A two-Position Nozzle

As we discussed in class, the highest thrust at a given external pressure p_a is obtained when the nozzle is sized such as to have an exit-plane pressure p_e that matches that external pressure. For rockets that must climb through the full range of atmospheric pressures from the ground to near vacuum (as in the case of all first stage engines), this matching cannot be achieved at each such pressure, and some compromise must be made. From the simplest to the most complex, these could be:

1. A fixed geometry bell nozzle (the most common approach). The nozzle is sized for matching at some intermediate pressure, and is over-expanded on the ground. The precise choice of area ratio or pressure ratio can be optimized through trajectory simulations, and it is common to find that this optimum would entail flow separation on the ground. Since this is undesirable for a robust engine, the choice is then to design for incipient separation at launch ($p_e / p_{a0} \simeq 0.4$).

2. A two-position nozzle, such as the one depicted in the sketch: the initial part of the ascent is made with a relatively low under-expansion nozzle, which could be designed as in (a), but at some point a nozzle extension is dropped into place, to provide additional thrust at the higher altitudes. The exit area for this extension and the point of transition are design parameters to be investigated in this HWK.

Figure removed due to copyright restrictions.

Fig. 14 Hagemann, Gerald, Hans Immich, *et al.* "Advanced Rocket Nozzles". *Journal of Propulsion and Power* 14, no. 5 (1998): 620-34.

3. A "rubber nozzle" that would be gradually extended so as to be always pressure-matched during the ascent. This is obviously impractical, but the concept is useful as a performance ideal.

(a) Consider a rocket whose chamber pressure is 70 atm., its chamber temperature is 3300K, the gas has a specific heat ratio $\gamma = 1.2$ and a molecular mass $M = 18\, g/mol$. The throat area is $A_t = 0.1 m^2$. Calculate the characteristic velocity $c*$ and the mass flow rate $\dot{m}$. These will be common to all the variants considered below.

The inner bell of its two-position nozzle is designed, as noted above, for $p_{e1}/p_{a0} \simeq 0.4$. Calculate the exit area A_{e1}, exit velocity u_{e1}, vacuum thrust, $F_{V1} = \dot{m}u_{e1} + p_{e1}A_{e1}$ and ground thrust, $F_0 = F_{V1} - p_a A_{e1}$.

(b) Let t_1 be the "transition time", at which the outer bell is dropped into place and t_b the stage burnout time. The initial mass is m_0, the mass at transition is m_1, and the burnout mass is m_b. Show that the velocities gained in vertical ascent before and after transition are given by

$$v_1 = \frac{F_{V1}}{\dot{m}}\ln(\frac{m_0}{m_1}) - gt_1 - A_{e1}\int_0^{t_1}\frac{p_a(t)}{m(t)}dt$$

$$v_b - v_1 = \frac{F_{V2}}{\dot{m}}\ln(\frac{m_1}{m_b}) - g(t_b - t_1) - A_{e2}\int_{t_1}^{t_b}\frac{p_a(t)}{m(t)}dt$$

where $m(t) = m_0 - \dot{m}t$. Assuming now that the exit area A_{e2} has been selected (and therefore the new exhaust velocity u_{e2} and exit pressure p_{e2} as well), show by differentiation that the final velocity v_b is maximized if the transition happens when the atmospheric pressure satisfies

$$p_a(t_1) = \frac{F_{V1} - F_{V2}}{A_{e2} - A_{e1}}$$

and that if this is satisfied, the thrust has no discontinuity when the extension is added.

(c) Assume the new nozzle configuration is also such as to start close to separation, i.e. $p_{e2} = 0.4\, p_a(t_1)$. This condition, together with the optimality condition above, determine p_{e2} or A_{e2}. Perform the calculation for the case under consideration (iteration or some other method will be needed).

(d) Show in tabular form the values of thrust on the ground, at the altitude where transition occurs, and in vacuum, for the fixed geometry bell, the two-position bell and the "rubber engine" bell. Using the approximation $p_a(atm) = e^{-z/6.8\,km}$, plot the three thrust profiles vs. z (up to about 60 km).

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