16.50   Lecture 15

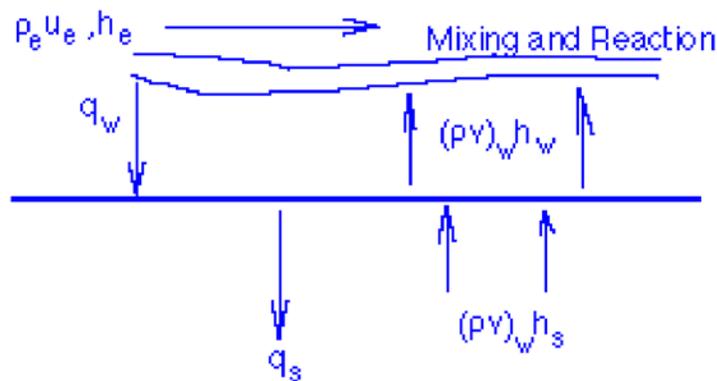**Subject: Ablative cooling**

By ablation we mean the recession of a surface due to heating, usually by a hot gas. It is the key process for

      a) Re-entry heat shields
      b) Solid propellant nozzles
      c) Rocket case insulation
      d) Fire-proofing skyscrapers' structures

Consider a hot gas flowing over a surface which can

      1) evaporate, and
      2) whose vapor can react with the external flow

A heat balance at the surface gives

$$q_s = q_w - (\rho v)_w \, \Delta h_w \qquad\qquad (1)$$

where the heat of ablation, $\Delta h_w = h_w - h_s$, in J/kg, can include a heat of vaporization and decomposition and q_w is the heat flux from the fluid boundary layer.

Let us <u>assume</u> that it is useful to write

$$q_w = \rho_e u_e St \left(H_e + \frac{u_e^2}{2} - H_w \right)$$

where H_w is the total enthalpy for the wall material at the wall temperature, and $H_e = c_{pg} T_{ge}$ is the gas specific enthalpy at the temperature T_{ge} just outside the boundary layer . Notice that this generalizes our previous expression for heat transfer by replacing the <u>total</u> enthalpy for the static enthalpy; this is of general validity for high-speed flows. By energy conservation in the core flow,

$$H_e + \frac{u_e^2}{2} = c_{pg} T_c .$$

Then the heat transfer <u>to the solid is</u>

$$q_s = \rho_e u_e St \left(H_e + \frac{u_e^2}{2} - H_w\right) - (\rho v)_w (h_w - h_s)$$

Since q_s is the heat flux available to heat the wall, it is clear that <u>the evaporation reduces the wall heat flux</u>, i.e. the heat flux into the solid.

Now what determines $(\rho v)_w$? First note that $(\rho v)_w = r\rho_s$ where r is our "recession rate". Consider a "thermal wave" propagating into the solid. We will now work <u>in the receding frame</u>, in which we can assume a steady situation. If the recession rate is r(m/s), we see solid material <u>moving</u> at $v_y = -r$, and convecting a heat flux . Then, if there is no local heat generation,

$$\frac{d}{dy}\left(-k_s \frac{dT}{dy} - r\rho_s c_s T\right) = 0$$

which integrates to $k_s \dfrac{dT}{dy} + r\rho_s c_s T = const. = r\rho_s c_s T_{w_\infty}$, or

$$\frac{d(T-T_{w\infty})}{dy} + \frac{r}{\alpha}(T - T_{w\infty}) = o \qquad \left(\alpha = \frac{k_s}{\rho_s c_s},\ \text{heat diffusivity, in m}^2/\text{s}\right)$$

$$\boxed{T - T_{w\infty} = (T_w - T_{w\infty})e^{-\frac{r}{\alpha}y}}$$

The heat flux at at y = o is $q_s = -k_s \dfrac{dT}{dy}\Big)_{y=o} = k_s \dfrac{r}{\alpha}(T_w - T_{w\infty})$, giving

$$\boxed{q_s = \rho_s c_s r(T_w - T_{w\infty})}, \qquad \text{or } \boxed{\rho_s r = \frac{q_s}{c_s(T_w - T_{w\infty})}}$$

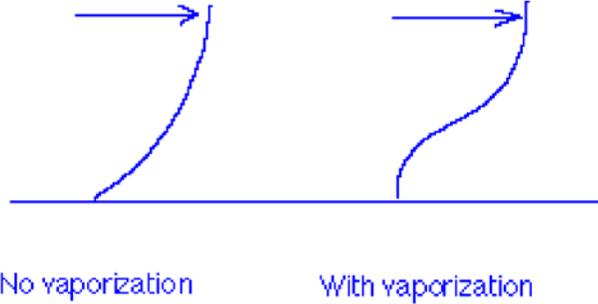So returning to our expression for the heat flux to the wall,

$$q_s = \rho_e u_e S_t c_{pg}(T_c - T_w) - \rho_s r \Delta h_w$$

$$\rho_s r c_s(T_w - T_{w\infty})$$

$$\Rightarrow \boxed{\rho_s r = \frac{\rho_e u_e c_{pg}(T_c - T_w)}{c_s(T_w - T_{w\infty}) + \Delta h_w} S_t}, \qquad \frac{\rho_s r}{\rho_e u_e S_t} = \frac{c_{pg}(T_c - T_w)}{c_s(T_w - T_{w\infty}) + \Delta h_w} \equiv \psi$$

You may compare this to Sutton pg. 510 Eq. 15-6

Now how does $(\rho v)_w$ influence S_t? Physically, we know that the vapor comes off the wall with u=0, so it will tend to retard the flow near the wall.

