16.512, Rocket Propulsion
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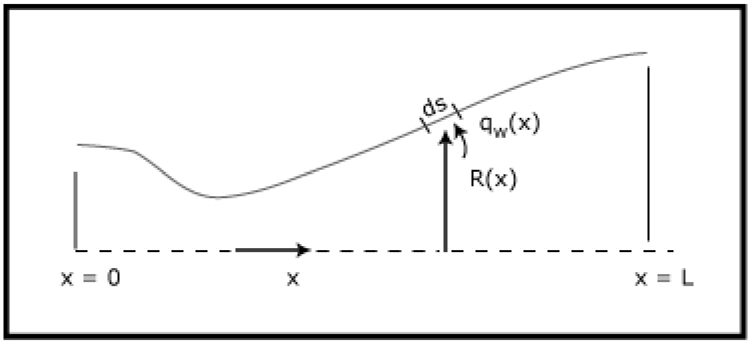
Lecture 8: Convective Heat Transfer: Other Effects

Overall Heat Loss and Performance Effects of Heat Loss

(1) Overall Heat Loss



The local heat loss per unit area is $q_w = \rho u c_p \left(T_{aw} - T_w\right) S_t$, and using $\dot{m} = \rho u \pi R^2$, the integrated heat loss is

$$Q_w \simeq \int_{x=0}^{L} q_w 2\pi R\, ds \;; \quad ds = \sqrt{1+\left(\frac{dR}{dx}\right)^2}\, dx \simeq dx \;\; \text{(small angles)} \qquad (1)$$

$$Q_w \simeq \int_0^L \frac{\dot{m}}{\pi R^2} c_p \left(T_{aw} - T_w\right) S_t\, 2\pi R\, dx = \dot{m}\int_0^L c_p \left(T_{aw} - T_w\right) S_t\, 2\frac{dx}{R} \qquad (2)$$

For an approximate evaluation, assume the quantity $c_p \left(T_{aw} - T_w\right) S_t$ is a weak function of x, and treat it as a constant. We then obtain

$$\frac{Q_w}{\dot{m} c_p T_c} \simeq \frac{T_{aw} - T_w}{T_c} 2S_t \int_0^L \frac{dx}{R(x)} \simeq \left(1 - \frac{T_w}{T_c}\right) 2S_t \int_0^L \frac{dx}{R(x)} \qquad (3)$$

For many rockets, $\left(\frac{L}{R}\right)_{eff} \equiv \int_0^L \frac{dx}{R(x)}$ is of the order of 6-10, and $\frac{T_w}{T_c} \sim \frac{1}{4} - \frac{1}{3}$, so the ratio $\frac{Q_w}{\dot{m} c_p T_c}$ (heat loss divided by total enthalpy flux) is of the order of 8-16 times the Stanton number. As we found before, S_t is itself ~ 0.001, leading to fraction heat losses of the order of 1-2%. While this is a small fraction, its absolute value may be large, because the total thermal power is enormous. As an example, for the SSME engine

$$\dot{m}\, c_p T_c = \frac{F}{C} c_p T_c \simeq \frac{2\times 10^6\ \text{N}}{4500\ \text{m/s}} \times 2770\ \frac{\text{J}}{\text{KgK}} \times 3600\,\text{K}\ ,$$

or $\dot{m}\, c_p T_c = 4.4\times 10^9\ \text{W}$ (the output power of four large power stations).

A 1.5% fraction of this means 66 MW lost to the walls (some 80,000 HP).

(2) Effect on Performance

As a starting guess, we could imagine that all of the losses $\left(Q_w\right)$ are reflected in an equal amount of kinetic energy loss in the exhaust. If u_{e_0} is the exist velocity with no losses,

$$\dot{m}\left(\frac{u_{e_0}^2}{2} - \frac{u_e^2}{2}\right) \simeq Q_w \tag{4}$$

But a little reflection shows that the kinetic energy loss must be less than Q_w.

