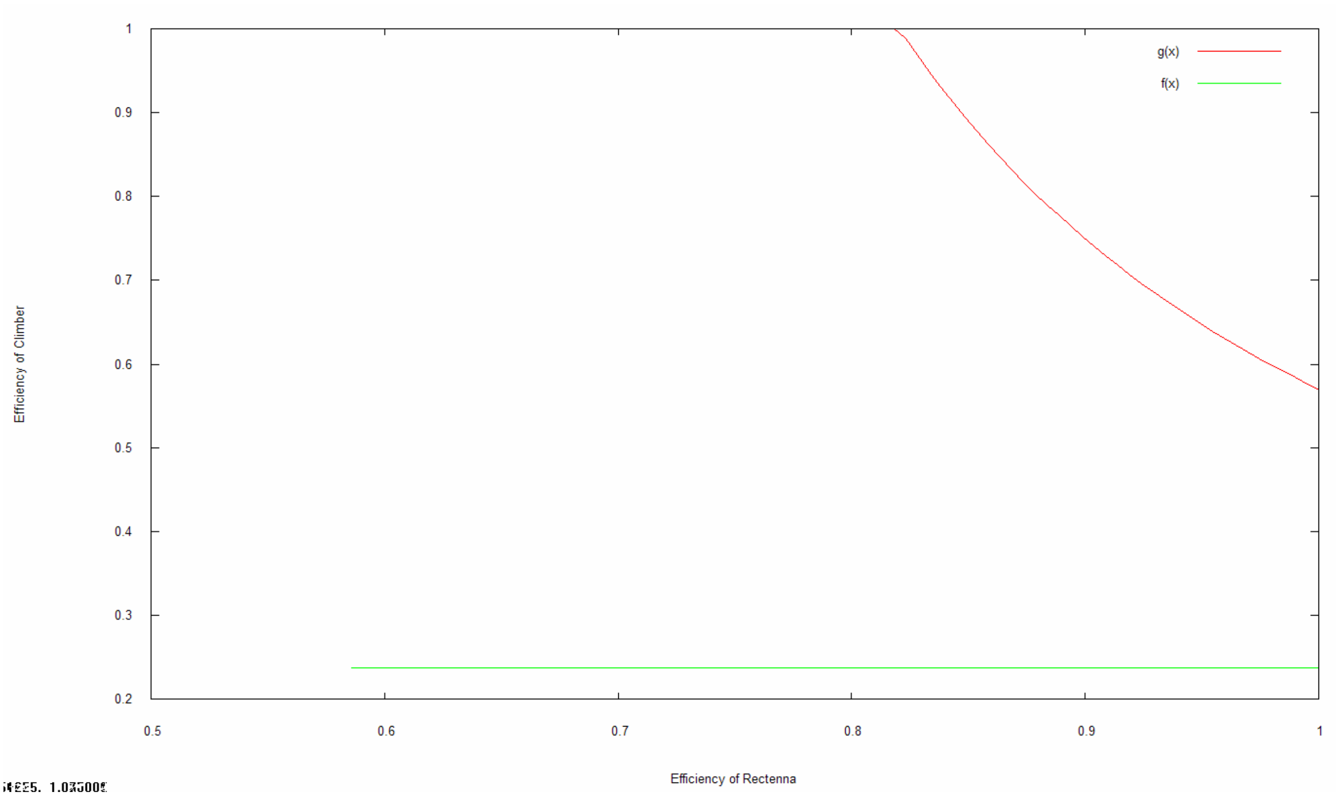
Finally

$$\varepsilon_{\text{climber}} = \frac{1}{4.205 * \varepsilon_{\text{rectenna}}}$$

$$\varepsilon_{\text{climber}} = \frac{1}{4.205 * \varepsilon_{\text{rectenna}} - 2.45}$$

