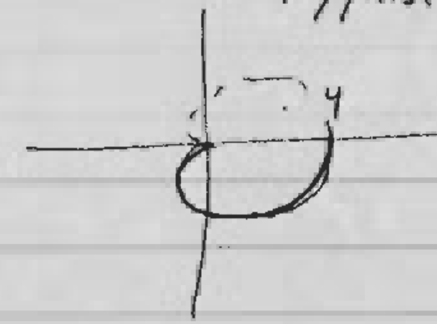
phil

1

Nyquist



$$G(s) = \frac{k}{(s+1)^2} \qquad K = 4$$

ω	Mag $\frac{4}{\omega^2+1}$	Phase deg	$-2\tan^{-1}\left(\frac{\omega}{1}\right)$ rad
0.5	3.2	-53.13	-0.9273
1	2	-90	-1.5708
2	0.8	-126.87	-2.2143
4	0.2353	-151.93	-2.6516

Glenn

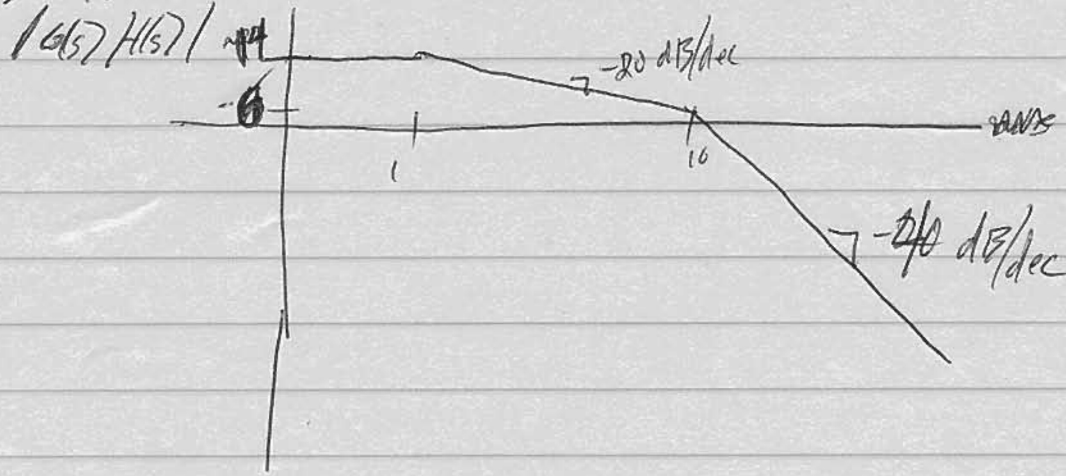
2. $G(s)H(s) = \dfrac{50}{s^2+11s+10}$

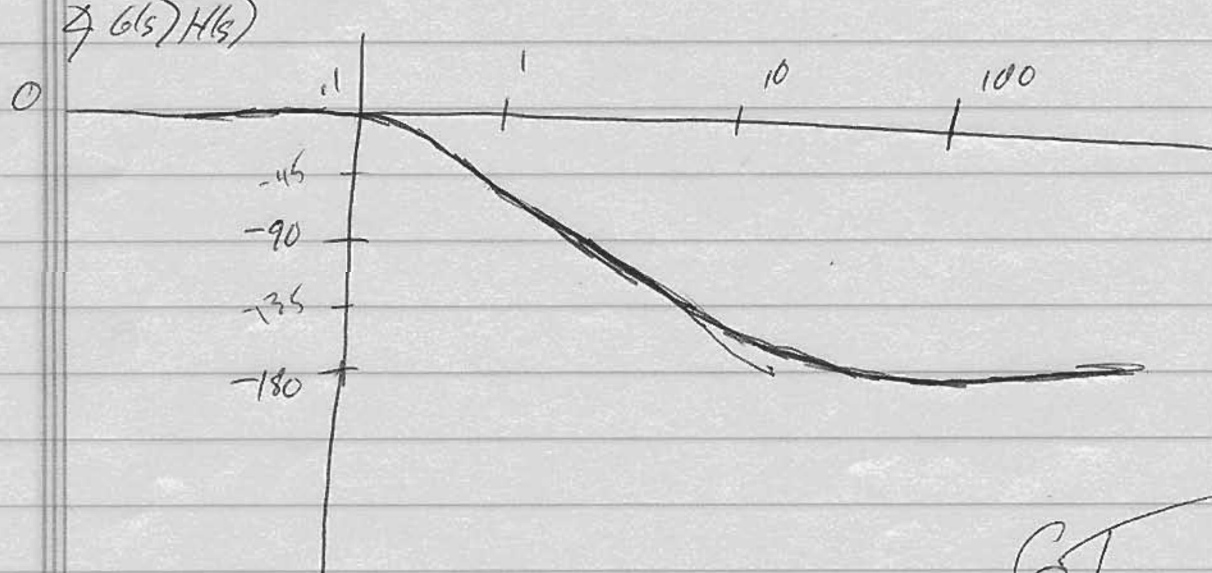
a.) corner freq ~~ω_n~~ $\omega = 1, 10$

