16.50 Lecture 2

Subjects: Rocket staging; Range of aircraft; Climb & Aceleration

1) Rocket Staging

The reason for staging is to avoid having to accelerate empty tanks. Assume for simplicity only two stages; one does not want to stage either too early (and then carry a heavy second stage tank) or too late (and carry a heavy first stage tank for a long time). It can be shown that for ideal symmetrical stages (same specific impulse, same structural fractions), the velocity increment should be divided equally between the stages, and this is a good first cut for more general cases.

In its simplest form, staging consists of just replacing the payload of the first stage by a complete second stage, which in turn has its own payload, as shown in the sketch.

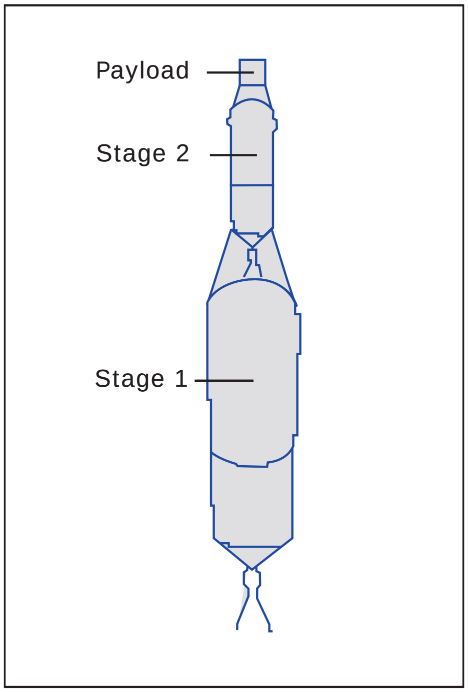


Image by MIT OpenCourseWare.

We go back to the rocket equation, and neglect gravity losses. Write for short $\varepsilon = m'_{str}$, and assume this structural fraction is the same for both stages (in each case the structural mass is normalized by the initial mass of that stage); assume also that the jet velocity c is the same for both stage engines (although typically the first stage engine will be larger and have a bigger thrust). The final payload is given by

$$(m_{pay})_2 = (m'_{pay})_2 (m_0)_2$$

and we also have

$$(m_0)_2 = (m_f)_1 - (m_{str})_1 = (m_0)_1 (e^{-\frac{\Delta V_1}{c}} - \varepsilon)$$

and $$(m'_{pay})_2 = (m'_f)_2 - (m'_{str})_2 = e^{-\frac{\Delta V_2}{c}} - \varepsilon$$

But the total velocity increment $\Delta V = \Delta V_1 + \Delta V_2$ is prescribed, and so we obtain

$$\frac{(m_{pay})_2}{(m_0)_1} = (e^{-\frac{\Delta V_1}{c}} - \varepsilon)(e^{-\frac{\Delta V - \Delta V_1}{c}} - \varepsilon)$$
$$= e^{-\frac{\Delta V}{c}} - \varepsilon(e^{-\frac{\Delta V_1}{c}} + e^{-\frac{\Delta V - \Delta V_1}{c}}) + \varepsilon^2$$

It is easy to se that this is maximum when $\Delta V_1 = \Delta V - \Delta V_1$, i.e., when $\Delta V_2 = \Delta V_1$. If this staging is selected, the overall payload fraction is

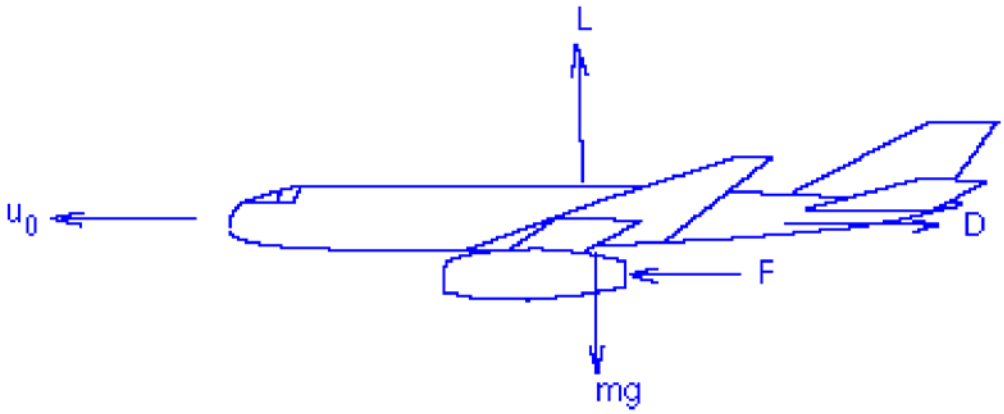
$$(\frac{(m_{pay})_2}{(m_0)_1})_{opt} = (e^{-\frac{\Delta V}{2c}} - \varepsilon)^2$$

This derivation can easily be extended to N>2 stages, but things are more complicated if the structural fractions or the jet velocities are different among stages.

