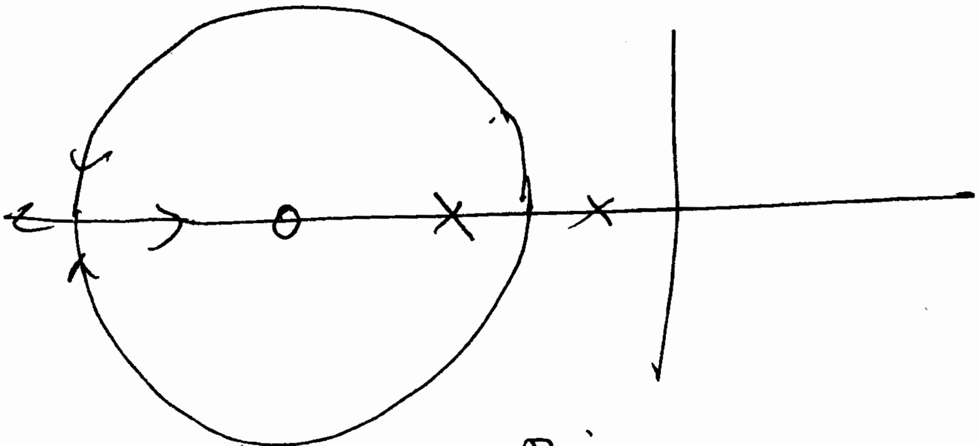
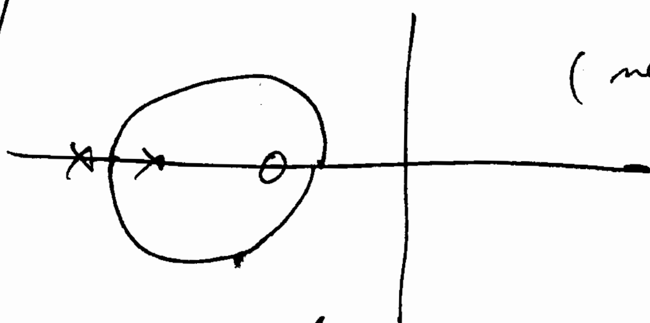
Problem set 3

– 2 poles, 1 zero.



or:



(negative gain)

why is this a circle?

Let:

$$H(s) = \frac{(s+Z)}{(s+p_1)(s+p_2)}$$

with $0 \leq p_1 \leq p_2 \leq Z$

$$1 + K H(s) = 0$$

$$\Leftrightarrow$$

$$1 + K \frac{(s+z)}{(s+p_1)(s+p_2)} = 0$$

$$\Leftrightarrow \quad (s+p_1)(s+p_2) + K(s+z) = 0.$$

$$\Leftrightarrow \quad s^2 + (p_1+p_2+K)s + p_1p_2 + Kz = 0$$

<u>Discriminant:</u>
$$\begin{aligned}\Delta &= (p_1+p_2+K)^2 \\ &\quad - 4(p_1p_2+Kz) \\ &= p_1^2+p_2^2+K^2+2p_1p_2 \\ &\quad + 2p_1K + 2p_2K \\ &\quad - 4p_1p_2 - 4Kz. \\ &= p_1^2+p_2^2 - 2p_1p_2 + 2p_1K + 2p_2K \\ &\quad - 4Kz + K^2. \\ &= (p_1-p_2)^2 + 2K(p_1+p_2-2z) + K^2.\end{aligned}$$

The range of K for which discriminant is negative is found by finding the roots of

$$(p_1 - p_2)^2 + 2K(p_1 + p_2 - 2z) + K^2 = 0.$$

vs. K -

the reduced discriminant of THAT equation is!

