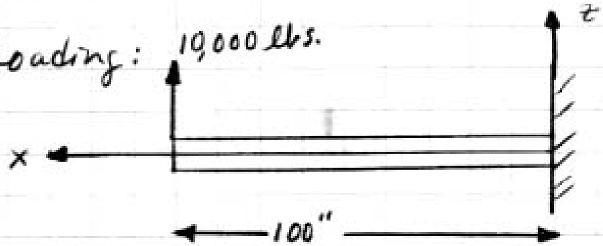
16.20 HANDOUT #4C

Fall, 2002

Single Cell Box Beam Example

→ Beam Loading:

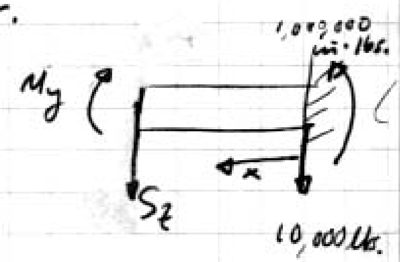


* Find M, S, T as function of x:

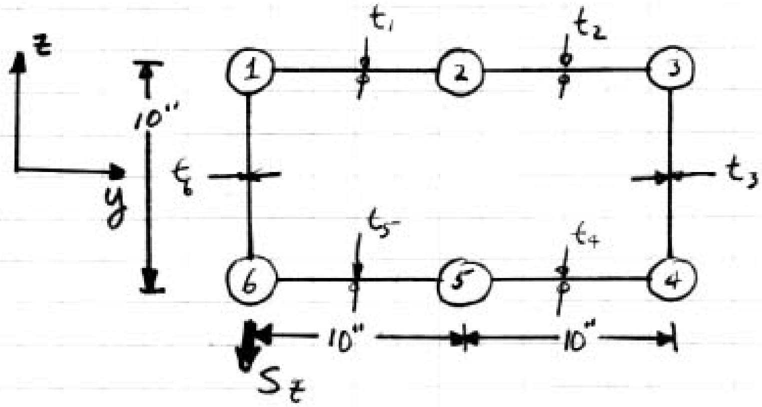
$$M_y = 10{,}000\,(100 - x) \text{ in}\cdot\text{lbs.}$$

$$S_z = -10{,}000 \text{ lbs.}$$

$$T(\text{applied}) = 0$$



→ Cross-Section



$A_1 = 1 \text{ in}^2$ $\quad t_1 = 0.02"$

$A_2 = 0.5 \text{ in}^2$ $\quad t_2 = 0.02"$

$A_3 = 0.5 \text{ in}^2$ $\quad t_3 = 0.04"$

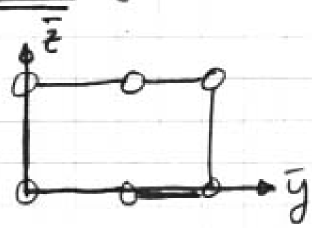
$A_4 = 0.5 \text{ in}^2$ $\quad t_4 = 0.02"$

$A_5 = 0.5 \text{ in}^2$ $\quad t_5 = 0.02"$

$A_6 = 1 \text{ in}^2$ $\quad t_6 = 0.06"$

Note: S_z applied at stringer 6.

(a) Axial stresses (due to bending)



Set axis system at same corner

Find centroid:

$$\bar{y}^* = \frac{\sum A_i^* \bar{y}_i}{\sum A_i^*}$$

$$\bar{z}^* = \frac{\sum A_i^* \bar{z}_i}{\sum A_i^*}$$

all same modulus, so $A_i^* = A_i$

Make a chart:

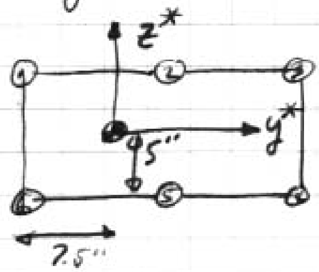
Stringer # i	A_i [in^2]	y_i [in]	z_i [in]
1	1.0	0	10
2	0.5	10	10
3	0.5	20	10
4	0.5	20	0
5	0.5	10	0
6	1.0	0	0

