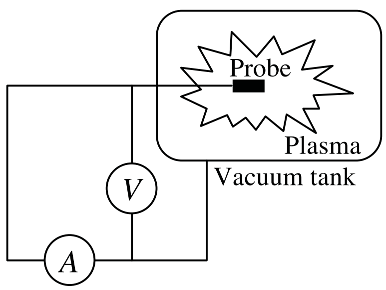
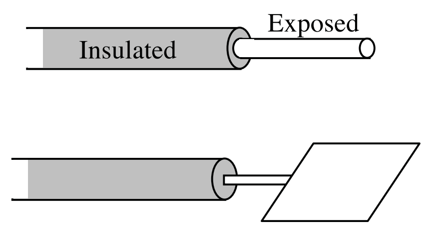
Electrostatic Probes

Among plasma diagnostics tools, electrostatic probes provide with a relatively simple way to obtain measurements of plasma properties. Under some peculiar conditions, probe theory is also straightforward. However, when such conditions are not met, or very detailed and specific information is desirable, then the interpretation of measurements becomes a difficult task. Because of this, probe development can still be considered as a prolific research area.

Langmuir developed the most simple of all examples in probe theory in 1924. Devices like the one he studied are referred to as Langmuir probes. They usually consist of a small electrode with an exposed conductive area A_p. This electrode is biased to some potential V with respect to a point in the system (for example, the vacuum tank where experiments are made). The probe is then immersed in the plasma and collected current I is measured through an amperimeter; in this way $I-V$ characteristics are generated and later analyzed to extract some plasma properties.



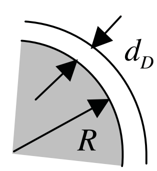
Typical connection for a single probe



Two types of probes

Langmuir probe theory assumptions:

1. The geometry of the probe is planar, i.e., the physical dimensions are much larger than the Debye length (locally).



$$\frac{d_D}{R} << 1$$

2. The plasma is collisionless
   $\lambda_{mfp} >> d_D$ and $\lambda_{mfp} >> R$

3. Quiescent plasma

Because of condition (1) we make use of our previous results concerning particle flux to a biased wall. The electron current flux to a Langmuir probe is

$$j_e = -en_e \begin{cases} e^{-\left(\frac{T_e+T_i}{2T_e}\right)} \sqrt{\dfrac{k(T_e+T_i)}{m_e}} & \text{for} \quad \phi \geq 0 \\ \dfrac{\bar{c}_e}{4} e^{\frac{e\phi}{kT_e}} & \text{for} \quad \phi < 0 \end{cases}$$

and for ions

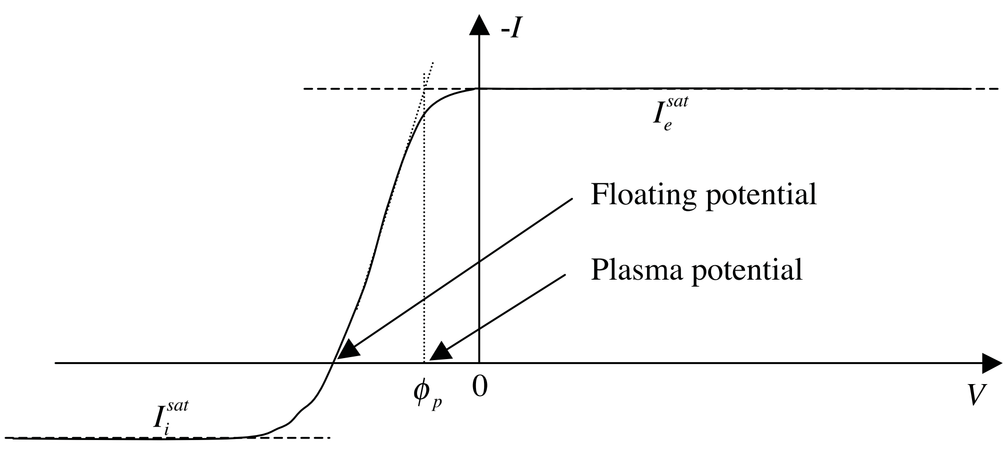
$$j_i = en_i \begin{cases} \frac{\bar{c}_i}{4} e^{-\frac{e\phi}{kT_i}} & \text{for} \quad \phi > 0 \\ e^{-\left(\frac{T_e+T_i}{2T_e}\right)} \sqrt{\frac{k(T_e+T_i)}{m_i}} & \text{for} \quad \phi \le 0 \end{cases}$$