$$(p_1 + p_2 - 2z)^2 - (p_1 - p_2)^2$$

$$= (\cancel{p_1} + p_2 - 2z - \cancel{p_1} + p_2)(p_1 \cancel{+p_2} - 2z + p_1 \cancel{-p_2})$$

$$= (2p_2 - 2z)(2p_1 - 2z)$$

$$= 4(z - p_2)(z - p_1) > 0.$$

Thus for $K \in \left[2z - (p_1+p_2) - 2\sqrt{(z-p_2)(z-p_1)} \, , \, 2z - (p_1+p_2) + 2\sqrt{(z-p_1)(z-p_2)} \right]$

The roots of the first equation are complex

Let us compute these roots:

<u>root 1,2</u>

$$x_{1,2} = \frac{-(p_1+p_2+K) \pm j\sqrt{-(p_1-p_2)^2 + 2K(2z-p_1-p_2) - K^2}}{2}.$$

~~the~~ Thus the distance of any of these roots to the zero is:

$$\left| \frac{(2z - p_1 - p_2 - K)}{2} + j\,\frac{\sqrt{-(p_1-p_2)^2 + 2K(2z-p_1-p_2) - K^2}}{2}. \right|$$

$$= \sqrt{\frac{(2z-p_1-p_2-K)^2}{4} + \frac{2K(2z-p_1-p_2) - K^2 - (p_1-p_2)^2}{4}}$$

$$= \sqrt{\frac{4z^2 + p_1^2 + p_2^2 + \cancel{K^2} - 2K(\cancel{2z-p_1-p_2}) + 2p_1p_2 - 4zp_1 - 4zp_2 \left[+2K(\cancel{2z-p_1-p_2}) - \cancel{K^2} - (p_1-p_2)^2\right]}{4}}$$

$$= \sqrt{\frac{4z^2+\rho_1^2+\rho_2^2+2\rho_1\rho_2-4z\rho_1-4z\rho_2-(\rho_1-\rho_2)^2}{4}}$$

$$= \sqrt{\frac{(2z-\rho_1-\rho_2)^2-(\rho_1-\rho_2)^2}{4}}$$

$$= \sqrt{\frac{(2z-\rho_1-\cancel{\rho_2}-\rho_1+\cancel{\rho_2})(2z-\cancel{\rho_1}-\rho_2+\cancel{\rho_1}-\rho_2)}{4}}$$

$$= \sqrt{(z-\rho_1)\,(z-\rho_2)}$$

$$\Downarrow$$

desired result . !

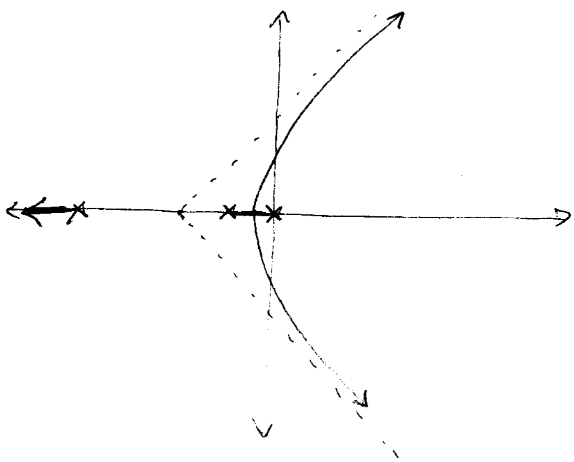
2

$$1 + \frac{K}{s(s+1)(s+5)}$$

(a) and (b)

$$\alpha = \frac{-1-5}{3} = -2$$

$$\phi_\ell = 60^\circ,\ 180^\circ,\ 300^\circ$$



(c)