b.) at low freq slope => 0 (0 dB/dec)

high freq slope => -2 (-40 dB/dec)

c.) Bode Plot





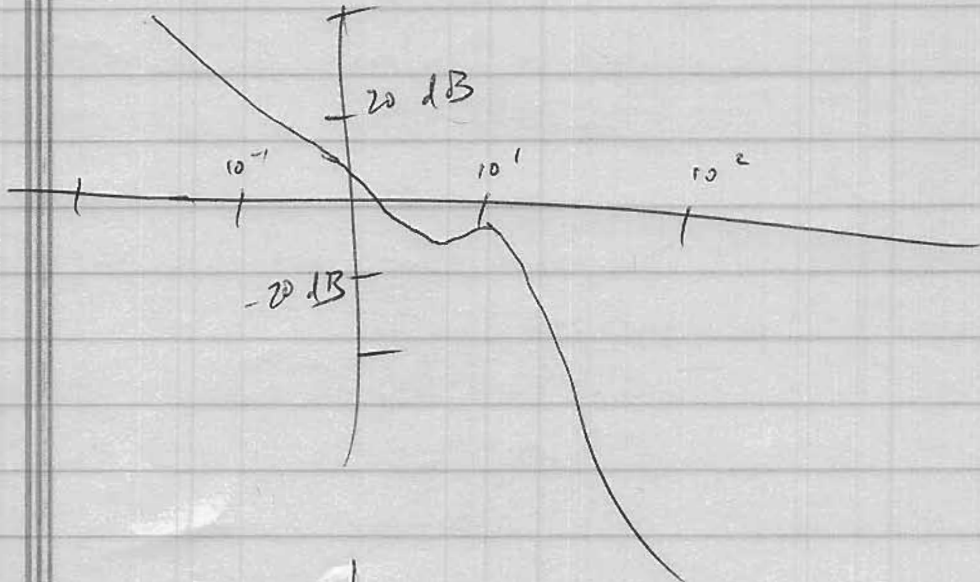
GT

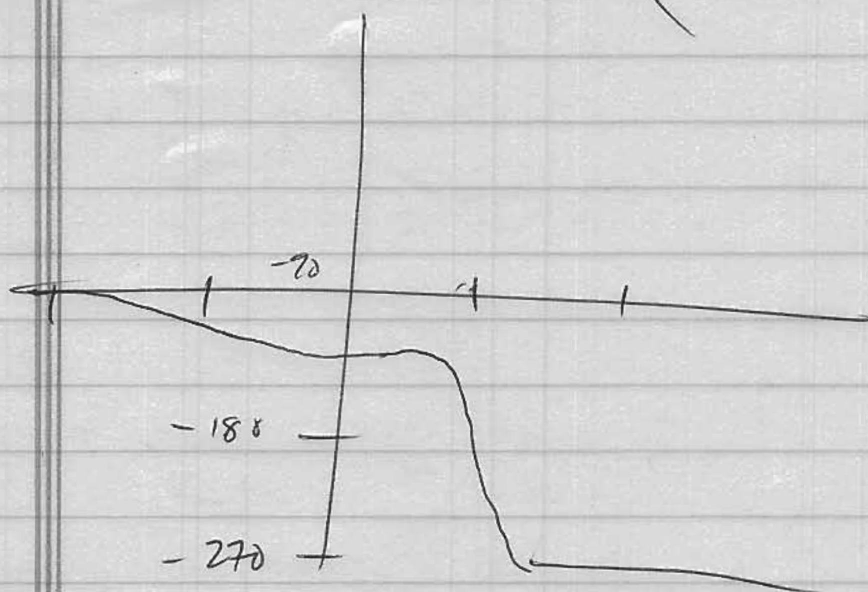
phil

3

GM 9,33 dB @ 7,7 rad/s

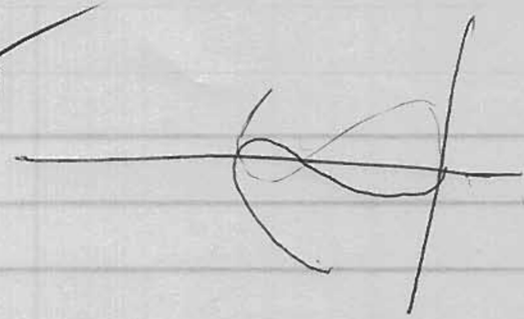
PM 51,2° @ 1.59 rad/s





#4

ϕ_1



4.) $C(s) = \dfrac{0.30(s+0.05)(s^2+1600)}{(s^2+0.05s+16)(s+70)}$

a.) $K(s) = 2 \qquad H(s) = 0.5$