So: $$\bar{y}^* = \frac{(1.0\,in^2)(0) + (0.5\,in^2)(10'') + (0.5\,in^2)(20'') + (0.5\,in^2)(20'') + (0.5\,in^2)(10'') + (1.0\,in^2)(0)}{1.0\,in^2 + 0.5\,in^2 + 0.5\,in^2 + 0.5\,in^2 + 0.5\,in^2 + 1.0\,in^2}$$

$$\bar{y}^* = \frac{30\,in^3}{4\,in^2} = 7.5''$$

and $\bar{z}^* = 5''$

Move axis system to centroid to find I's:



Make another chart (as referenced to centroid):

Stringer # i	A_i [in²]	y_i^* [in]	z_i^* [in]
1	1.0	-7.5	5.0
2	0.5	2.5	5.0
3	0.5	12.5	5.0
4	0.5	12.5	-5.0
5	0.5	2.5	-5.0
6	1.0	-7.5	-5.0

Find:

$$I_{yy}^* = \sum A_i^* z_i^{*2} = (1.0\,in^2)(5.0")^2 + (0.5\,in^2)(5.0")^2 + (0.5\,in^2)(5.0")^2 + (0.5\,in^2)(-5.0")^2 + (0.5\,in^2)(-5.0")^2 + (1.0\,in^2)(-5.0")^2$$

$$\Rightarrow I_{yy}^* = 100\,in^4$$

and:

$$I_{yz}^* = \sum A_i^* y_i^* z_i^* =$$

$$= (1.0\,in^2)(-7.5")(5.0") + (0.5\,in^2)(2.5")(5.0") + (0.5\,in^2)(12.5")(5.0") + (0.5\,in^2)(12.5")(-5.0") + (0.5\,in^2)(2.5")(-5.0") + (1.0\,in^2)(-7.5")(-5.0")$$

$= 0 \Rightarrow$ section is symmetric

$$I_{zz}^* = \sum A_i^* y_i^{*2} = (1.0\,in^2)(-7.5")^2 + (0.5\,in^2)(2.5")^2 + (0.5\,in^2)(12.5")^2 + (0.5\,in^2)(12.5")^2 + (0.5\,in^2)(2.5")^2 + (1.0\,in^2)(-7.5")^2$$

$$\Rightarrow I_{zz}^* = 275\,in^4$$

For a symmetrical section with $M_z = 0$:

$$\sigma_x = \frac{-M_y z}{I_y}$$

$\therefore$

$$(\sigma_x)_1 = (\sigma_x)_2 = (\sigma_x)_3 = -\frac{10{,}000(100-x)\ \text{in}\cdot\text{lbs}\,(5'')}{100\ \text{in}^4}$$

$$= -500\,(100-x)\ \text{psi}$$

(x in [inches])

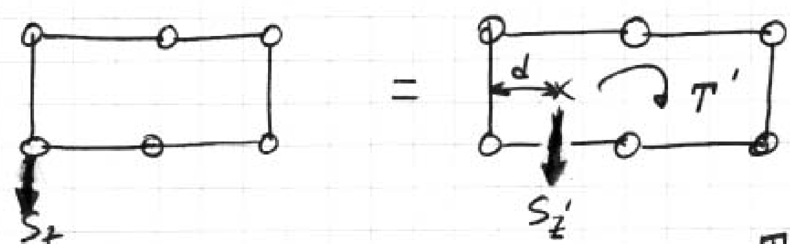
$$(\sigma_x)_4 = (\sigma_x)_5 = (\sigma_x)_6 = \frac{-10{,}000(100-x)\ \text{in}\cdot\text{lbs}\,(-5'')}{100\ \text{in}^4}$$

$$= +500\,(100-x)\ \text{psi}$$

(x in [inches])

(b) Shear Stresses

Resolve problem into "Pure Shear" and "Pure Torsion"

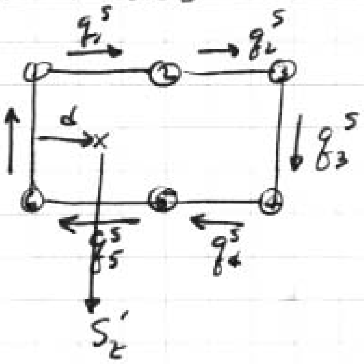


$T' = -S_z d$

$S_z' = S_z = -10{,}000$ lbs.