$$j\omega\,(j\omega+1)(j\omega+5) + K = 0$$

$$j\omega\,(-\omega^2 + 6j\omega + 5) + K = 0$$

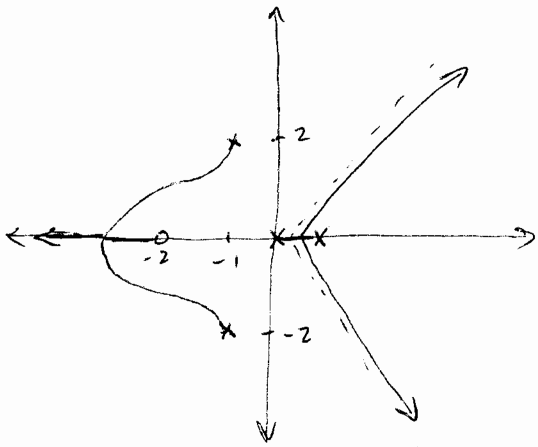
$$-\omega^3 j - 6\omega^2 + 5j\omega + K = 0$$

$$\boxed{\begin{aligned} &\text{Im}: \quad \omega(-\omega^2+5) = 0 \longrightarrow \omega = \pm\sqrt{5} \\ &\text{Re}: \quad K - 6\omega^2 = 0 \longrightarrow K_{crit} = 30 \end{aligned}}$$

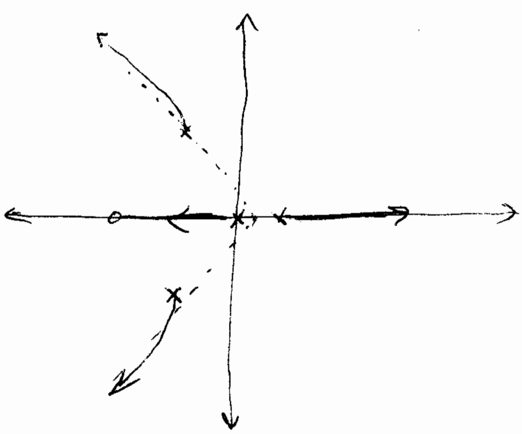
3

a) $KG(s) = \dfrac{K(s+2)}{s(s-1)(s^2+2s+5)}$

$k > 0$



$k < 0$



$K > 0$

3) Asymptotes

$$\alpha = \frac{(-1-1+1)-(-2)}{3} = \boxed{\frac{1}{3}}$$

$$\phi_l = 60°, 180°, 300°$$

4) Departure angles

$$-\gamma_1 + 63.4 - 116.6 - 135 - 90 = \pm(2i+1)180$$

$$\boxed{\gamma_1 = -98.2°}$$

5) no $j\omega$-axis crossing

6) $b\dfrac{da}{ds} - a\dfrac{db}{ds} =$

$$= s^4 + s^3 + 3s^2 - 5s$$

$$- 4s^4 - 11s^3 - 12s^2 - 7s + 10 =$$

$$- 3s^4 - 10s^3 - 9s^2 - 12s + 10 = 0$$

$$\boxed{\begin{array}{l} \text{breakin @} \approx -2.91 \\ \text{breakout @} \approx 0.51 \end{array}}$$

$K < 0$

3) $\alpha = \frac{1}{3}$

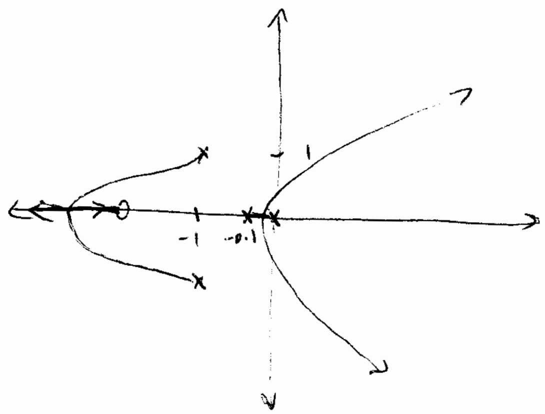
$\phi_l = 0°, 120°, 240°$

4) $\gamma_1 = 81.8°$

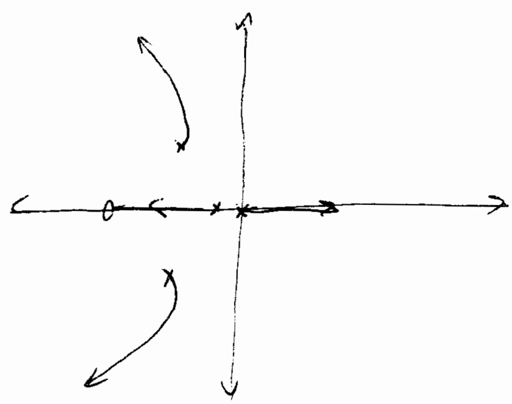
#3 (cont'd)