$g_m = \infty$

$\phi_m = 79.3^\circ \quad 13.2 \text{ rad/sec}$

b.) $K(s) = K_1 + K_2/s \qquad K_2/K_1 = 0.5$

$= \dfrac{K_1 s + K_2}{s}$

$= K_2 \dfrac{\left(K_1/K_2\, s + 1\right)}{s}$

$w = 4$

$w = 0.16$

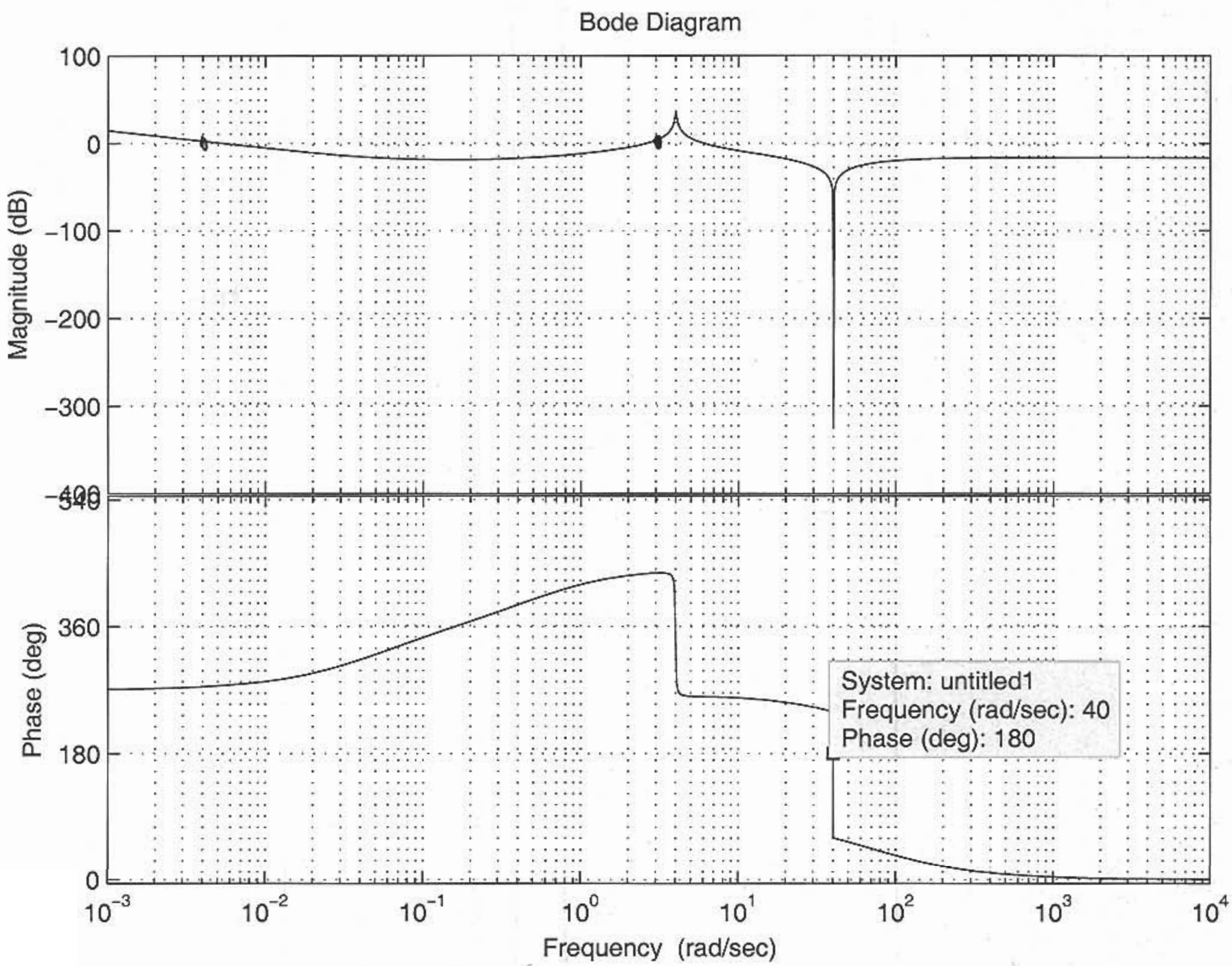
$$10^{-1/2} = \frac{0.30 \cdot (w^2+0.05^2)^{1/2}(1600-w^2)((2\cdot w)^2+1)^{1/2}}{w((16-w^2)^2+(0.05w)^2)^{1/2}(70^2+w^2)^{1/2}}$$