No vaporization   With vaporization

Quantitatively, Lees says, for small surface blowoff effect,

$$\frac{St}{St_o} = \frac{B}{e^B - 1} \quad c_{pg} \simeq \frac{1.2}{0.2}\frac{8,320}{20} \approx 2500\,J/Kg/K\,, \qquad B = \frac{(\rho v)_w}{\rho_e u_e St_o}$$

while Sutton quotes Lees as giving for the larger blowoffs

$$\frac{St}{St_o} = 1.27\,B^{-.77} \qquad 5 < B < 100$$

These give the same trends, but somewhat different numbers. We adopt here Lees' formulation: We had $\dfrac{\rho_s r}{\rho_e u_e S_t} = \psi$, so $B = \psi\dfrac{S_t}{S_{to}} = \psi\dfrac{B}{e^B - 1}$

$$\text{or } e^B - 1 = \psi \implies B = \ell n(1 + \psi)$$

so finally,
$$\rho_s r = \left\{\ell n\left[1 + \frac{c_{pg}(T_c - T_w)}{c_s(T_w - T_{w\infty}) + \Delta h_w}\right]\right\}\rho_e u_e S_{to} \tag{1}$$

and then

$$q_s = (\rho_s r)c_s(T_w - T_{w\infty}) = \rho_e u_e S_{t0}\, c_s(T_w - T_{w\infty})\ell n\left[1 + \frac{c_{pg}(T_c - T_w)}{c_s(T_w - T_{w\infty}) + \Delta h_w}\right] \tag{1'}$$

At the throat, $(\rho_e u_e)_t = \dfrac{\dot m}{A_t} = \dfrac{p_c}{c^*}$. This is where q_w, and hence r, is maximum.

Numerically, one often sees $\Delta h_w \gg c_s(T_w - T_{w\infty})$, and also $\psi \ll 1$. This would leave us with the approximations

$$\boxed{\rho_s r \cong \frac{c_{pg}(T_c - T_w)}{\Delta h_w}\rho_e u_e S_{to}} \tag{2}$$

$$S_t \cong S_{to} \tag{3}$$

and going back to the surface heat balance, $\boxed{q_s \cong o}$, i.e., to the first order, <u>the heat does not penetrate below the ablating layer,</u> which is as intended.

Equations (1) or (2) can be used to select the proper thickness of the ablative (sacrificial) layer, once the burn time is given: $\delta_{Abl} = r\, t_b$.
Now let us look at some numerical examples, to show the magnitudes of the terms:

A. For <u>carbon</u>, $\Delta Hf_g = 172$ kcal/gmole$=60 \times 10^6 \dfrac{\text{Joule}}{\text{kg}}$

$$\approx h_w - h_s = \frac{172000}{12} = 15,000\,\frac{\text{cal}}{\text{g}}$$

$$c_{pg} \simeq \frac{1.2}{0.2}\frac{8,320}{20} \approx 2500\ J/Kg/K \qquad, \qquad \rho_s \approx 2000\ kg/m^3\ kg/m^3$$

and assume $T_c - T_w = 1,000K$.

Now $\rho_e u_e \approx\ = \Gamma\dfrac{p}{\sqrt{RT}} \approx 0.65\dfrac{p}{\sqrt{RT}}$

$$\approx \frac{10^5(p_c atm)}{\sqrt{\dfrac{8.32x10^3}{20}(3000)}} \approx 58\ (p_c\ \text{atm})\,\frac{\text{kg}}{\text{m}^2\text{s}}$$

Taking $p_c = 100$ atm and $S_t = 0.001$, we then calculate

$$r \approx \frac{2,500*1,000}{2,000\times60\times10^6}\times 58\times100\times0.001 = 1.2\times10^{-4}\,m/s = 0.12\,mm/s$$

So in 100 sec., the change in the surface is ≈ 12 mm $= 1.2$ cm.

This is of the right magnitude, but most important is to see <u>what</u> it <u>depends on</u>.

B. For <u>rubber</u> $\underline{h_w\text{-}h_s} \approx 10,000$ cal/gm mole. For an approximate empirical formula $C_{10}\,H_{20}$, this gives per unit mass

$$\frac{10000}{120+20} = \frac{10000}{140} \approx 70\,\frac{\text{cal}}{\text{g}} \approx 280,000\,\frac{\text{Joule}}{\text{kg}}$$

Also $c_p \approx 1$ cal/g $\approx 4,000$ Joule/kg K

So now we would get about 100 times the regression rate for carbon. In this case though, if the rubber is used in the casing of a solid rocket engine then $\rho_e u_e \ll (\rho_e u_e)_{\text{throat}}$, maybe by $\dfrac{1}{100}$, so the net regression may end up being similar.

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