The total current collected by the probe is then $I = jA_p = (j_e + j_i)A_p$

$$I = en_e A_p \left(\frac{\bar{c}_i}{4} e^{-\frac{e\phi}{kT_i}} - e^{-\left(\frac{T_e+T_i}{2T_e}\right)} \sqrt{\frac{k(T_e+T_i)}{m_e}} \right) \quad \text{for} \quad \phi > 0$$

$$I = en_e A_p \left(e^{-\left(\frac{T_e+T_i}{2T_e}\right)} \sqrt{\frac{k(T_e+T_i)}{m_i}} - \frac{\bar{c}_e}{4} e^{\frac{e\phi}{kT_e}} \right) \quad \text{for} \quad \phi < 0$$

where quasineutrality was assumed. We observe that for very large positive potentials the ion current is retarded to a point where it is practically stopped. In this situation, the current collected reaches the constant value of $I_e^{sat} = -en_e A_p e^{-\left(\frac{T_e+T_i}{2T_e}\right)} \sqrt{\frac{k(T_e+T_i)}{m_e}}$, the electron saturation current. The opposite is true for large negative potentials. In this case the electronic component is canceled and what remains is the ion saturation current $I_i^{sat} = en_e A_p e^{-\left(\frac{T_e+T_i}{2T_e}\right)} \sqrt{\frac{k(T_e+T_i)}{m_i}}$. A typical $I-V$ characteristic could look similar to:



In general we note that the electronic current is much larger than the ion current. This can be seen after calculating the ratio

$$\left|\frac{I_e^{sat}}{I_i^{sat}}\right| = \sqrt{\frac{m_i}{m_e}} \geq 43$$

The floating potential ϕ_{fp} is defined as the voltage where the collected current is identically zero. This is equivalent to have a non-biased probe immersed in the plasma. In this configuration, more electrons will arrive to the surface than ions, charging it with a negative potential. Because of this, we solve $I = 0$ for the case when $\phi < 0$

$$\phi_{fp} = \phi\big|_{I=0} = \frac{kT_e}{2e}\left[\ln\left(2\pi\frac{m_e}{m_i}\left(\frac{T_i+T_e}{T_e}\right)\right) - \frac{(T_i+T_e)}{T_e}\right]$$

Experimentally, one would measure the floating potential with respect to, for example, the tank, V_{fp}. This last expression gives the floating potential with respect to the plasma potential ϕ_p; they are simply related by $\phi_{fp} = V_{fp} - \phi_p$.

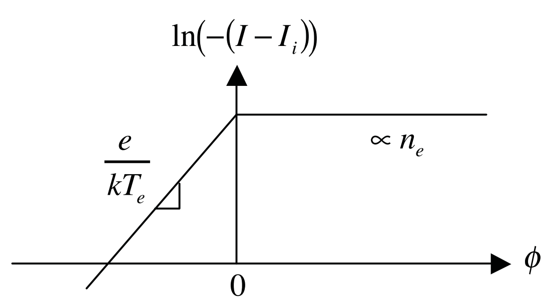
The plasma potential ϕ_p is defined as the probe potential in which the charges inside the sheath change from a positive cloud to a negative one. A positive cloud exists as long as ions reach the probe. Coming from the ion saturation condition, as the probe potential is increased, less and less ions reach the surface (but more electrons arrive, increasing the amount of negative current measured) and therefore the cloud gradually losses its positive charge. At some point, as the probe potential is increased further, the cloud becomes negative since the thermal motion of electrons start to dominate, even if some ions still reach the probe. Keep in mind that all the equations given above, where ϕ appears, are given with respect to the plasma potential.

One could subtract the ion current from the total current in the experimental measurements and obtain, after taking the natural logarithm:

$$\ln(-(I-I_i)) = \ln\left(en_eA_p\sqrt{\frac{k(T_i+T_e)}{m_e}}\right) - \left(\frac{T_i+T_e}{2T_e}\right) \qquad \text{for} \quad \phi > 0$$

$$\ln(-(I-I_i)) = \ln\left(A_p\frac{en_e\bar{c}_e}{4}\right) + \frac{e}{kT_e}\phi \qquad \text{for} \quad \phi < 0$$

Apart from a weak dependence on temperature inside of the logarithm for the case when $\phi < 0$, the slope of the line is very close to $\frac{e}{kT_e}$. Therefore, from measuring the slope, the plasma temperature T_e can be estimated.