$w = 0.16 \qquad K_2 = .669 \qquad K_1 = 2K_2$

$w = 4 \qquad K_2 = .0011 \qquad K_1 = .002314$

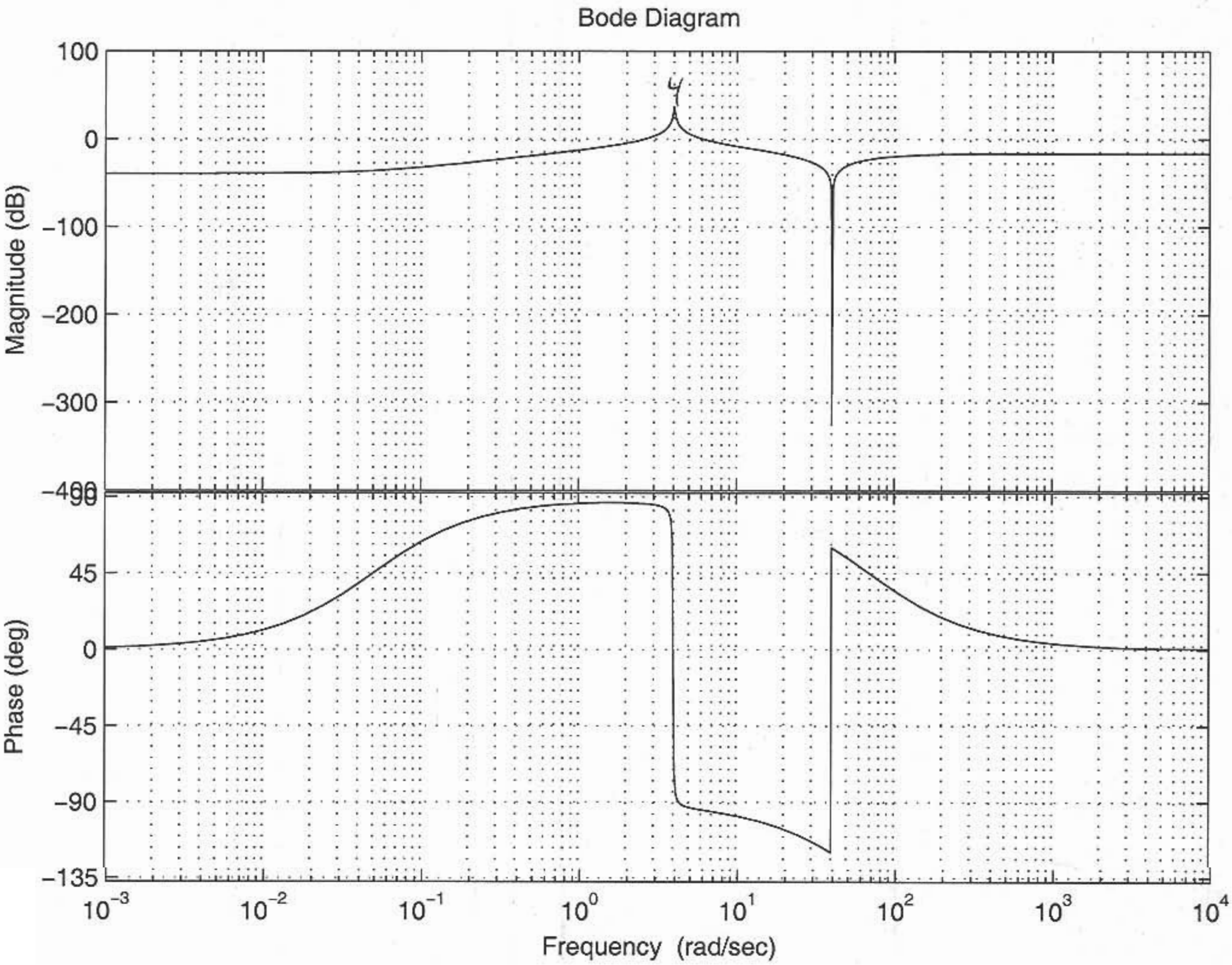
w/ controller

#4



w/o controller

#4



#5

Need integrator. Need zero to get phase back. Lead gives better overall response, but not necessary.

6T

Final solutions

Problem 6.