Range of Aircraft

For aircraft, the simplest measure of performance is cruise fuel consumption and the resulting range. At one time this was also the critical performance measure for transports, bombers and fighters. This is less true now for transports because the ranges accessible with modern engines and airframes are in the order of 8,000 miles. For bombers aerial refueling extends the range to the extent that again range is no longer such a challenge (although refueling is quite expensive). Range is still important for fighters because the requirements for high speed and maneuverability conflict with those for long range.

Consider an aircraft in straight, level flight



$$F = D = \frac{L}{L/D} = \frac{mg}{L/D}$$

We define the Specific Impulse by:

$$F = gI\dot{m}$$

where $\dot{m}$ is the fuel mass flow, so that $\dot{m} = -\frac{dm}{dt}$. Note that the inverse of I is the 'Specific Fuel Consumption, SFC, in appropriate units. From this definition and the force balances we obtain

$$\frac{mg}{L/D} = -gI\frac{dm}{dt}$$

$$\frac{dm}{m} = -\frac{D}{LI}dt$$

If we <u>assume</u> D/L and I are constant (Here we are implying <u>models</u> for both the propulsion system and the aircraft), then

$$\ell n\frac{m(t)}{m_0} = -\frac{D}{LI}t$$

The range R = u_0t where u_0 is the flight velocity, so

$$R = u_0 I\frac{L}{D}\ell n\frac{m_0}{m(R)}$$

where m(R) is the aircraft mass at the range R. This is the <u>Breguet Range Equation</u>.

As in the case of the rocket, it is useful to divide the mass m_0 into structural, payload, engines and fuel:

$$m_0 = m_{struct} + m_{eng} + m_{fuel} + m_{pay}$$

or

$$1 = m'_{struct} + m'_{eng} + m'_{fuel} + m'_{pay}$$

If the fuel is expended at R,

$$\frac{m(R)}{m_0} = m'_{struct} + m'_{eng} + m'_{pay} = 1 - m'_{fuel}$$

For a fixed m'_{struct} and m'_{eng}, we can trade off between m'_{pay} and m'_{fuel}, hence between Range and Payload. If we define:

$$m'_{empty} = m'_{struct} + m'_{eng}$$

we can write

$$\frac{m(R)}{m_0} = m'_{empty} + m'_{pay}$$

and

$$R = u_0 I \frac{L}{D} \ell n \frac{1}{m'_{empty} + m'_{pay}}$$

from which

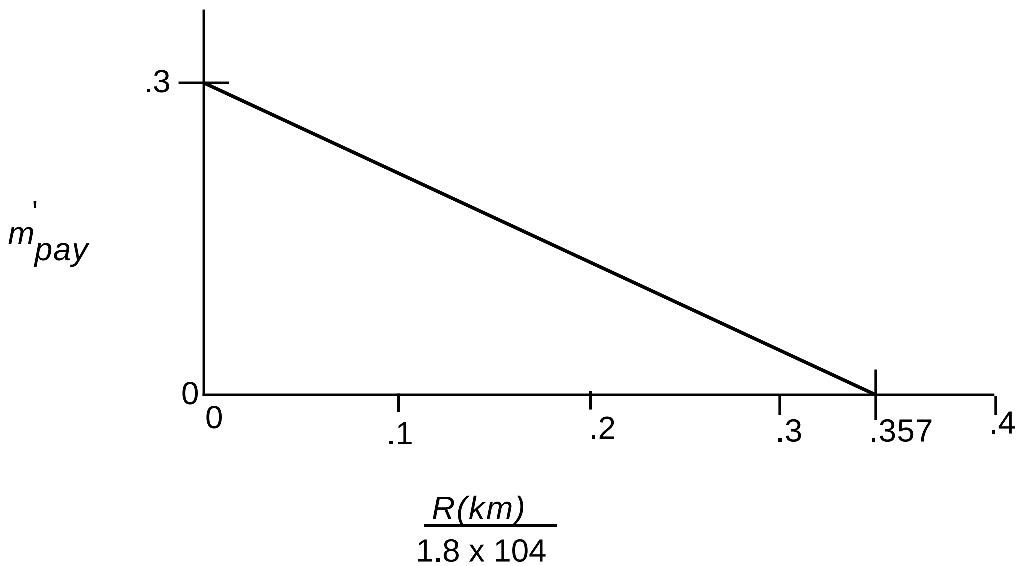
$$m'_{pay} = e^{-\frac{R}{U_0 I(L/D)}} - m'_{empty}$$