Step ① "Pure Shear" case

label the shear flows :



(q_i^s : shear flow in skin i due to "Pure Shear" case)

a) Apply joint equilibrium

Joint equilibrium: $q_{out} - q_{in} = \dfrac{Q_y S_z}{I_{yy}} = \dfrac{A z S_z}{I_{yy}}$

At joint 1: $q_1^s - q_6^s = \dfrac{(1\text{ in}^2)(5'')(-10{,}000\text{ lbs})}{100\text{ in}^4} = -500$ lbs/in (*1)

At joint 2: $q_2^s - q_1^s = \dfrac{(0.5\text{ in}^2)(5'')(-10{,}000\text{ lbs})}{100\text{ in}^4} = -250$ lbs/in (*2)

At joint 3: $q_3^s - q_2^s = \dfrac{(0.5\text{ in}^2)(5'')(-10{,}000\text{ lbs})}{100\text{ in}^4} = -250$ lbs/in (*3)

At joint 4: $q_4^s - q_3^s = \dfrac{(0.5\text{ in}^2)(-5'')(-10{,}000\text{ lbs})}{100\text{ in}^4} = 250$ lbs/in (*4)

At joint 5: $q_5^s - q_4^s = \dfrac{(0.5\text{ in}^2)(-5'')(-10{,}000\text{ lbs})}{100\text{ in}^4} = 250$ lbs/in (*5)

At joint 6: $q_6^s - q_5^s = \dfrac{(1\text{ in}^2)(-5'')(-10{,}000\text{ lbs})}{100\text{ in}^4} = 500$ lbs/in (*6)

(one of these is not independent)

b) Use <u>Torque</u> <u>Boundary</u> <u>Condition</u>

$$\Sigma T_{internal} = T_{applied} \qquad \overset{+}{\curvearrowright} \text{ about stringer } ⑥$$

$$T_{applied} = S_z d = (10{,}000 \text{ lbs})\, d$$

$$\Sigma T_{internal} = \Sigma (q_i^s)(\text{skin length})_i (\text{moment arm})_i$$

$$= q_1^s(10'')(10'') + q_2^s(10'')(10'') + q_3^s(10'')(20'') + q_4^s(10'')(0) + q_5^s(10'')(0) + q_6^s(10'')(0)$$

$$\Rightarrow (-10{,}000 \text{ lbs})\, d = 100 \text{ in}^2 (q_1^s + q_2^s + 2q_3^s) \qquad (*)\ (7)$$

c) Apply <u>No</u> <u>Twist</u> <u>Condition</u>

$$\oint \frac{q}{t}\, ds = 0$$

$$\Rightarrow \sum \frac{q_i}{t_i} (\text{skin length})_i = 0$$

$$\frac{q_1^s(10'')}{0.02''} + \frac{q_2^s(10'')}{0.02''} + \frac{q_3^s(10'')}{0.04''} + \frac{q_4^s(10'')}{0.02''} + \frac{q_5^s(10'')}{0.02''} + \frac{q_6^s(10'')}{0.06''} = 0 \qquad (*)\ ⑧$$

We now need to solve the $(*)$ equations (7 of them) for the unknowns (7 of them: d and 6 q_i^s)

It is convenient to express all the q_i^s in terms of one q_i.