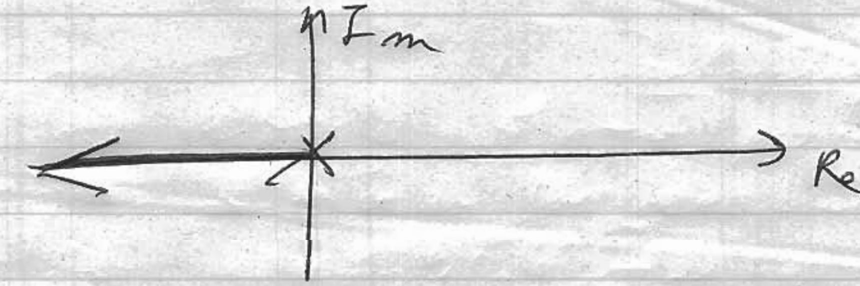
a) - Assume the velocity is zero - If the external applied force has amplitude less than D, ~~then~~ (say positive), then the switch in the feedback path reacts first with a force of amplitude $-D$ (backwards) to any ~~f~~forward move of the mass: thus the mass cannot move forward - It cannot move backwards either, because as soon as it tries and do that, the reaction force on the ~~[illegible]~~ feedback path becomes $+D$ and stops the mass from trying to move backwards -

- If the velocity is ~~now~~ strictly positive, then the reaction force in the feedback path is negative, of amplitude $-D$ as intuition suggests

And vice versa if the velocity is strictly negative -

— We can use describing function to see, indeed, if there are uncontrolled oscillations — This is <u>not</u> a rigorous argument, but it's OK to use it since not much else is available to you —

As we have seen, in the case of a switch nonlinearity, there is a limit cycle if the root locus crosses the $j\omega$ axis away from zero — In our case, the root locus is;



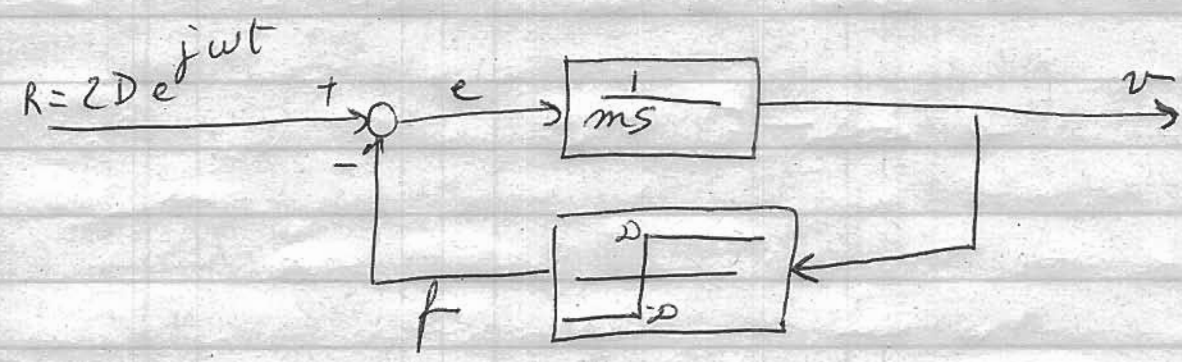
So it never crosses the $j\omega$ axis —

$\Rightarrow$ there is no limit cycle.

(c)

(b) The system is subject to a forcing sinusoidal function of amplitude 2D

The steady - state velocity response should be a periodic function.



(b) Let us assume that the only signals of interest are sinusoids -

we have: $e = 2D\,e^{j\omega t} - f$

we approximate f by : $f = f_0\, e^{j(\omega t + \phi_1)}$

v by : $v = v_0\, e^{j(\omega t + \phi_2)}$

and $\left| f_0 = \dfrac{4D}{\pi v_0}\, v_0 \right.$, $\phi_1 = \phi_2$

and $v = \dfrac{1}{jm\omega}\, e$ or: $v_0 = \dfrac{1}{m\omega} |e|$

~~Thus:~~ $\cancel{e = 2De^{j\omega t} - \frac{4D}{\pi v_0} \frac{e}{jm\omega}}$ and: $\phi_1 = \phi_2 = -90^\circ$

(d)

Thus:

$$f = \frac{4D \,\cancel{m\omega}}{\pi \,\cancel{|e|}} \frac{1}{j\cancel{m\omega}} \cancel{|e|} = \frac{4D}{\pi j}$$

so: ~~e = 2D~~

$$e = \left(2D - \frac{4D}{\pi j}\right) e^{j\omega t}$$

or:
$$\frac{e}{2D} = \left(1 - \frac{2}{\pi j}\right) e^{j\omega t}$$

~~So: in general the DF for dry friction is: from R to e:~~

~~$$\left(1 - \frac{2}{\pi j}\right)$$~~

So at the output, we have a "sinusoid" of amplitude $\left|\left(\frac{1}{mj\omega}\left(1 - \frac{2}{\pi j}\right)\right)\right| 2D$.

e

(c) In general, we have:

$$e = R\,e^{j\omega t} - f$$

with $f = \dfrac{4D}{\pi j}\,e^{j\omega t}$.