Indeed, heat losses that occur near the nozzle exit plane are almost irrelevant for performance, because the thermodynamic efficiency of the remaining expansion from the point of loss to the exhaust is very small, so very little of that loss is reflected in a kinetic energy decrease.

So, for the time being, we simply acknowledge this by writing

$$\dot{m}\left(\frac{u_{e_0}^2}{2} - \frac{u_e^2}{2}\right) < Q_w \tag{5}$$

$$\text{or}\ \ u_e > \sqrt{u_{e_0}^2 - \frac{2Q_w}{\dot{m}}}\ \ \text{or}\ \ \frac{u_e}{u_{e_0}} > \sqrt{1 - \frac{2Q_w}{\dot{m}\, u_{e_0}^2}} \tag{6}$$

Remembering that $u_{e_0}^2 / 2 = c_p\left(T_c - T_e\right) = c_p T_c \left[1 - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{2}}\right]$,

$$\frac{u_e}{u_{e0}} > \sqrt{1 - \frac{\left(Q_w \big/ \dot{m}\, c_p T_c\right)}{1 - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}}} \tag{7}$$

If the fractional loss $\frac{Q_w}{\dot{m}\, c_p T_c}$ is of order 1.5% and the expansion efficiency

$\eta = 1 - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}$ is of order 75%, then $\frac{u_e}{u_{e0}} \gtrsim 1 - \frac{1}{2}\,\frac{0.015}{0.75} = 1 - 0.01$ (i.e., a loss of less than 1% in specific impulse, ignoring the exit pressure contribution).

The calculation can be made more precise by tracking the evolution of the gas temperature. The total energy equation, accounting for the losses, is

$$\dot{m}\frac{dh_t}{dx} = \rho u A\left(c_p \frac{dT}{dx} + u\frac{du}{dx}\right) \simeq -q_w 2\pi R = -\rho u c_p \left(\underset{\simeq T_c}{T_{aw}} - T_w\right) S_t\, 2\pi R$$

$$\text{or } c_p \frac{dT}{dx} + u\frac{du}{dx} = -\frac{2S_t}{R} c_p \left(T_c - T_w\right) \tag{8}$$

But the momentum equation (ignoring, somewhat inconsistently, the effects of friction), gives $\rho u \frac{du}{dx} + \frac{dp}{dx} = 0$, or $u\frac{du}{dx} = -\frac{1}{\rho}\frac{dp}{dx}$. Substituting in (8),

$$c_p \frac{dT}{dx} = \frac{1}{\rho}\frac{dp}{dx} - \frac{2S_t}{R} c_p \left(T_c - T_w\right)$$

Divide by $c_p T$ and note that $\frac{1}{\rho c_p T}\frac{dp}{dx} = \frac{\gamma - 1}{\gamma}\frac{1}{p}\frac{dp}{dx}$

$$\frac{1}{T}\frac{dT}{dx} = \frac{\gamma - 1}{\gamma}\frac{1}{p}\frac{dp}{dx} - \frac{2S_t}{R}\frac{T_c - T_w}{T} \tag{9}$$

Without the heat loss term, this would integrate to $\frac{T}{T_c} = \left(\frac{P}{P_c}\right)^{\frac{\gamma-1}{\gamma}}$, the ideal flow (isentropic) relation. More generally now,

$$\frac{T}{T_c} = \left(\frac{P}{P_c}\right)^{\frac{\gamma-1}{\gamma}} \exp\left[-\int_0^L \frac{2S_t}{R}\,\frac{T_c - T_w}{T}\,dx\right] \qquad (10)$$

and since the exponent is a small number,

$$\frac{T}{T_c} = \left(\frac{P}{P_c}\right)^{\frac{\gamma-1}{\gamma}} \left[1 - \int_0^L \frac{2S_t}{R}\,\frac{T_c - T_w}{T}\,dx\right] \qquad (11)$$