b) $KG(s) = \dfrac{K(s+2)}{s(s+0.1)(s^2+2s+2)}$

k > 0



k < 0



k > 0

3) $\alpha = \frac{-1}{30}$ $\quad \phi_\ell = 60°, 180°, 300°$

4) $\gamma_1 = -132°$

5) $\omega = 0.43$ $\quad (k_{crit} = 0.185)$

6) breakin @ ≈ −2.6
breakout @ ≈ −0.05

k < 0

3) $\alpha = -\frac{1}{30}$, $\quad \phi_\ell = 0°, 120°, 240°$

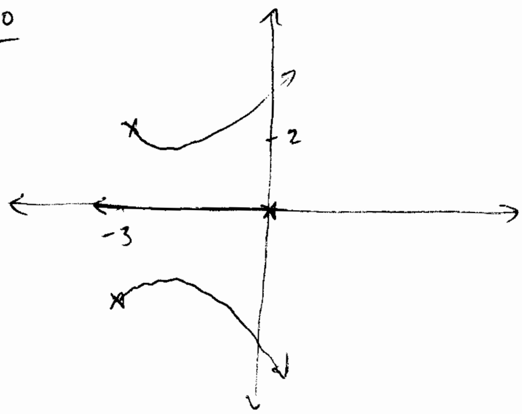
4) $\gamma_1 = 48°$

5) no $j\omega$-axis crossing

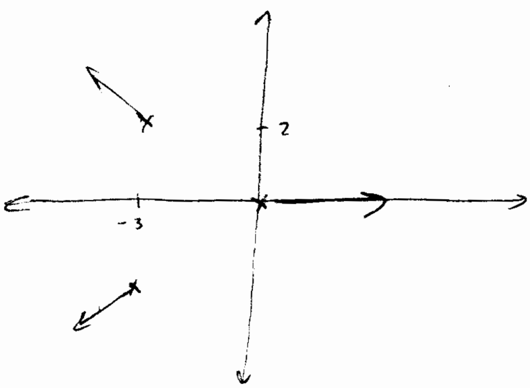
\# 3 (cont'd)

c) $KG(s) = \dfrac{2K}{s(s^2+6s+13)}$

k > 0



k < 0



k > 0

3) $\alpha = -2$, $\phi_\ell = 60°, 180°, 300°$

4) $\gamma_1 = -56.7°$

5) $\omega = \pm 3.6$ ($k_{crit} \approx 38.9$)

k < 0

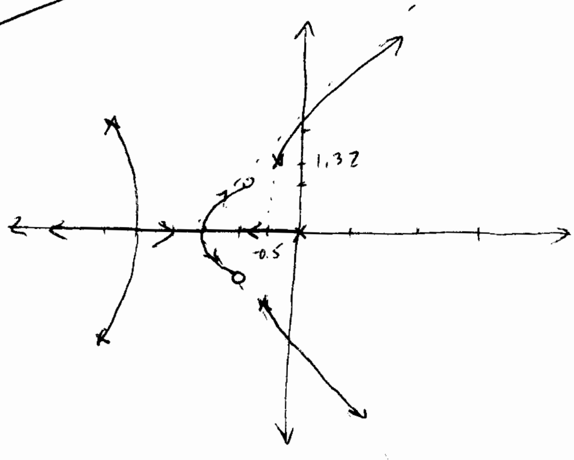
3) $\alpha = -2$, $\phi_\ell = 0°, 120°, 240°$