so: and $v = \dfrac{1}{mj\omega}\,e$.

$$\text{so: } v = \frac{1}{mj\omega}\left(R - \frac{4D}{\pi j}\right)e^{j\omega t}.$$

$$\text{or: } \frac{v}{Re^{j\omega t}} = \frac{1}{mj\omega R}\left(R - \frac{4D}{\pi j}\right).$$

Thus:

$$\boxed{N(A,\omega) = \left(\frac{1}{m\omega}\right)\left(-j + \frac{4D}{\pi R}\right).}$$

P

Problem 7

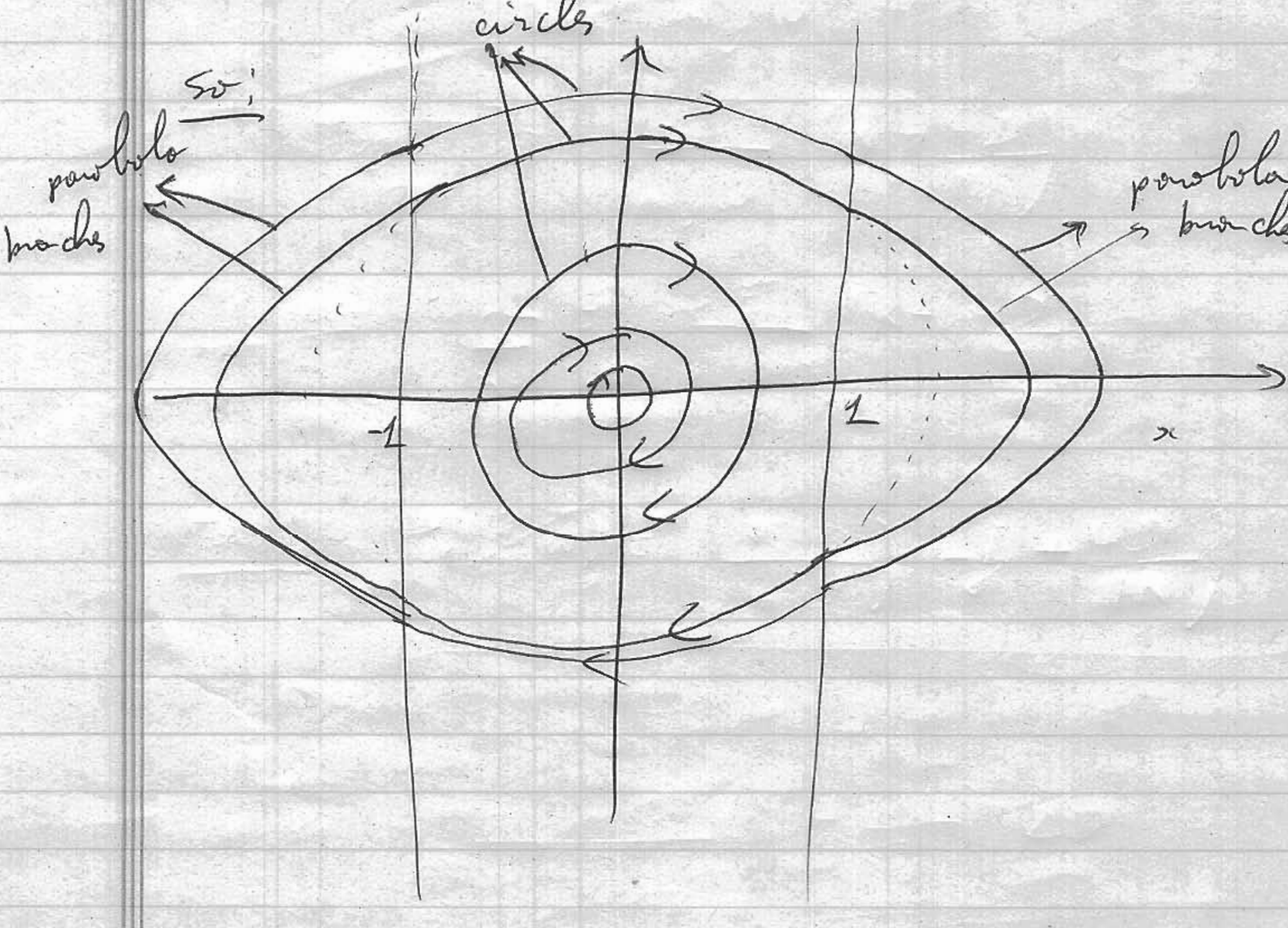
a) The equation of motion is;

$\ddot{x} = -x$ for $|x| < 1 \to$ circle

$\ddot{x} = -1$ for $x > 1 \to$ parabola

$\ddot{x} = 1$ for $x < -1 \to$ parabola

So;



$\boxed{b}$ $\Omega = 0.1$ rad / sec.

the centrifugal acceleration is:

$x \, 0.01$ m/sec^2

~~at~~ and is equal to 1 when $x = 100$m.