To evaluate the correction term, we use for T the <u>undisturbed</u> T, as if no heat loss had happened. This gives

$$\frac{T_c - T_w}{T} = \frac{T_c - T_w}{T_c}\,\frac{T_c}{T} \simeq \left(1 - \frac{T_w}{T_c}\right)\left(\frac{P_c}{P}\right)^{\frac{\gamma-1}{\gamma}}$$

and we also assume that $S_t\left(1 - \frac{T_w}{T_c}\right)$ is nearly constant:

$$\frac{T}{T_c} \simeq \left(\frac{P}{P_c}\right)^{\frac{\gamma-1}{\gamma}} \left[1 - 2S_t\left(1 - \frac{T_w}{T_c}\right)\int_0^L \left(\frac{P_c}{P}\right)^{\frac{\gamma-1}{\gamma}} \frac{dx}{R}\right] \qquad (12)$$

and, in particular, at the exit plane,

$$\frac{T_e}{T_c} \simeq \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}} \left[1 - 2S_t\left(1 - \frac{T_w}{T_c}\right)\int_0^L \left(\frac{P_c}{P}\right)^{\frac{\gamma-1}{\gamma}} \frac{dx}{R}\right] \qquad (13)$$

We now express the exit kinetic energy as

$$u_e^2 = 2c_p\left(T_{t_e} - T_e\right) \qquad (14)$$

where both T_{t_e} and T_e are affected by the losses. For the total energy loss, we have

$$\dot{m}\,c_p\left(T_c - T_{t_e}\right) = Q_w$$

and so

$$\frac{T_{t_e}}{T_c} = 1 - \frac{Q_w}{\dot{m}\, c_p T_c} = 1 - 2S_t\left(1 - \frac{T_w}{T_c}\right)\int_0^L \frac{dx}{R} \tag{15}$$

where we have used the result in equation (3). For the loss of static energy, we have the result in (13). Using both in (14),

$$u_e^2 = 2c_p T_c\left[1 - 2S_t\left(1 - \frac{T_w}{T_c}\right)\int_0^L \frac{dx}{R} - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}} + 2S_t\left(1 - \frac{T_w}{T_c}\right)\int_0^L \left(\frac{P_e}{P}\right)^{\frac{\gamma-1}{\gamma}} \frac{dx}{R}\right]$$

$$\text{or} \quad u_e^2 = 2c_p T_c\left\{1 - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}} - 2S_t\left(1 - \frac{T_w}{T_c}\right)\int_0^L \left[1 - \left(\frac{P_e}{P}\right)^{\frac{\gamma-1}{\gamma}}\right]\frac{dx}{R}\right\} \tag{16}$$

We see now that the factor $1 - \left(\frac{P_e}{P}\right)^{\frac{\gamma-1}{\gamma}}$ occurring in the integral of (16) is just the "thermodynamic relief" we had mentioned earlier, which makes the loss of kinetic energy be less than the heat loss. Indeed, this factor becomes zero as $P \to P_e$, so, as anticipated, heat losses near the exit are irrelevant.

To simplify the writing, use $u_{e_0}^2 = 2c_p T_c\left[1 - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]$

and define

$$\eta_{0,e} = 1 - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{and} \quad \eta_{x,e} = 1 - \left(\frac{P_e}{P(x)}\right)^{\frac{\gamma-1}{\gamma}} \quad :$$

$$\boxed{\frac{u_e^2}{u_{e_0}^2} = 1 - 2S_t\left(1 - \frac{T_w}{T_c}\right)\int_0^L \left(\frac{\eta_{x,e}}{\eta_{0,e}}\right)\frac{dx}{R}} \tag{17}$$

and, again, the right-hand-side minus the $\left(\frac{\eta_{x,e}}{\eta_{0,e}}\right)$ factor would be the relative heat loss (equation 3). Numerical evaluation shows that the modified integral in (17) is about $\frac{2}{3}$ of the original integral $\int_0^L \frac{dx}{R}$. Remembering our earlier estimate of the relative I_{sp} loss ($\lesssim 1\%$), we conclude that a better estimate is about 0.67%. This amounts to 3 sec. out of $I_{sp} \simeq 400s$.