Use q_1^s with the joint equilibrium equations (1-6)

$$q_2^s = q_1^s - 250 \text{ lbs/in}$$
$$q_3^s = q_2^s - 250 \text{ lbs/in} = q_1^s - 500 \text{ lbs/in}$$
$$q_4^s = q_3^s + 250 \text{ lbs/in} = q_1^s - 250 \text{ lbs/in}$$
$$q_5^s = q_4^s + 250 \text{ lbs/in} = q_1^s$$
$$q_6^s = q_1^s - 500 \text{ lbs/in}$$

Place these into the "No-Twist" equation (#8) and solve for q_1^s:

$$q_1^s + (q_1^s - 250 \text{ lbs/in}) + \tfrac{1}{2}(q_1^s - 500 \text{ lbs/in})$$
$$+ (q_1^s - 250 \text{ lbs/in}) + q_1^s + \tfrac{1}{3}(q_1^s + 500 \text{ lbs/in}) = 0$$

gives: $4.83\, q_1^s = 583$ lbs/in

$\Rightarrow q_1^s = 121$ lbs/in

Using these in the above:

$q_2^s = -129$ lbs/in $\qquad q_4^s = -129$ lbs/in

$q_3^s = -379$ lbs/in $\qquad q_5^s = 121$ lbs/in

$q_6^s = 621$ lbs/in

Finally, use these in the Torque B.C. equation (#7)

$$d = -\frac{1}{100}\frac{\text{in}^2}{\text{lbs}}\left(121 - 129 + 2[-379]\right)\frac{\text{lbs}}{\text{in}}$$

$\Rightarrow d = 7.66''$

→ We check the results by checking the equipollence of the system

$$\sum \text{Internal Shear Loads} = \text{Applied Shear}$$

Horizontal ($\xrightarrow{+}$): $q_1^s(10") + q_2^s(10") - q_4^s(10") - q_5^s(10") \stackrel{?}{=} 0$

$121 \text{ lbs/in} - 129 \text{ lbs/in} + 129 \text{ lbs/in} - 121 \text{ lbs/in} = 0$

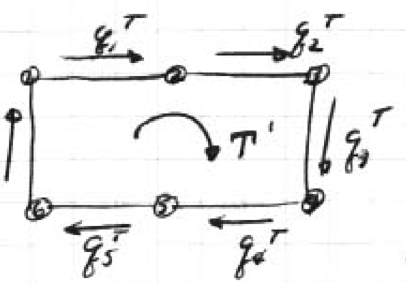
✓ checks

Vertical ($+\uparrow$): $-q_3^s(10") + q_6^s(10") \stackrel{?}{=} 10{,}000 \text{ lbs}$

$379 \text{ lbs/in}(10") + 621 \text{ lbs/in}(10") \stackrel{?}{=} 10{,}000 \text{ lbs}$

$3790 \text{ lbs} + 6210 \text{ lbs} = 10{,}000 \text{ lbs.}$ ✓ checks

Step ② "Pure Torsion" Case



(q_i^T = shear flow in skin i due to "Pure Torsion" case)

$$T' = -S_z d$$
$$= -(-10{,}000 \text{ lbs})(7.66")$$
$$\Rightarrow T' = 76{,}600 \text{ in-lbs}$$

a) Apply joint equilibrium

for "Pure Torsion" Case: $q_{out} - q_{in} = 0$

At joint 1 : $q_1^T - q_6^T = 0$ (*) (11)

At joint 2 : $q_2^T - q_1^T = 0$ (*) (12)

At joint 3 : $q_3^T - q_2^T = 0$ (*) (13)

At joint 4 : $q_4^T - q_3^T = 0$ (*) (14)

At joint 5 : $q_5^T - q_4^T = 0$ (*) (15)

At joint 6 : $q_6^T - q_5^T = 0$ (*) (16)