The equations of motion are:

$\ddot{x} = -0.99x$ for $|x| < 1$ $\Rightarrow$ circle.

$\ddot{x} = -1 + 0.01x$ for $x > 1$

$\ddot{x} = 1 + 0.01x$ for $x < -1$

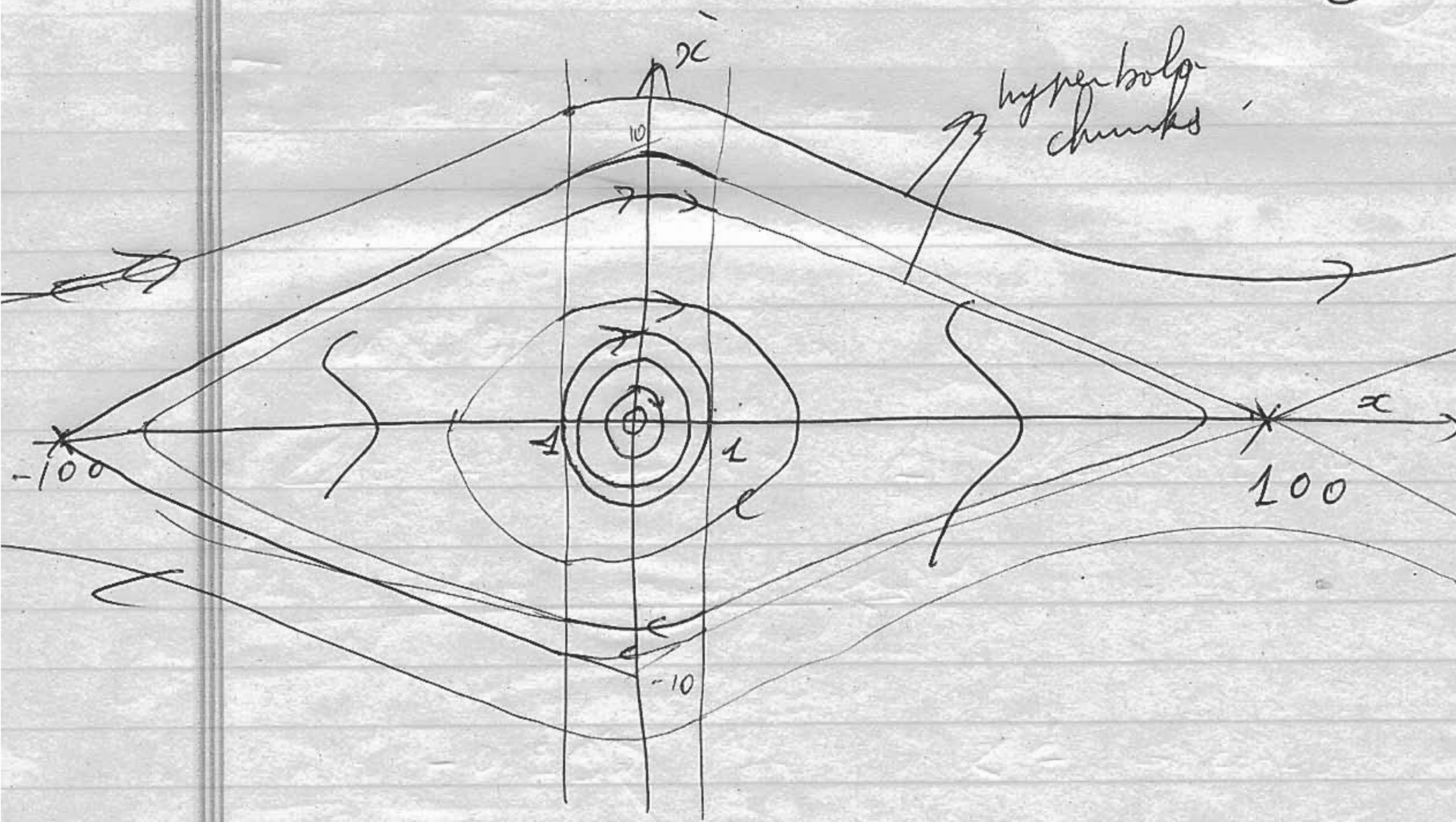
what's this?

Analyze: $\ddot{x} = -1 + 0.01x$ in more detail -

we have an equilibrium at $\dot{x} = 0$, $x = 100$.

and this is the equation of an inverted pendulum.