4) $\gamma_1 = 123.3°$

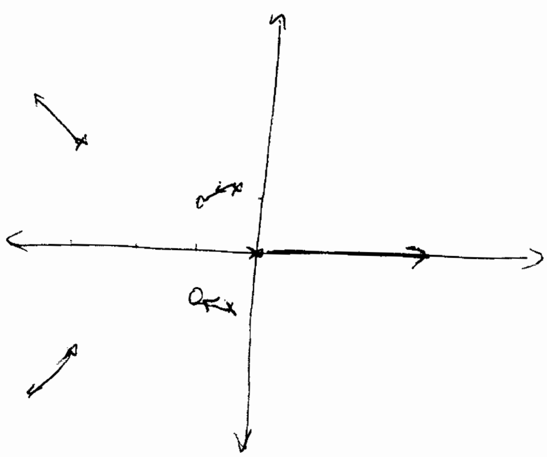
3 (cont'd)

d) $$KG(s) = \frac{2K(s^2+2s+2)}{s(s^2+6s+13)(s^2+s+2)}$$

K > 0



K < 0



K > 0

3) $\alpha = -\frac{5}{3}$, $\phi_\ell = 60°, 180°, 300°$

4) cx pair near origin: $\gamma_1 = 51°$
other cx pair: $\gamma_2 = -71°$

5) $\omega = \pm 2.75$

6) breakin @ ≈ -2.6
breakaway @ ≈ -1.6

K < 0

3) $\alpha = -\frac{5}{3}$, $\phi_\ell = 0°, 120°, 240°$

4) $\gamma_1 = -129°$
$\gamma_2 = 109°$

#3 (cont'd)