(quite similar to what we obtained for a rocket).

From this we can construct a Range vs. Payload chart. As an example, suppose

$$m'_{empty} = 0.7; \qquad u_0 I \frac{L}{D} = (300\frac{m}{s})(4000s)(15) = 1.8x10^7 m$$

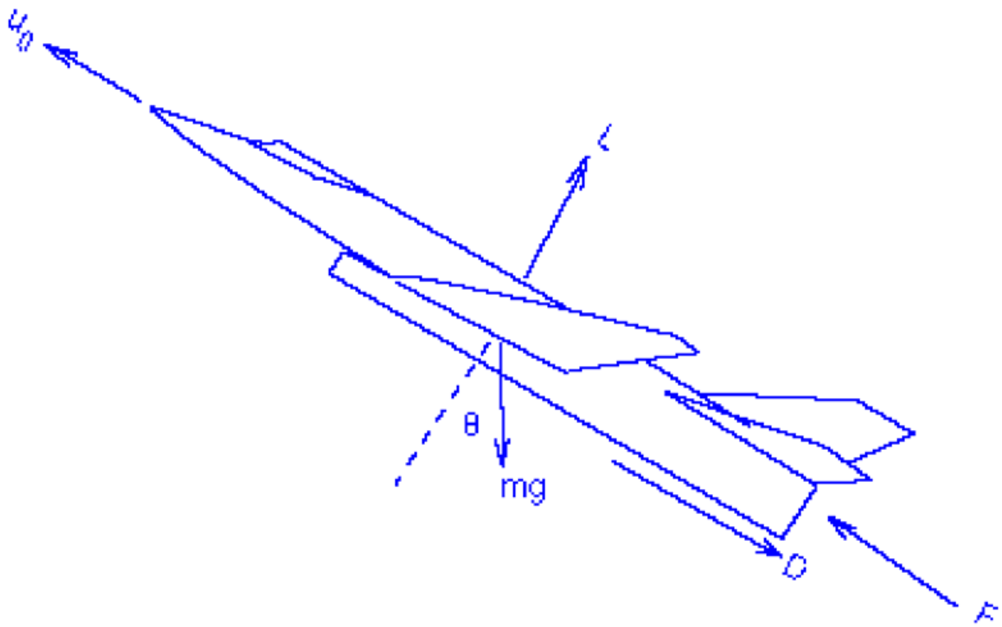
$$m'_{pay} = e^{-\frac{R(km)}{1.8x10^4}} - 0.7$$



Note this is not a straight line, although it is close.

Climb & Acceleration

Sometimes we are more interested in climb and maneuver rather than cruise, as for fighter aircraft. Then an Energy Approach is most helpful.



Suppose the aircraft is climbing at angle θ from the horizontal. The equation of motion along the path is

$$m\frac{du_0}{dt} = F - D - mgSin\theta$$

$$u_0\frac{du_0}{dt} = \frac{(F-D)u_0}{m} - gu_0Sin\theta$$

but $u_0Sin\theta = \frac{dh}{dt}$, so it follows that

$$g\frac{dh}{dt} + u_0\frac{du_0}{dt} = \frac{(F-D)u_0}{m} \equiv \frac{dE}{dt}$$

Were we define a Total Energy $E = gh + \frac{u_0^{\ 2}}{2}$ per unit mass.

If we had a simple model for F and D as a function of u_0 and h we could integrate this as we did for the rocket. But, as we shall see later in the semester these dependencies are much more complex for the aircraft engine than for the rocket. Thus for the two classes of engine we have these quite different thrust characteristics:

Rocket Engine
- Nearly independent of its environment, atmosphere and speed.

Aircraft Engine
- Strongly dependent on flight speed and atmosphere.