(h)



c) assume now Ω spins up!
The generic equations of motion are!

$$\ddot{x} = (-1 + \Omega^2)\, x \qquad \text{for } |x| < 1.$$

$$\ddot{x} = -1 + \Omega^2 x \qquad \text{for } x > 1$$

$$\ddot{x} = 1 + \Omega^2 x \qquad \text{for } x < -1.$$

assume: $\Omega^2 < 1$.

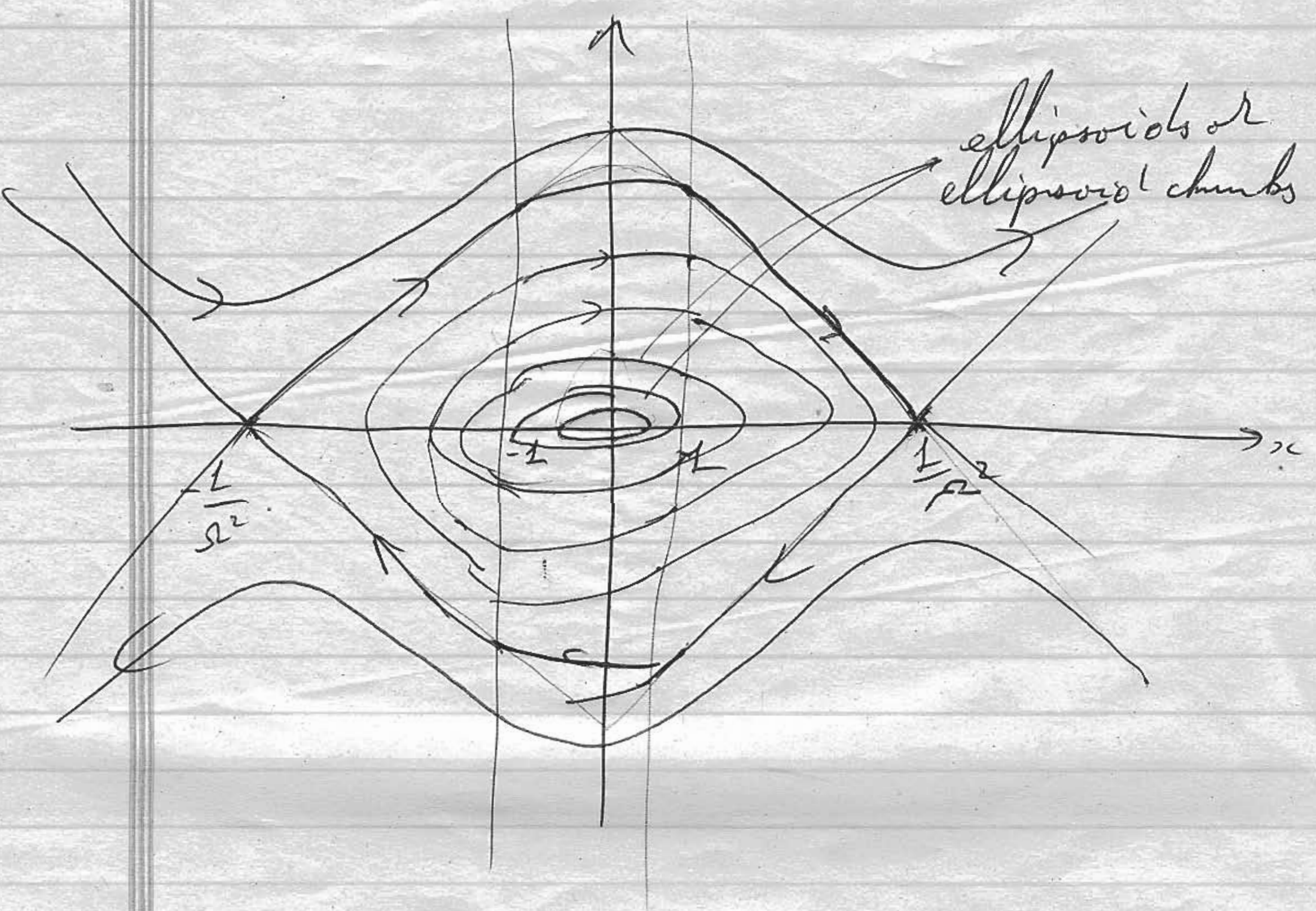
Then: $\ddot{x} = (-1 + \Omega^2)x$ is stable for $|x| < 1$

and: $\ddot{x} = -1 + \Omega^2 x$

has equilibrium at $x = 0$

$x = \frac{1}{\Omega^2}$.

The phase plane looks like:



Assume now: $\Omega^2 > 1$.

Then $\ddot{x} = (-1 + \Omega^2)\, x$ is unstable too!

The phase plane now looks like: