16.225 - Computational Mechanics of Materials
Homework assignment # 5
Handed out: 11/24/03
Due: 12/8/03

Endow your version of sumMIT with dynamics capability. To this end, implement Newmark's algorithm, both in implicit ($\beta \neq 0$) and explicit ($\beta = 0$) form. In all cases use a lumped mass matrix computed by the row-sum method. As an example of application, consider the square sheet problem of HA04. Take as initial conditions the final deformation obtained by static stretching, and set the initial velocities to zero. Estimate a stable time step for explicit dynamics calculations as $\triangle t_s = \triangle x_{\min}/c$, where $\triangle x_{\min}$ is one half of the minimum element side among all elements in the mesh (you may estimate this value by creating your mesh and determining the lengths of the sides by inspection) and $c = \sqrt{k/\rho_0}$. Keep the mesh and the time step constant throughout the calculations. Take the calculations up to time $T = 5L/c$. Consider the following combination of parameters:

1. $\beta = 1/2$, $\gamma = 1$, $\triangle t = 10\triangle t_s$ and $\triangle t = 100\triangle t_s$
2. $\beta = 1/4$, $\gamma = 1/2$, $\triangle t = 10\triangle t_s$ and $\triangle t = 100\triangle t_s$.
3. $\beta = 0$, $\gamma = 1/2$, $\triangle t = 0.2\triangle t_s$, $\triangle t = \triangle t_s$ and $\triangle t = 10\triangle t_s$.

Plot deformed meshes at various stages of deformation. Plot the x_1 coordinate of the center point of the right edge of the plate against time. Use for comparison an accurate solution obtained, e. g., by taking $\beta = 1/4$, $\gamma = 1/2$ and $\triangle t = 0.01\triangle t_s$. Comment on the conclusions you draw on the accuracy, stability and dissipation properties of the specific algorithm resulting from each set of algorithm parameters.