(one of these is not independent)

$$\Rightarrow q_1^T = q_2^T = q_3^T = q_4^T = q_5^T = q_6^T$$

b) Use <u>Torque Boundary Condition</u>

$$\Sigma T_{internal} = T_{applied} \quad \circlearrowright^{+} \text{ about stringer ⑥}$$

$$T_{applied} = 76{,}600 \text{ in}\cdot\text{lbs.}$$

as before:

$$\Sigma T_{internal} : q_1^T(10'')(10'') + q_2^T(10'')(10'') + q_3^T(10'')(20'') + q_4^T(10'')(0'') + q_5^T(10'')(0'') + q_6^T(10'')(0'')$$

$$\Rightarrow 100\text{in}^2(q_1^T + q_2^T + 2q_3^T) = 76{,}600 \text{ in}\cdot\text{lbs.}$$

using $q_1^T = q_2^T = q_3^T$

$$\Rightarrow 4q_1^T = 766 \text{ lbs/in}$$

$$\Rightarrow q_1^T = 192 \text{ lbs/in}$$

So: $q_1^T = q_2^T = q_3^T = q_4^T = q_5^T = q_6^T = 192$ lbs/in

Can check these via: $\sum F_{Horizontal} = 0$

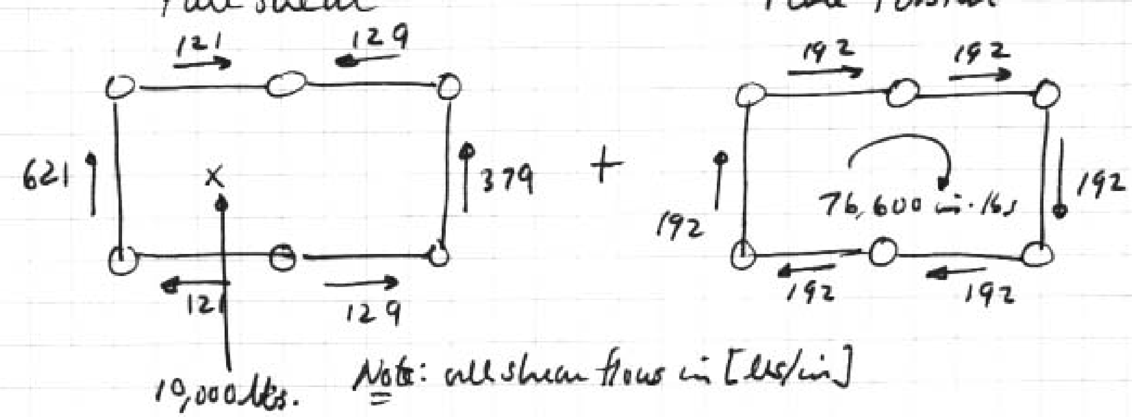
$$\xrightarrow{+} \quad q_1^T(10") + q_2^T(10") - q_4^T(10") - q_5^T(10") = 0 \checkmark \text{ checks}$$

$$\sum F_{vertical} = 0$$

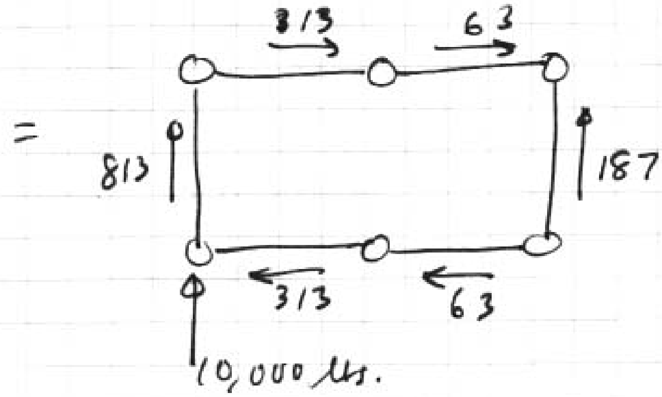
$$+\uparrow \quad -q_3^T(10") + q_6^T(10") = 0 \checkmark \text{ checks}$$

Step ③ Sum results for "Pure Shear" and "Pure Torsion"

"Pure Shear" + "Pure Torsion



Note: all shear flows in [lbs/in]