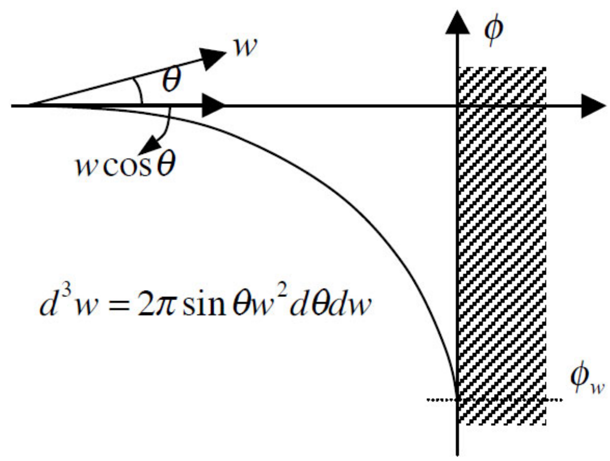
When the potential is increased to $\phi > 0$ the measurement is proportional to the plasma density n_e, which now can be calculated from our previous knowledge of T_e, if we assume that T_i is known.

Method of Medicus

Perhaps one of the most relevant features of electrostatic probes is that they can be used to measure the distribution function itself, provided it is isotropic, although not necessarily Maxwellian.

Say that we want to find the distribution function of electrons $f_e(w)$. The first step is to compute the electronic current to a negatively biased Langmuir probe. In spherical coordinates the current is given by

$$I_e = 2\pi e A_p \int_{w_{\min}}^{\infty} w^3 f_e(w)dw \int_0^{\theta_{\max}} \cos\theta \sin\theta d\theta$$



Only those electrons with enough energy will be able to reach the wall $\frac{1}{2}m_e(w\cos\theta)^2 \geq -e\phi_w$, therefore

$$\theta \leq \cos^{-1}\left(\sqrt{\frac{-2e\phi_w}{m_e w^2}}\right) = \cos^{-1}\left(\frac{w_{\min}}{w}\right) = \theta_{\max}$$

The angular integral is obtained immediately

$$\int_0^{\theta_{\max}} \cos\theta \sin\theta d\theta = -\frac{1}{2}\cos^2\theta d\theta\Big|_0^{\theta_{\max}} = \frac{1}{2}\left(1 - \frac{2e(-\phi_w)}{m_e w^2}\right)$$

And the current is then

$$I_e = \pi e A_p \int_{w_{\min}}^{\infty} w\left(w^2 - \frac{2e(-\phi_w)}{m_e}\right) f_e(w)dw$$

Experimentally one would measure the current as a function of the probe potential ϕ_w. The distribution function is inside of the integral symbol so we need to find a way to extract it from the current. The derivative of a definite integral can be written as (this is a consequence of Leibnitz law)

$$\frac{d}{dt}\int_{a(t)}^{b(t)} f(x,t)dx = f[b(t),t]\frac{db(t)}{dt} - f[a(t),t]\frac{da(t)}{dt} + \int_{a(t)}^{b(t)}\frac{\partial f}{\partial t}(x,t)dx$$

Considering that the distribution function vanishes when $w \to \infty$, the first derivative of the current with respect to the probe potential is

$$\frac{dI_e}{d\phi_w} = \pi e A_p \left[-w_{\min} \left(w_{\min}^2 - \frac{2e(-\phi_w)}{m_e} \right) f_e(w_{\min}) \frac{dw_{\min}}{d\phi_w} + \int_{w_{\min}}^{\infty} \frac{2e}{m_e} w f_e(w) dw \right]$$

The first term inside the bracket is zero from the definition of $w_{\min}$. We are left we the integral in the right. Taking the second derivative of the current

$$\frac{d^2 I_e}{d\phi_w^2} = \pi e A_p \left[-\frac{2e}{m_e} w_{\min} f_e(w_{\min}) \frac{dw_{\min}}{d\phi_w} \right] \quad \text{where} \quad w_{\min} = \sqrt{\frac{-2e\phi_w}{m_e}}$$

We finally obtain the expression

$$\frac{d^2 I_e}{d\phi_w^2} = \frac{2\pi e^3 A_p}{m_e^2} f_e\left(\sqrt{\frac{-2e\phi_w}{m_e}} \right)$$

This is a powerful technique, but also a difficult one, since one must be able to extract numerically with enough precision the two differentials from experimental data to reconstruct the shape of the distribution function.

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16.55 Ionized Gases
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