Homework 6

Eric Feron

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1. A 564,000 lbs Boeing 747 is approaching land at seal level (flaps and landing gear down). Assuming a velocity of 221 ft/sec (Mach 0.198), the lateral-directional perturbation equations are

$$\begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{p} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -0.0890 & -2.19 & 0 & 0.319 \\ 0.0760 & -0.217 & -0.166 & 0 \\ -0.602 & 0.327 & -0.975 & 0 \\ 0 & 0.15 & 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \\ p \\ \phi \end{bmatrix} + \begin{bmatrix} 0.0327 \\ -0.151 \\ 0.0636 \\ 0 \end{bmatrix} \delta r$$

$$y = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}.$$

   (a) Compute the corresponding transfer function linking r, the yaw rate (expressed in rad/sec), and δr, the rudder angle (expressed in rad).
   (b) Draw the uncompensated root-locus of this system.
   (c) Draw the Frequency response of this system.
   (d) Design a compensator for this system. What maximum bandwidth are you comfortable with?
   (e) The rudder is limited to $\pi/6$ rad maximum deflection. Assume the B747 is subject to a wind gust throwing it at an initial yaw rate of 0.2 rad/sec. What closed loop settling time (to get yaw rate back to zero) are you comfortable designing for?

2. Satellite Attitude control systems very often use reaction wheels to provide angular motion. The equations of motion for the system are

$$\begin{aligned} \text{Satellite:} \quad & I\ddot{\phi} = T_c + T_{ex} \\ \text{Wheel:} \quad & J\dot{r} = -T_c \\ \text{Measurement:} \quad & \dot{Z} = \dot{\phi} - aZ \\ \text{Control:} \quad & T_c = -D(s)(Z - Z_d). \end{aligned}$$

with

$$
\begin{array}{rcl}
T_c & = & \text{Control torque} \\
T_{ex} & = & \text{Disturbance torque} \\
\phi & = & \text{Attitude angle to be controlled} \\
Z & = & \text{Measurement from sensor} \\
Z_d & = & \text{Reference angle} \\
I & = & \text{Satellite inertia } (1500 \text{ kg/m}^2) \\
a & = & \text{Sensor constant } (0.5 \text{ rad/sec}), \\
D(s) & = & \text{Compensation}
\end{array}
$$

(a) Assume $D(s) = K_0$ a constant gain. Draw the root-locus with respect to K_0 for the resulting closed-loop system.

(b) For what range of K_0 is the closed-loop system stable?

(c) Add a lead controller with a pole at -1 so the closed-loop system has a bandwidth $\omega_{\text{BW}} = 0.04$ rad/sec and a damping ratio of $\zeta = 0.5$ and the compensation is given by

$$D(s) = K_1 \frac{s + z}{s + 1}.$$

Where should the zero of the lead compensator be located? Draw the root-locus and Bode plot of the compensated system. What value of K_1 allows the specifications to be met?

(d) For what range of K_1 is the system stable?

(e) What is the steady-state error (difference between Z and Z_d) to a constant disturbance torque T_{ex} for your design?

(f) What are the phase and gain margins for your design?