Or:

$$q_1 = q_1^S + q_1^T = 121 \text{ lbs/in} + 192 \text{ lbs/in} = 313 \text{ lbs/in}$$

$$q_2 = q_2^S + q_2^T = -129 \text{ lbs/in} + 192 \text{ lbs/in} = 63 \text{ lbs/in}$$

$$q_3 = q_3^S + q_3^T = -379 \text{ lbs/in} + 192 \text{ lbs/in} = -187 \text{ lbs/in}$$

$$q_4 = q_4^S + q_4^T = -129 \text{ lbs/in} + 192 \text{ lbs/in} = 63 \text{ lbs/in}$$

$$q_5 = q_5^S + q_5^T = 121 \text{ lbs/in} + 192 \text{ lbs/in} = 313 \text{ lbs/in}$$

$$q_6 = q_6^S + q_6^T = 621 \text{ lbs/in} + 192 \text{ lbs/in} = 813 \text{ lbs/in}$$

Make a final check on these via:

- $\Sigma F_{horizontal} = 0$

$$\xrightarrow{+} \quad q_1(10'') + q_2(10'') - q_4(10'') - q_5(10'') \stackrel{?}{=} 0$$

$$313 \text{ lbs/in} + 63 \text{ lbs/in} - 63 \text{ lbs/in} - 313 \text{ lbs/in} = 0 \checkmark$$

checks

- $\Sigma F_{vertical} = 10{,}000 \text{ lbs} \quad (S_z)$

$$+\uparrow \quad -q_3(10'') + q_6(10'') = S_z$$

$$187 \text{ lbs/in}(10'') + 813 \text{ lbs/in}(10'') \stackrel{?}{=} 10{,}000 \text{ lbs.}$$

$$1870 \text{ lbs} + 8130 \text{ lbs.} = 10{,}000 \text{ lbs.} \checkmark$$

checks

- $\Sigma T_{about ⑥} = T_{applied} = 0 \quad \curvearrowright +$

$$q_1(10'')(10'') + q_2(10'')(10'') + q_3(10'')(20'') + q_4(10'')(0'') + q_5(10'')(0'') + q_6(10'')(0'') \stackrel{?}{=} 0$$

$\Rightarrow (313 \text{ lbs/in})(100 \text{ in}^2) + (63 \text{ lbs/in})(100 \text{ in}^2) + (-187 \text{ lbs/in})(200 \text{ in}^2) \stackrel{?}{=} 0$

$31300 \text{ in}\cdot\text{lbs} + 6300 \text{ in}\cdot\text{lbs} - 37400 \text{ in}\cdot\text{lbs} = 0$

✓ checks
(within roundoff error)

Find shear stresses via $\tau_{xs} = \frac{q}{t}$

$(\tau_{xs})_1 = \frac{313 \text{ lbs/in}}{0.02''} = 15,650 \text{ psi}$

$(\tau_{xs})_2 = \frac{63 \text{ lbs/in}}{0.02''} = 4,680 \text{ psi}$

$(\tau_{xs})_3 = \frac{-187 \text{ lbs/in}}{0.04''} = 4,675 \text{ psi}$

$(\tau_{xs})_4 = \frac{63 \text{ lbs/in}}{0.02''} = 3,150 \text{ psi}$

$(\tau_{xs})_5 = \frac{313 \text{ lbs/in}}{0.02''} = 15,650 \text{ psi}$

$(\tau_{xs})_6 = \frac{813 \text{ lbs/in}}{0.06''} = 13,550 \text{ psi}$

SECTION PROPERTIES

(to find I_{yy}, I_{zz}, I_{yz})

$$A_{yy} = \frac{1}{\int \frac{(\bar{q}_y)^2}{t} ds} \qquad A_{zz} = \frac{1}{\int \frac{(\bar{q}_z)^2}{t} ds} \qquad A_{yz} = \frac{1}{\int \frac{\bar{q}_y \bar{q}_z}{t} ds}$$

$\bar{q}_y$: shear flow due to unit S_y (= 1 lb)