e) $KG(s) = \dfrac{K(s+0.5)(s+15)}{s(s^2+2s+2)(s+5)(s+15)}$

K > 0

3) $\alpha = -6.66$, $\phi_\ell = 60°, 180°, 300°$

4) $\gamma_1 = 116°$

5) $\omega = 8.42$

K < 0

3) $\alpha = -6.66$, $\phi_\ell = 0°, 120°, 240°$

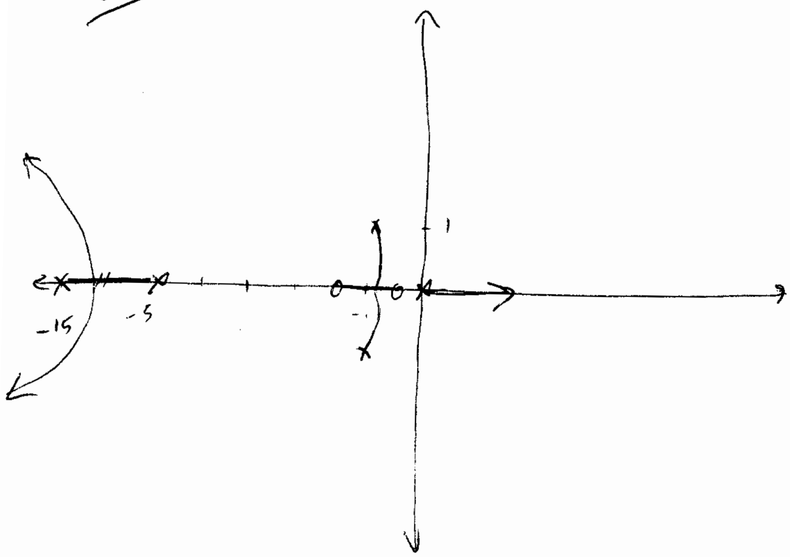
4) $\gamma_1 = -64°$

5) no $j\omega$-axis crossing

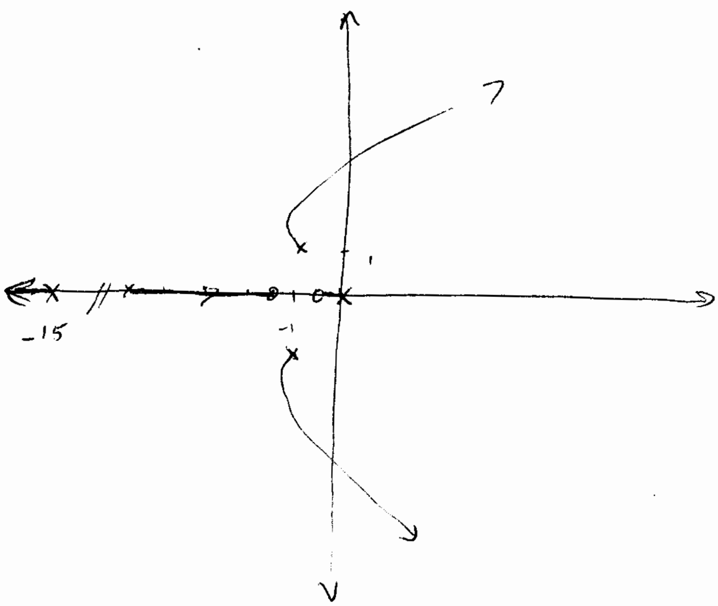
6) breakin @ ≈ -11

breakout @ ≈ -0.92

K < 0



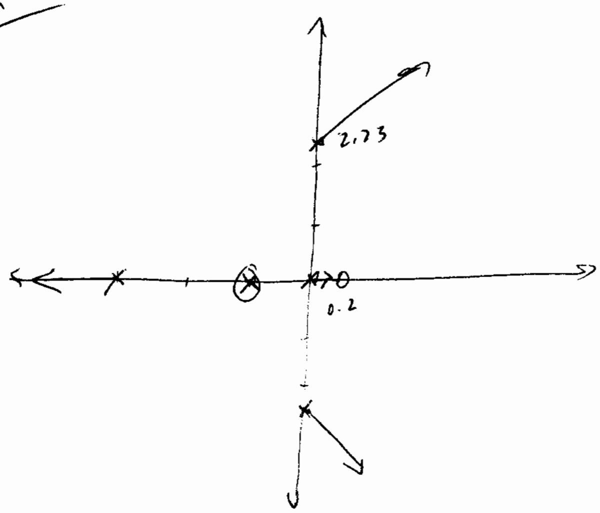
K > 0



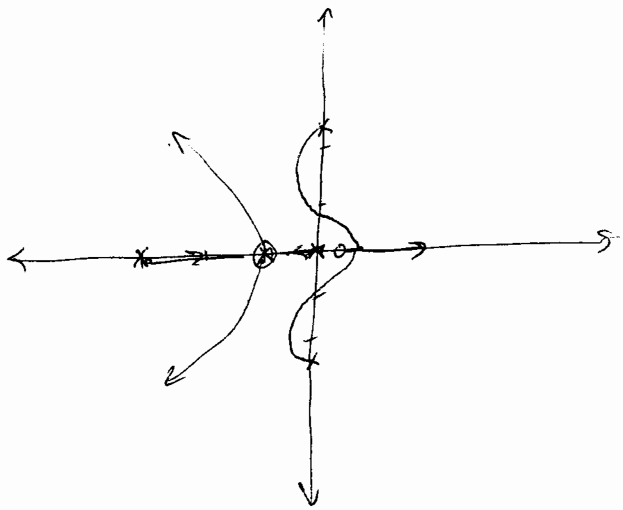
#3 (cont'd)

f) $kG(s) = \dfrac{K(s+1)(s-0.2)}{s(s+1)(s+3)(s^2+5)}$

$k > 0$



$k < 0$



$k > 0$

3) $\alpha = -1.07$, $\phi_\ell = 60°, 180°, 300°$

4) $\gamma_1 = 57°$

5) $\omega = \pm\sqrt{5}$

$k < 0$

3) $\alpha = -1.07$, $\phi_\ell =$

4) $\delta_1 = -123°$

5) $\omega \approx 0.73$

6) breakout @ -1

breakin @ approx. 0.73