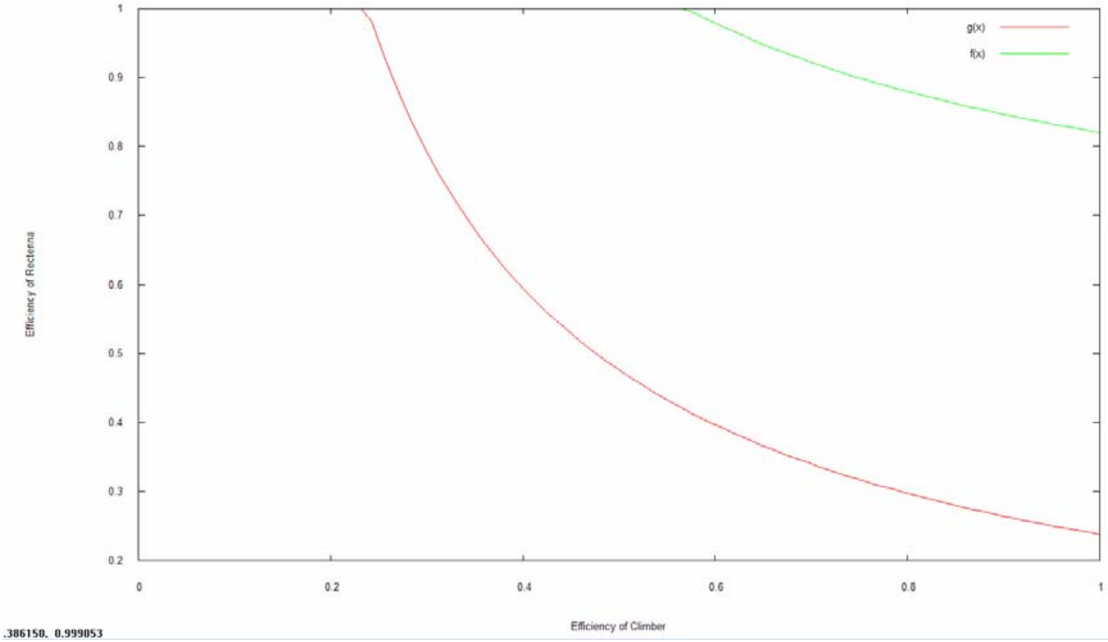
which gives the following plot

Range of Climber Efficiencies as Driven by Rectenna Efficiency



- Green = lower bound, Red= upper bound
- More specifically this shows
  - o that the efficiency of the rectenna cannot be below .5826 percent
  - o that for a given rectenna efficiency, the climber must fit within a given range of efficiencies
- This analysis, while numerically interesting can be done in reverse, with efficiency of the motor being the driving variable and the efficiency of the rectenna being the variable with imposed limits.
- $2500J / \varepsilon_{\text{climber}} \le 10512.5J * \varepsilon_{\text{rectenna}} \le 2500J / \varepsilon_{\text{climber}} + 6125J$
  $.2378 / \varepsilon_{\text{climber}} \le \varepsilon_{\text{rectenna}} \le .2378 / \varepsilon_{\text{climber}} + .5826$

Range of Rectenna Efficiencies as Driven by Climber Efficiency



- This graph demonstrates the range of values a rectenna must fall in to win the competition based on the efficiency of the climber. (assuming a constant speed of 2 m/s and the current electricmagnetic field)
- The upper cap is put on by the capacitors, the lower cap by the speed of the climber
- How would you go about changing this graph?
    - Increasing the speed of the climber is the easiest (physically) way to change rectenna restrictions.
    - Changes the time of travel which changes the amount of energy the rectenna can receive at any one time
- Currently outside the scope of this analysis, factors into the scoring equation which is the optimization that is most important.

In conclusion, using the above graph it is possible to get a range of values for the needed efficiency of the rectenna array based upon the efficiency of the climber (which already exists)
This means once rectenna design is done and tested it will be possible to know if it is sufficient to power the climber. It should be noted here that this assumes only one layer of panels and a constant speed of 2 m/s. The actual constraints would be tighter due to the energy needed to run the brakes, start the motors and lost through the capacitor array

It was decided that a CAD model simulation of the panel holding the rectenna was not necessarily useful, as the actual interfaces with the climber are not completely defined (there are potential tarps, ribbons, connection points and stresses that are not known yet)