$\bar{q}_z$: shear flow due to unit S_z (= 1 lb)

$$\bar{q}_{z_i} = \frac{q_i^s}{S_z} = \frac{q_i^s}{-10{,}000 \text{ lbs}}$$

$$\Rightarrow \bar{q}_{z_1} = \frac{121 \text{ lbs/in}}{-10{,}000 \text{ lbs}} = -0.0121 \text{ 1/in}$$

$$\bar{q}_{z_2} = \frac{-129 \text{ lbs/in}}{-10{,}000 \text{ lbs}} = 0.0129 \text{ 1/in}$$

$$\bar{q}_{z_3} = \frac{-379 \text{ lbs/in}}{-10{,}000 \text{ lbs}} = 0.0379 \text{ 1/in}$$

$$\bar{q}_{z_4} = \frac{-129 \text{ lbs/in}}{-10{,}000 \text{ lbs}} = 0.0129 \text{ 1/in}$$

$$\bar{q}_{z_5} = \frac{121 \text{ lbs/in}}{-10{,}000 \text{ lbs}} = -0.0121 \text{ 1/in}$$

$$\bar{q}_{z_6} = \frac{621 \text{ lbs/in}}{-10{,}000 \text{ lbs}} = -0.0621 \text{ 1/in}$$

$$A_{zz} = 1 \Big/ \left\{ \frac{(-0.0121 \text{ 1/in})^2}{0.02''}(10'') + \frac{(0.0129 \text{ 1/in})^2}{0.02''}(10'') + \frac{(0.0379 \text{ 1/in})^2}{0.04''}(10'') + \frac{(0.0129 \text{ 1/in})^2}{0.02''}(10'') + \frac{(-0.0121 \text{ 1/in})^2}{0.02''}(10'') + \frac{(-0.0621 \text{ 1/in})^2}{0.06''}(10'') \right\}$$

$\Rightarrow A_{zz} = 0.76 \text{ in}^2$

To find A_{yy}, A_{yz}, would need to apply S_y and do a solution

Torsional Stiffness :
$$J = \frac{2A}{\oint \frac{\bar{q}}{t} ds}$$

where $\bar{q}$ = shear flow due to T of unit (1 in · lb)

here: $\bar{q} = \frac{q_i^T}{T'} = \frac{q_i^T}{76{,}600 \text{ in} \cdot \text{lbs.}}$

$$\Rightarrow \bar{q}_1 = \frac{192 \text{ lbs/in}}{76{,}600 \text{ in} \cdot \text{lbs.}} = 0.00251 \text{ 1/in}^2$$
$$\bar{q}_2 = \frac{192 \text{ lbs/in}}{76{,}600 \text{ in} \cdot \text{lbs}} = 0.00251 \text{ 1/in}^2$$
$$\bar{q}_3 = \frac{192 \text{ lbs/in}}{76{,}600 \text{ in} \cdot \text{lbs}} = 0.00251 \text{ 1/in}^2$$
$$\bar{q}_4 = \frac{192 \text{ lbs/in}}{76{,}600 \text{ in} \cdot \text{lbs.}} = 0.00251 \text{ 1/in}^2$$
$$\bar{q}_5 = \frac{192 \text{ lbs/in}}{76{,}600 \text{ in} \cdot \text{lbs.}} = 0.00251 \text{ 1/in}^2$$
$$\bar{q}_6 = \frac{192 \text{ lbs/in}}{76{,}600 \text{ in} \cdot \text{lbs.}} = 0.00251 \text{ 1/in}^2$$

A = Area enclosed = 10" × 20" = 200 in²

$$\Rightarrow J = \frac{2(200 \text{ in}^2)}{0.00251 \text{ 1/in}^2 (10") \left\{ \frac{1}{0.02"} + \frac{1}{0.02"} + \frac{1}{0.04"} + \frac{1}{0.02"} + \frac{1}{0.02"} + \frac{1}{0.06"} \right\}}$$

$\Rightarrow J = 66.0 \text{ in}^4$