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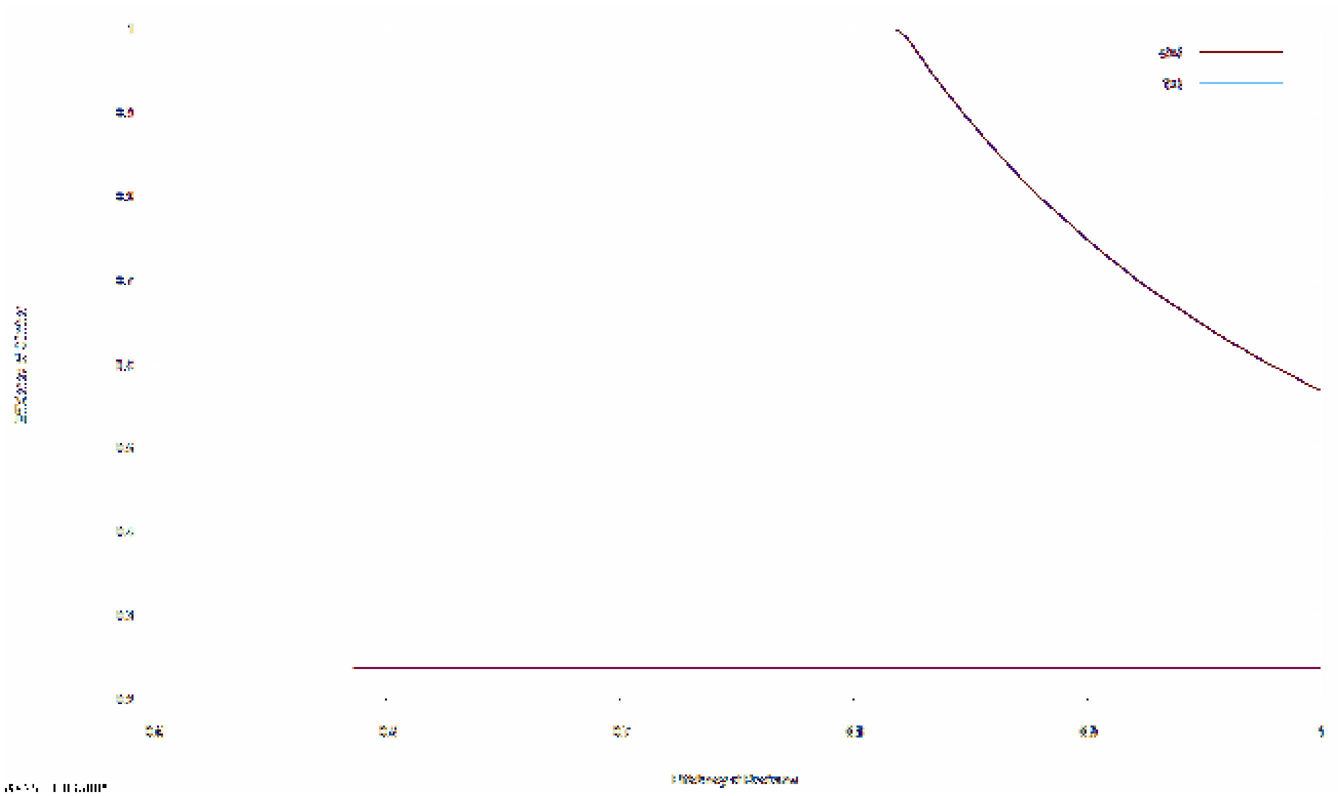

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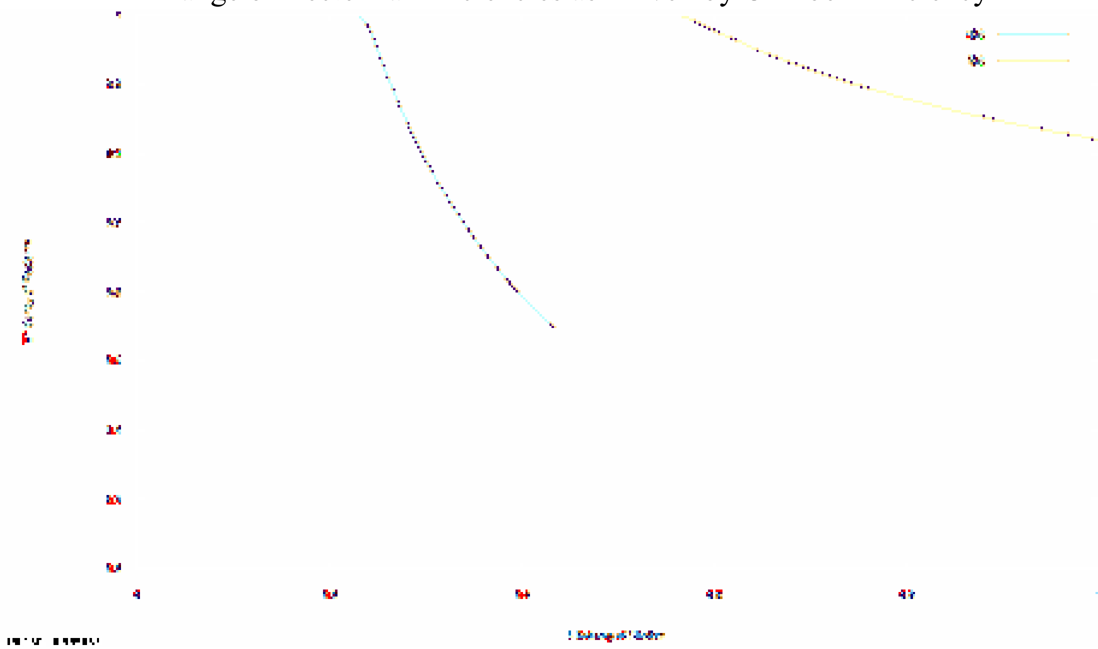

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